Semi-plenary at CDC08: Distributed Control using Decompositions and Games

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Distributed Control using Decompositions and Games

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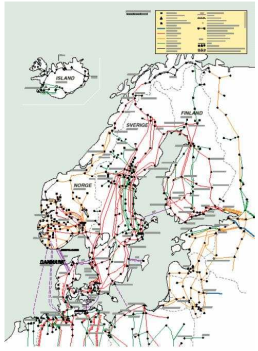
Needs for distributed control theory

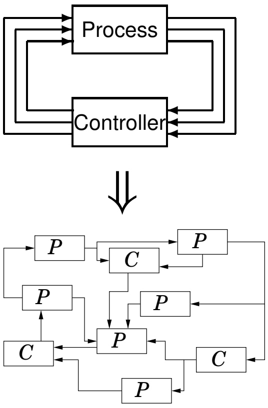
Three major challenges:

- Rapidly increasing complexity
- Dynamic interaction
- Information is decentralized



Building theoretical foundations for distributed control

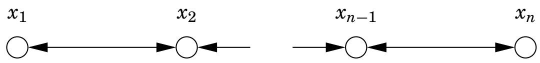
A centralized paradigm dominates theory and curriculum today



We need methodology for

- Decentralized specifications
- Decentralized design
- Validation of global behavior

A "Wind Farm" Case Study



$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ \vdots \\ x_n(t+1) \end{bmatrix} = \begin{bmatrix} 0.6 & 0.1 & & 0 \\ 0.3 & \ddots & \ddots & \\ & \ddots & \ddots & 0.1 \\ 0 & & 0.3 & 0.6 \end{bmatrix} \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ \vdots \\ x_n(t+1) \end{bmatrix} + \begin{bmatrix} u_1(t)+w_1(t) \\ u_2(t)+w_2(t) \\ \vdots \\ u_n(t)+w_n(t) \end{bmatrix}$$

Minimize $V = \mathbf{E}\sum_{i=1}^n \left(|x_i|^2 + |u_i|^2\right)$

Approximating the Centralized Controller

Bellman's equation $|x|_P^2 = \min_u \left(|Ax+Bu|_P^2 + |x|^2 + |u|^2\right)$ gives $u = -Lx$ where

$$L = \begin{bmatrix} 0.3420 & 0.0737 & 0.0046 & 0.0002 \\ 0.1839 & 0.3448 & 0.0736 & 0.0047 \\ 0.0103 & 0.1840 & 0.3447 & 0.0726 \\ 0.0008 & 0.0104 & 0.1808 & 0.3296 \end{bmatrix}$$

Diagonal dominance of L suggests natural approximations:

$$\bar{L}_0 = \begin{bmatrix} 0.34 & 0 & 0 & 0 \\ 0 & 0.34 & 0 & 0 \\ 0 & 0 & 0.34 & 0 \\ 0 & 0 & 0 & 0.33 \end{bmatrix} \qquad \bar{L}_1 = \begin{bmatrix} 0.34 & 0.07 & 0 & 0 \\ 0.18 & 0.34 & 0.07 & 0 \\ 0 & 0.18 & 0.34 & 0.07 \\ 0 & 0 & 0.18 & 0.33 \end{bmatrix}$$

Today's challenges: Distributed controller validation
Distributed control synthesis

Inspiration from other fields

- Congestion control in networks
- Collective motion in biology
- Oscillator synchronization in physics
- Parallelization in optimization theory
- Saddle points and equilibria in economics
- Cooperative and non-cooperative game theory

Much focus on convergence to equilibria, less on dynamic performance.

Outline

- Introduction
- **Game theory and dual decomposition**
- Dynamic dual decomposition
- Distributed validation for wind farm example
- Distributed synthesis

50 years old idea: Dual decomposition

$$\min_{x,y,z,w}[V_1(x,y) + V_2(x,z) + V_3(x,w)]$$
$$= \max_{p,q} \min_{x_1,x_2,x_3,y,z,w}[V_1(x_1,y) + V_2(x_2,z) + V_3(x_3,w) + p(x_1-x_2) + q(x_2-x_3)]$$

The optimum is a Nash equilibrium of the following game:

