Models and Methods in Optical Astrometry

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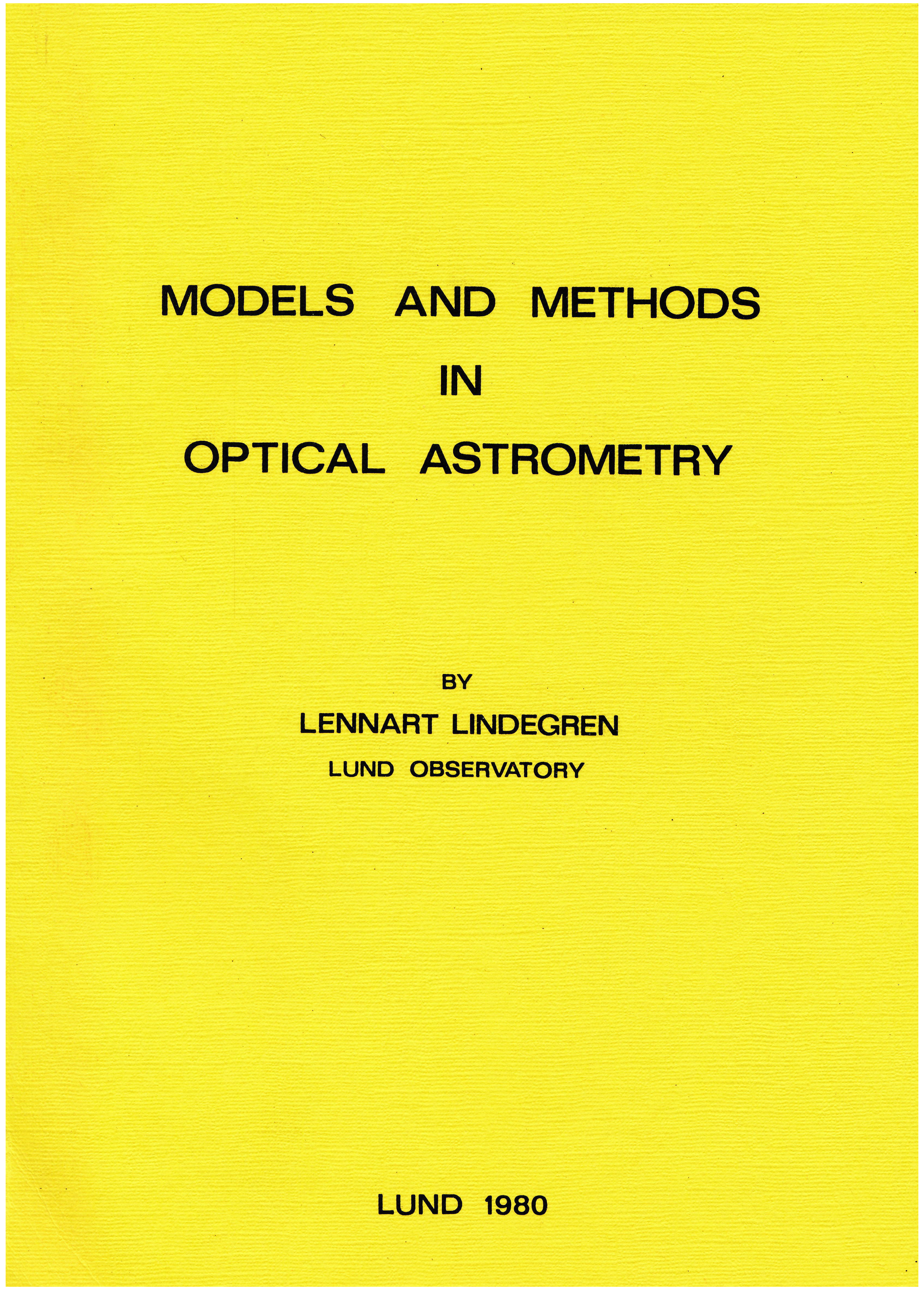
PhD thesis (1980)



