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Active Distances and Cascaded Convolutional Codes

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Abstract — A family of active distances for convolutional codes is introduced. Lower bounds are derived for the ensemble of periodically time-varying convolutional codes.

I. INTRODUCTION
The "extended distances" were introduced by Thommesen and Justesen [1] for unit memory (UM) convolutional codes. We present (non-trivial) extensions to encoder memories \( m \) and call them active distances since they stay "active" in the sense that we consider only those codewords which do not pass two consecutive zero states [2].

II. ACTIVE DISTANCES
Consider the ensemble of binary, rate \( R = b/c \), periodically time-varying convolutional codes encoded by a polynomial generator matrix of memory \( m \) and period \( T \),

\[
G(t) = \begin{pmatrix}
G_0(t) & \cdots & G_m(t + m) \\
G_0(t+1) & \cdots & G_m(t + m + 1) \\
\vdots & & \ddots \\
\end{pmatrix}
\]

(1)
in which each digit in each of the matrices \( G_i(t + T) \) for \( 0 \leq i \leq m \) and \( 0 \leq t \leq T - 1 \), is chosen independently and equally likely to be 0 and 1.

Let \( U_{[1-m,t+j+m]} \) be the set of information sequences \( u_{t-1, t} \ldots u_{t+j+m} \) such that the first \( m \) and the last \( m \) subblocks are zero and they do not contain \( m + 1 \) consecutive zero subblocks.

Let \( U_{[t-m,t+j]} \) be the set of information sequences \( u_{t-1, t} \ldots u_{t+j} \) such that the first \( m \) subblocks are zero and they do not contain \( m + 1 \) consecutive zero subblocks.

Let \( U_{[t-1, t+j]} \) be the set of information sequences \( u_{t-1, t} \ldots u_{t+j} \) such that at least one subblock is nonzero and they do not contain \( m + 1 \) consecutive zero subblocks.

Next we introduce the truncated time-varying generator matrix

\[
G_{[t,t+j]} = \begin{pmatrix}
G_m(t) \\
G_0(t) & \cdots & G_m(t + j) \\
\vdots & & \ddots \\
G_0(t+j) & \cdots & G_m(t + j) \\
\end{pmatrix}
\]

III. CASCADED CODES
Consider a scheme with two convolutional codes in cascade.

Theorem 1 There exist cascaded convolutional codes in the ensemble of periodically time-varying cascaded convolutional codes whose active distance satisfies

\[
\delta_{t}^{(l)} = \frac{a_j^{(l)}}{mc} \geq \left(1 + \frac{1}{l+1}R\right) - O\left(\frac{\log_2 m}{m}\right)
\]

for \( l \geq l_0 = O\left(\frac{1}{m}\right) \),

\[
\delta_{t}^{(l)} = \frac{a_j^{(l)}}{mc} \geq \left(1 + \frac{1}{l+1}R\right) - O\left(\frac{\log_2 m}{m}\right)
\]

for \( l \geq l_0 = O\left(\frac{\log_2 m}{m}\right) \), and

\[
\delta_{t}^{(l)} = \frac{a_j^{(l)}}{mc} \geq \left(1 + \frac{1}{l+1}R\right) - O\left(\frac{\log_2 m}{m}\right)
\]

for \( l \geq l_0 = \frac{1}{l+1}R + O\left(\frac{\log_2 m}{m}\right) \).

By minimizing the lower bound on the active row distance we obtain nothing but the main term in Costello's lower bound on the free distance, viz., \( \frac{R}{\log_2(2^{m-1})} \).

REFERENCES

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