The three computers try to minimize their respective costs

Computer 1:	$\min_{x_1,y}[V_1(x_1,y) + px_1]$
Computer 2:	$\min_{x_2,z}[V_2(x_2,z) - px_2 + qx_2]$
Computer 3:	$\min_{x_3,w}[V_3(x_3,w) - qx_3]$

while the "market makers" try to maximize their payoffs

Between computer 1 and 2:	$\max_p[p(x_1-x_2)]$
Between computer 2 and 3:	$\max_q[q(x_2-x_3)]$

Potential game

The three computers try to minimize the *potential function*

$$V_1(x_1, y) + V_2(x_2, z) + V_3(x_3, w) + p(x_1 - x_2) + q(x_2 - x_3)$$

while the market makers try to maximize it.

Finding a *Nash equilibrium* (where no player has a incentive to change strategy) is greatly simplified by existence of a potential function.

The saddle point algorithm

Update in gradient direction:

Computer 1:	$\begin{cases} \dot{x}_1 = -\partial V_1/\partial x - p \\ \dot{y} = -\partial V_1/\partial y \end{cases}$
Computer 1 and 2:	$\dot{p} = x_1 - x_2$
Computer 2:	$\begin{cases} \dot{x}_2 = -\partial V_2/\partial x + p - q \\ \dot{z} = -\partial V_2/\partial z \end{cases}$
Computer 2 and 3:	$\dot{q} = x_2 - x_3$
Computer 3:	$\begin{cases} \dot{x}_3 = -\partial V_3/\partial x + q \\ \dot{w} = -\partial V_3/\partial w \end{cases}$

Globally convergent if V_i convex! [Arrow, Hurwicz, Usawa 1958]

Lyapunov function: $\mathbf{V} = \dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 + \dot{y}^2 + \dot{z}^2 + \dot{w}^2 + \dot{p}^2 + \dot{q}^2$

Decentralized Bounds on Suboptimality

Given any $p, q, \bar{x}, \bar{y}, \bar{z}, \bar{w}$, the distributed test

$$V_1(\bar{x}, \bar{y}) + p\bar{x} \le \alpha \min_{x_1, y} [V_1(x_1, y) + px_1]$$
$$V_2(\bar{x}, \bar{z}) - p\bar{x} + q\bar{x} \le \alpha \min_{x_2, z} [V_2(x_2, z) - px_2 + qx_2]$$
$$V_3(\bar{x}, \bar{w}) - q\bar{x} \le \alpha \min_{x_3, w} [V_3(x_3, w) - qx_3]$$

implies that the globally optimal cost J^* is bounded as

$$V_1(\bar{x}, \bar{y}) + V_2(\bar{x}, \bar{z}) + V_3(\bar{x}, \bar{w}) \le \alpha \min_{x,y,z,w} \left[V_1(x, y) + V_2(x, z) + V_3(x, w)\right]$$

Proof: Add both sides up!

What do we achieve?

- Performance criteria for individual nodes
- Suboptimality bounds indicate *where* things went wrong
- Prices show the relative importance of different terms
- Sparsity structure useful for efficient computations

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A General Optimal Control Problem

Minimize $V(u) = \mathbf{E} \sum_i \ell_i(x_i(t), u_i(t))$

subject to

$$\begin{cases} x_1(t+1) = f_1(x_1, v_{1j}, u_1, w_1) \\ \vdots \\ x_J(t+1) = f_J(x_J, v_{Jj}, u_J, w_J) \end{cases}$$

where

$$v_{ij} = x_j$$

holds for all i, j.

Decomposing the Cost Function

$$\max_p \min_{u,v} \sum_i \mathbf{E}\left[\ell_i(x_i(t), u_i(t)) + 2\textstyle\sum_j (p_{ij})^T (x_j - v_{ij})\right]$$
$$= \max_p \sum_i \min_{u_i, v_{ij}} \mathbf{E}\left[\ell_i(x_i(t), u_i(t)) - 2\textstyle\sum_j (p_{ij})^T v_{ij} + 2(\textstyle\sum_j p_{ji})^T x_i\right]$$

so agent i should minimize the stationary value of

$$\mathbf{E}\Big(\underbrace{\ell_i(x_i(t), u_i(t))}_{\text{his own cost}} \overbrace{-2\textstyle\sum_j [p_{ij}(t)]^T v_{ij}(t)}^{\text{what he expects others to pay him}} \underbrace{+2[\textstyle\sum_j p_{ji}(t)]^T x_i(t)}_{\text{what he pays others}} \Big)$$