MODELS AND METHODS

IN

OPTICAL ASTROMETRY

BY

LENNART LINDEGREN

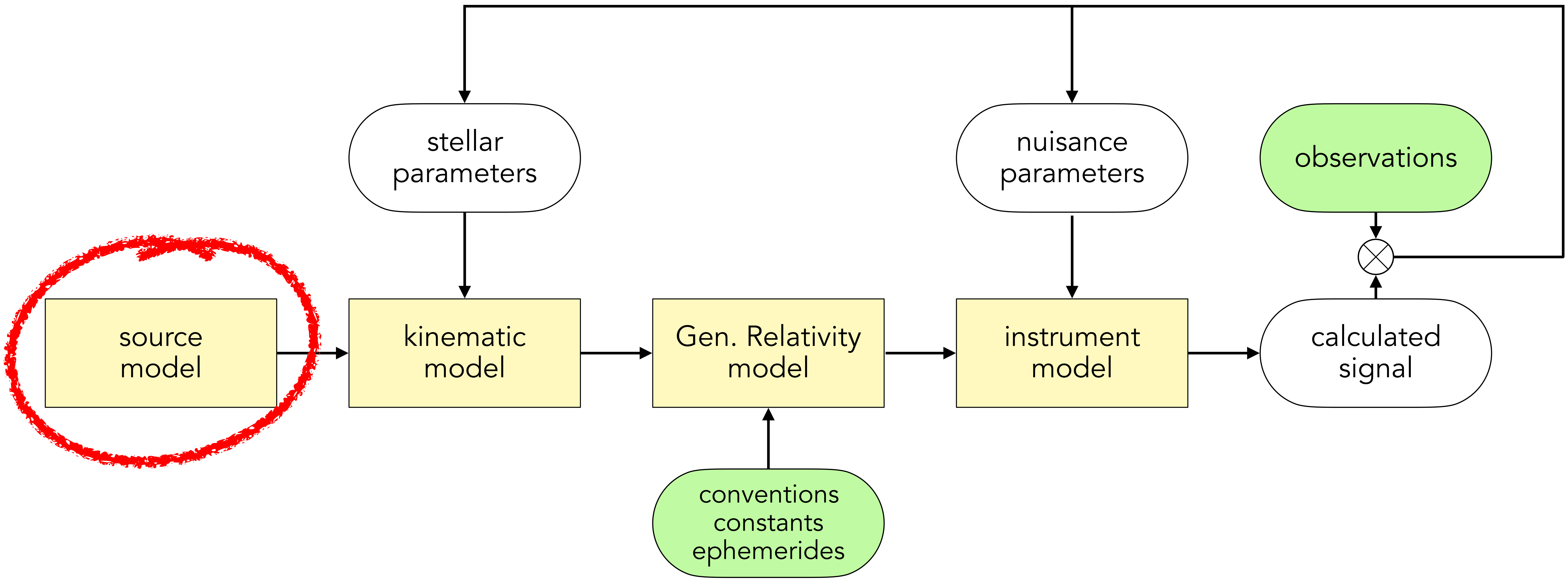
LUND OBSERVATORY

LUND 1980

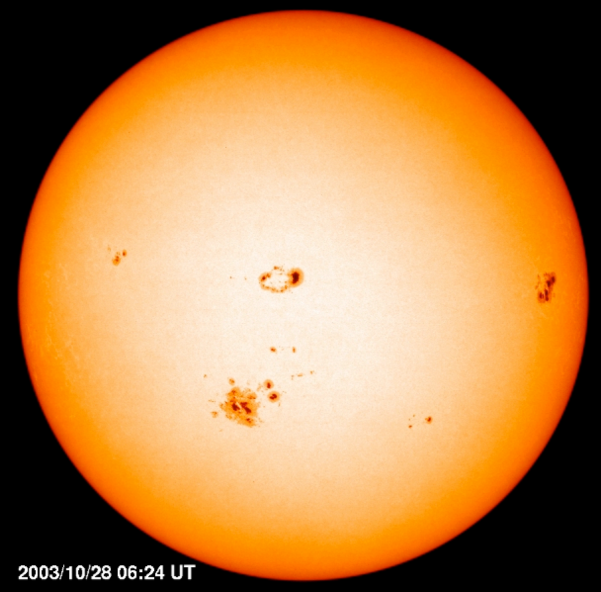
"All models are wrong, but some are useful"

George E. P. Box (1976)