Distributed Verification

$$\max_p \sum_i \min_{u_i, v_{ij}} \mathbf{E} \underbrace{\left[\ell_i(x_i(t), u_i(t)) - 2\textstyle\sum_j (p_{ij})^T v_{ij} + 2(\textstyle\sum_j p_{ji})^T x_i\right]}_{J_i(x_i, u_i, v_{\{i\}}, p)}$$

Each agent i makes the comparison

$$\underbrace{\mathbf{E} J_i(\bar{x}_i, \bar{u}_i, \bar{x}_j, \bar{p})}_{\text{Actual cost in node } i} \le \alpha \underbrace{\min_{x_i, u_i, v_{ij}} \mathbf{E} J_i(x_i, u_i, v_{ij}, \bar{p})}_{\text{Optimal cost in node } i}$$

where minimization is subject to the local dynamics

$$x_i(t+1) = f_i(x_i, v_{ij}, u_i, w_i)$$

If no actual cost exceeds the expected cost by more than 10%, then the global cost is within 10% from optimal.

Theorem on Verification

Consider control laws $\bar{u}_i = \mu_i(\bar{x})$ and stationary solutions to

$$\bar{x}_i(t+1) = f_i(\bar{x}_i, \bar{x}_j, \mu_i(\bar{x}), w_i)$$

where $w_i(t)$ is stationary white noise. If $\alpha \geq 0$, then (I) implies (II) :

(I) There exists $\bar{p} = \lambda(\bar{x})$ satisfying

$$\mathbf{E}J_i(\bar{x}_i, \bar{u}_i, \bar{x}_j, \bar{p}) \leq \alpha \min_{x_i, u_i, v_{ij}} \mathbf{E}J_i(x_i, u_i, v_{ij}, \bar{p})$$

when minimizing over stationary solutions to

$$x_i(t+1) = f_i(x_i, v_{ij}, u_i, w_i)$$

(II) $\sum_i \mathbf{E}\ell_i(\bar{x}_i, \bar{u}_i) \leq \alpha \min_u \sum_i \mathbf{E}\ell_i(x_i, u_i)$ when minimizing over stationary solutions to

$$\begin{cases} x_1(t+1) = f_1(x_1, x_j, u_1, w_1) \\ \quad\vdots \\ x_J(t+1) = f_J(x_J, x_j, u_J, w_J) \end{cases}$$

If dynamics is linear, $\ell_i \geq 0$ convex and $\alpha = 1$, then (II) implies (I).

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- **Distributed validation for wind farm example**
- Distributed synthesis

A "Wind Farm" Case Study

Minimize $V = \mathbf{E}\sum_{i=1}^4 \left(|x_i|^2 + |u_i|^2\right)$

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \end{bmatrix} = \begin{bmatrix} 0.6 & 0.1 & 0 & 0 \\ 0.3 & 0.6 & 0.1 & 0 \\ 0 & 0.3 & 0.6 & 0.1 \\ 0 & 0 & 0.3 & 0.6 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} u_1(t) + w_1(t) \\ u_2(t) + w_2(t) \\ u_3(t) + w_3(t) \\ u_4(t) + w_4(t) \end{bmatrix}$$

Today's challenges: Distributed controller validation
Distributed control synthesis

$$\bar{L}_0 = \begin{bmatrix} 0.34 & 0 & 0 & 0 \\ 0 & 0.34 & 0 & 0 \\ 0 & 0 & 0.34 & 0 \\ 0 & 0 & 0 & 0.33 \end{bmatrix} \qquad \bar{L}_1 = \begin{bmatrix} 0.34 & 0.07 & 0 & 0 \\ 0.18 & 0.34 & 0.07 & 0 \\ 0 & 0.18 & 0.34 & 0.07 \\ 0 & 0 & 0.18 & 0.34 \end{bmatrix}$$

Validation Using Centralized Model

The variance $\mathbf{E}\sum_{i=1}^4 \left(|x_i|^2 + |u_i|^2\right)$ for the optimal centralized controller becomes

$$V_* = 4.9904$$

while the values for the decentralized approximations become

$$V_0 = 5.2999 \qquad\qquad V_1 = 4.9917$$

These numbers were calculated using a global model.

We will next use dual decomposition to see that the control laws can be both validated and synthesized in a distributed way.

Decomposing the turbine dynamics

Minimize $\mathbf{E}\sum_{i=1}^4 \left(|x_i|^2 + |u_i|^2\right)$

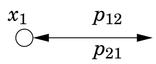
subject to

$$\begin{bmatrix} x_1^+ \\ x_2^+ \\ x_3^+ \\ x_4^+ \end{bmatrix} = \underbrace{\begin{bmatrix} 0.6 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.6 \end{bmatrix}}_{\bar{A}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 \end{bmatrix}}_{\widetilde{A}} \begin{bmatrix} v_{12} \\ v_{21} \\ v_{23} \\ v_{32} \\ v_{34} \\ v_{43} \end{bmatrix} + \begin{bmatrix} u_1 + w_1 \\ u_2 + w_2 \\ u_3 + w_3 \\ u_4 + w_4 \end{bmatrix}$$

when

$$\begin{bmatrix} v_{12} \\ v_{21} \\ v_{23} \\ v_{32} \\ v_{34} \\ v_{43} \end{bmatrix} = \underbrace{\left[\begin{array}{cccc} 0 & I & 0 & 0 \\ \hline I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ \hline 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \\ \hline 0 & 0 & I & 0 \end{array}\right]}_{S} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Problem solved by the first turbine

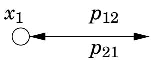


Minimize $\mathbf{E}(|x_1|^2 + |u_1|^2 + 2p_{12}v_{12} - 2p_{21}x_1)$
when $x_1^+ = 0.6x_1 + 0.1v_{12} + u_1 + w_1$

using measurements of x and knowledge of the joint spectral density of x, w, p_{13} and p_{21}.

Notice: Once the price sequences $p_{12}(t)$, $p_{21}(t)$ are given, no other knowledge of the outside world is relevant. However, since future prices are usually not available, knowledge of other states can be useful for price prediction.

Problem solved by the first turbine



Minimize $\mathbf{E}(|x_1|^2 + |u_1|^2 + 2p_{12}v_{12} - 2p_{21}x_1)$
when $x_1^+ = 0.6x_1 + 0.1v_{12} + u_1 + w_1$

Test for suboptimality:

$$\begin{aligned} &\mathbf{E}(|x_1|^2 + |u_1|^2 + 2p_{12}x_2 - 2p_{21}x_1) \\ &\leq \alpha \min_{u_1, v_{12}} \mathbf{E}(|x_1|^2 + |u_1|^2 + 2p_{12}v_{12} - 2p_{21}x_1) \end{aligned}$$

Performance degradation due to decentralization

$$\bar{L}_0 = \begin{bmatrix} 0.34 & 0 & 0 & 0 \\ 0 & 0.34 & 0 & 0 \\ 0 & 0 & 0.34 & 0 \\ 0 & 0 & 0 & 0.34 \end{bmatrix} \quad \bar{L}_1 = \begin{bmatrix} 0.34 & 0.07 & 0 & 0 \\ 0.18 & 0.34 & 0.13 & 0 \\ 0 & 0.13 & 0.34 & 0.18 \\ 0 & 0 & 0.07 & 0.34 \end{bmatrix} \quad \bar{M} = \begin{bmatrix} 0.03 & 0.26 & 0 & 0 \\ 0.55 & 0.10 & 0 & 0 \\ 0 & 0.07 & 0.28 & 0 \\ 0 & 0.28 & 0.07 & 0 \\ 0 & 0 & 0.10 & 0.55 \\ 0 & 0 & 0.26 & 0.03 \end{bmatrix}$$

Compare expected and actual costs for the two control laws:

$u = -\bar{L}_0 x$ and $\bar{p} = \bar{M}x$:

$$\begin{aligned} 1.5647 &\leq 1.5350\,\alpha \\ 1.0853 &\leq 0.8558\,\alpha \\ 1.0853 &\leq 0.8558\,\alpha \\ 1.5647 &\leq 1.5350\,\alpha \end{aligned}$$

$$1.062 = \frac{V}{V_*} \leq \alpha = 1.27$$

$u = -\bar{L}_1 x$ and $\bar{p} = \bar{M}x$:

$$\begin{aligned} 1.5741 &\leq 1.5740\,\alpha \\ 0.9132 &\leq 0.9217\,\alpha \\ 0.9132 &\leq 0.9217\,\alpha \\ 1.5741 &\leq 1.5740\,\alpha \end{aligned}$$

$$1.0003 = \frac{V}{V_*} \leq \alpha = 1.0094$$

Outline

- Introduction
- Game theory and dual decomposition
- Dynamic dual decomposition
- Distributed validation
- **Distributed synthesis**

"Wind Farm" Revisited

Minimize $V = \mathbf{E}\sum_{i=1}^{n}\left(|x_i|^2 + |u_i|^2\right)$

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ \vdots \\ x_n(t+1) \end{bmatrix} = \begin{bmatrix} 0.6 & 0.1 & & 0 \\ 0.3 & \ddots & \ddots & \\ & \ddots & \ddots & 0.1 \\ 0 & & 0.3 & 0.6 \end{bmatrix} \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ \vdots \\ x_n(t+1) \end{bmatrix} + \begin{bmatrix} u_1(t)+w_1(t) \\ u_2(t)+w_2(t) \\ \vdots \\ u_n(t)+w_n(t) \end{bmatrix}$$

We will optimize a tri-diagonal control structure

$$\bar{L} = \begin{bmatrix} * & * & & 0 \\ * & & \ddots & \\ & \ddots & & * \\ 0 & & * & * \end{bmatrix}$$

Optimal Prices by Dynamic Programming

Optimal control problem:

Minimize $\mathbf{E}(|x|^2 + |u|^2)$
when $x^+ = \bar{A}x + \widetilde{A}v + Bu + w$ and $v = Sx$

Dynamic programming gives control law as well as prices:

$$|x|_P^2 = \max_p \min_{u,v}\left[|\bar{A}x + \widetilde{A}v + Bu|_P^2 + |x|^2 + |u|^2 - 2p^T(v - Sx)\right]$$

$$p(t) = \begin{bmatrix} p_{12}(t) \\ p_{21}(t) \\ p_{23}(t) \\ p_{32}(t) \\ p_{34}(t) \\ p_{43}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0.0342 & 0.2574 & 0.0010 & 0.0002 \\ 0.5545 & 0.1013 & 0.0382 & 0.0038 \\ 0.0364 & 0.0676 & 0.2755 & 0.0025 \\ 0.0025 & 0.2755 & 0.0676 & 0.0364 \\ 0.0038 & 0.0382 & 0.1013 & 0.5545 \\ 0.0002 & 0.0010 & 0.2574 & 0.0342 \end{bmatrix}}_{M} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

The same P and $u(t) = -Lx(t)$ as in classical solution.

Prices by distributed gradient iteration

Diagonal dominance suggests a tri-diagonal structure for M

$$\begin{bmatrix} \bar{p}_{12}(t) \\ \bar{p}_{21}(t) \\ \bar{p}_{23}(t) \\ \bar{p}_{32}(t) \\ \bar{p}_{34}(t) \\ \bar{p}_{43}(t) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & m_{32} & m_{33} & 0 \\ 0 & m_{42} & m_{43} & 0 \\ 0 & 0 & m_{53} & m_{54} \\ 0 & 0 & m_{63} & m_{64} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

After running the system with fixed prices and control laws during a time interval $t = 1, \ldots, T$, the correlation between state measurements and constraint violations can be estimated as

$$\mathbf{E}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(v_{12} - x_2) \quad\approx\quad \frac{1}{T}\sum_{t=1}^{T}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}[v_{12}(t) - x_2(t)]$$

If the correlation is non-zero, the prices $\begin{bmatrix} m_{11} & m_{12} \end{bmatrix}$ are adjusted.