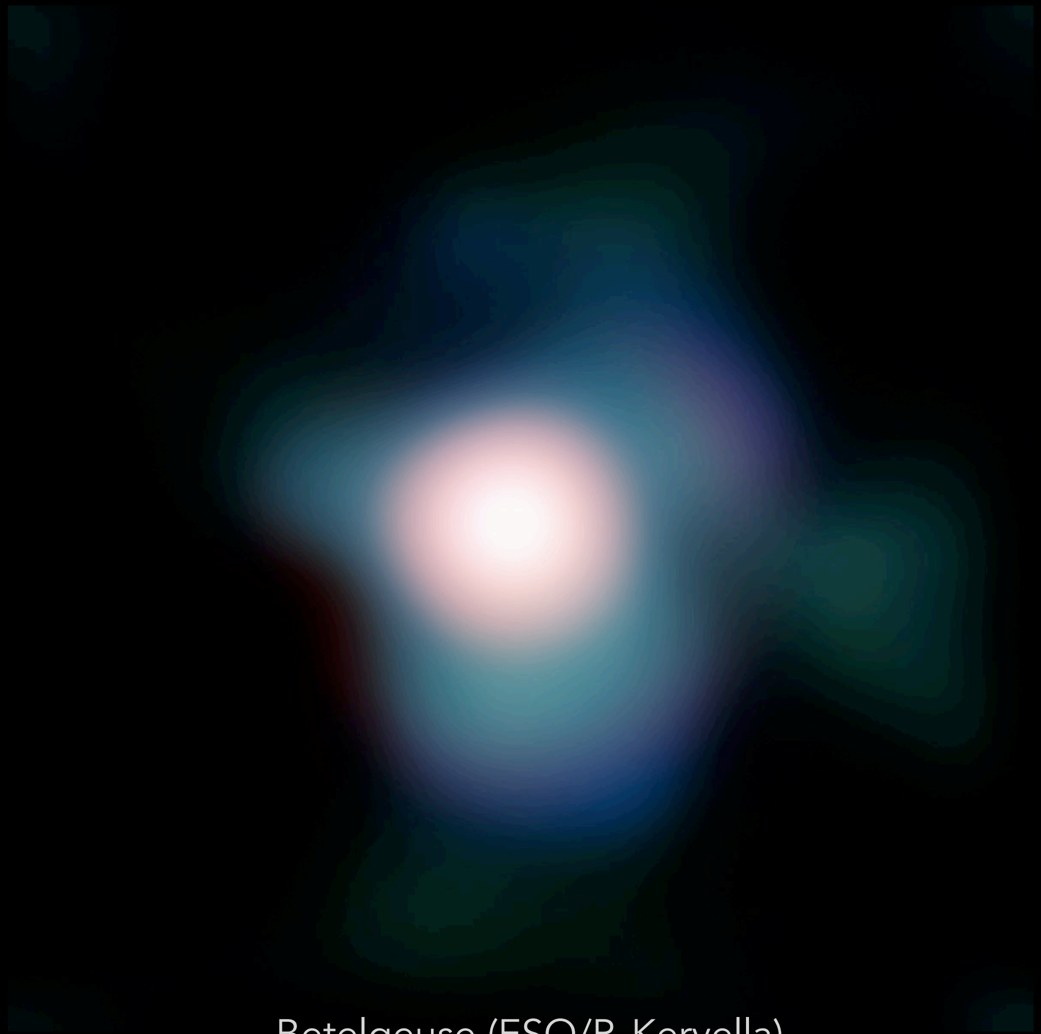
Forward modelling (basic Hipparcos, Gaia)



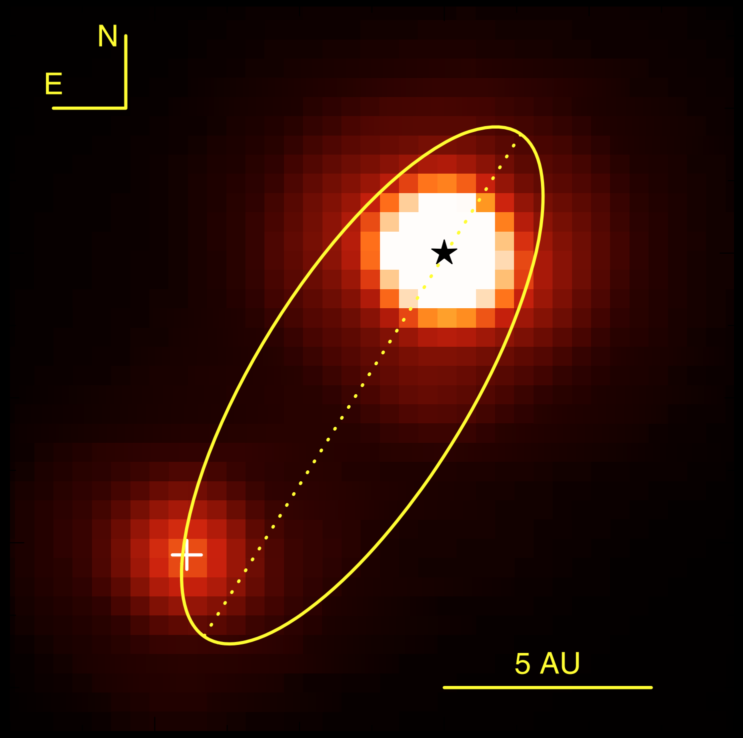
Model: Objects are point sources



The Sun (SOHO, ESA & NASA)



Betelgeuse (ESO/P. Kervella)



Binary β CrB (ESO/Bruntt et al.)