Distributed gradient iteration for control law

By the maximum principle, optimal solutions to

Minimize $\mathbf{E}(|x_1|^2 + |u_1|^2 + 2p_{12}v_{12} - 2p_{21}x_1)$
when $x_1^+ = 0.6x_1 + 0.1v_{12} + u_1 + w_1$

must minimize the Hamiltonian

$$\mathbf{E}\left[|x_1|^2 + |u_1|^2 + 2p_{12}v_{12} - 2p_{21}x_1 - \lambda_1(0.6x_1 + 0.1v_{12} + u_1 + w_1)\right]$$

This allows us to modify the control law

$$u_1 = \begin{bmatrix} l_{11} & l_{12} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

in the gradient direction using correlation estimates from the time interval $t = 1, \ldots, T$.

Gradient iteration for the wind park

```
cost =

    7.4944


L =

    0.0138    0.0195         0         0         0
    0.0162    0.0283    0.0294         0         0
         0    0.0264    0.0333    0.0294         0
         0         0    0.0264    0.0283    0.0195
         0         0         0    0.0162    0.0138
```

Gradient iteration for the wind park

```
cost =

    5.3183


L =

    0.0366    0.0411         0         0         0
    0.0386    0.0623    0.0546         0         0
         0    0.0555    0.0686    0.0544         0
         0         0    0.0554    0.0620    0.0405
         0         0         0    0.0385    0.0363
```

Gradient iteration for the wind park

```
cost =

    4.4277


L =

    0.0709    0.0629         0         0         0
    0.0666    0.1025    0.0749         0         0
         0    0.0853    0.1070    0.0744         0
         0         0    0.0851    0.1016    0.0611
         0         0         0    0.0662    0.0697
```

Gradient iteration for the wind park

```
cost =

    3.9476


L =

    0.1187    0.0812         0         0         0
    0.0987    0.1494    0.0885         0         0
         0    0.1146    0.1509    0.0879         0
         0         0    0.1144    0.1479    0.0777
         0         0         0    0.0976    0.1155
```

Gradient iteration for the wind park

```
cost =

    3.6674


L =

    0.1820    0.0903         0         0         0
    0.1324    0.2041    0.0920         0         0
         0    0.1419    0.2032    0.0917         0
         0         0    0.1416    0.2023    0.0853
         0         0         0    0.1296    0.1743
```

Gradient iteration for the wind park

```
cost =

    3.5166


L =

    0.2654    0.0777         0         0         0
    0.1611    0.2684    0.0755         0         0
         0    0.1607    0.2674    0.0759         0
         0         0    0.1604    0.2672    0.0731
         0         0         0    0.1549    0.2479
```

Gradient iteration for the wind park

```
cost =

    3.4732


L =

    0.2347    0.0393         0         0         0
    0.1152    0.2363    0.0449         0         0
         0    0.1187    0.2393    0.0444         0
         0         0    0.1189    0.2369    0.0410
         0         0         0    0.1103    0.2131
```

Gradient iteration for the wind park

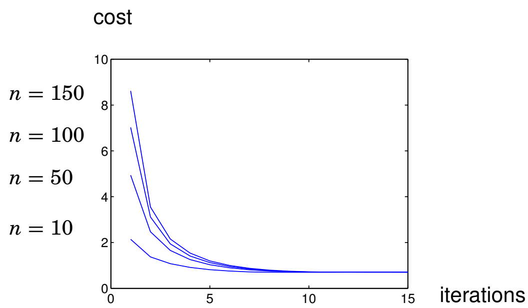
```
cost =

    3.4949


L =

    0.2579    0.0679         0         0         0
    0.1464    0.2673    0.0704         0         0
         0    0.1507    0.2676    0.0702         0
         0         0    0.1504    0.2664    0.0664
         0         0         0    0.1414    0.2389
```

Convergence rate versus state dimension



For a fixed number of iterations and fixed sparsity structure of L, M, the computational cost grows linearly with n!

Conclusions

We have seen dynamic dual decomposition used for

- Distributed validation
- Distributed synthesis

Benefits to be obtained

- Reduced complexity
- Control structure reflects plant structure
- Flexibility and robustness

We have the tools to deal with dynamics!

Welcome to join the efforts!

Much (most) remains to be done and much is happening already at this conference!

See [Rantzer CDC07]

[Rantzer ACC09] covers much of this lecture. Working paper on `www.control.lth.se/user/anders.rantzer`

Lund University funds postdocs and will also hire new faculty members to complement the competence of our current staff.

Acknowledgements

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