Angular resolution and astrometric precision

Angular resolution: $\boxed{R = \dfrac{\lambda_{\mathrm{eff}}}{D}}$

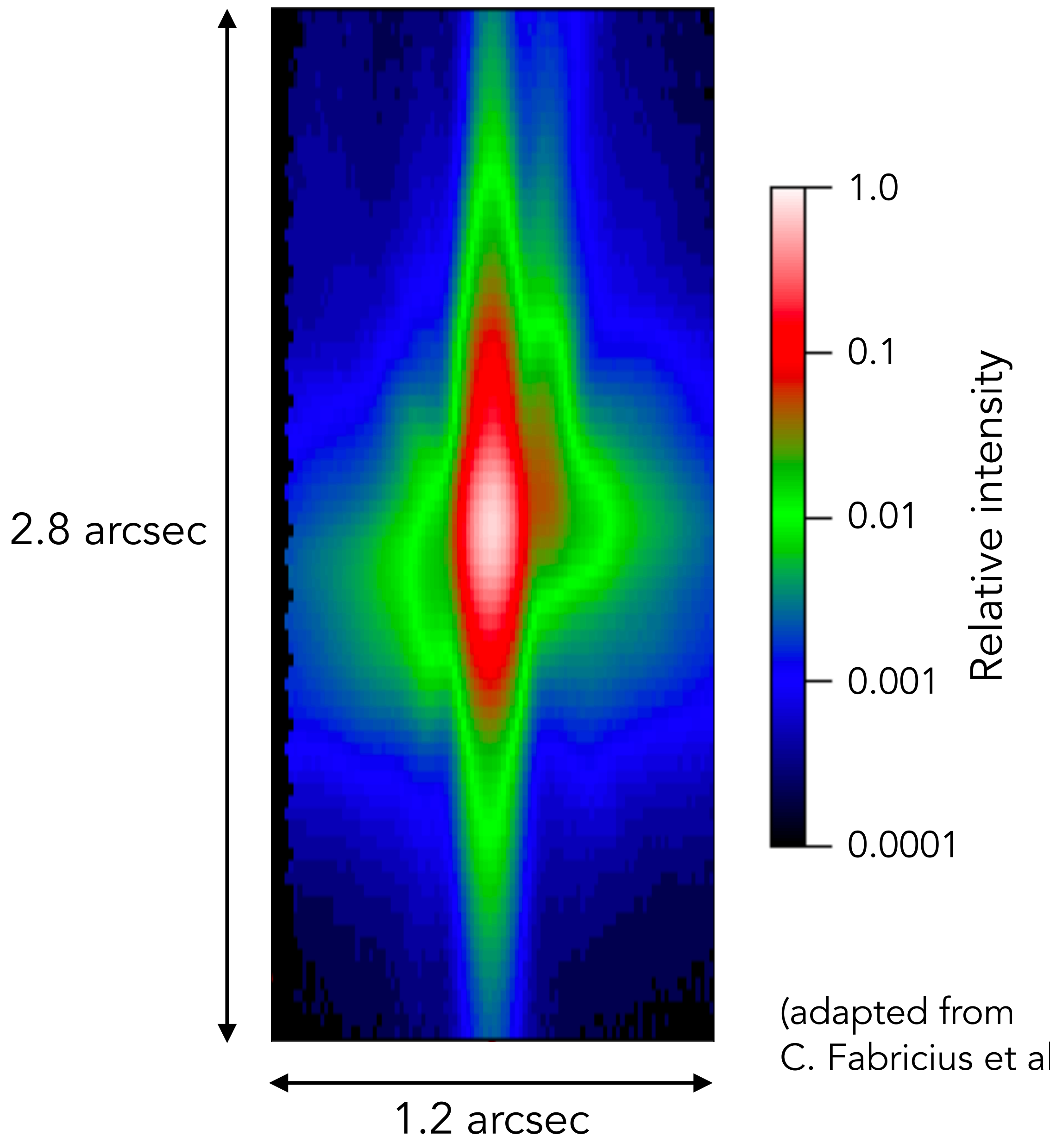
$$\left.\begin{aligned} D &= 1.45\ \mathrm{m} \\ \lambda_{\mathrm{eff}} &\simeq 700\ \mathrm{nm} \end{aligned}\right\} \quad \Rightarrow \quad R \simeq 0.1\ \mathrm{arcsec}$$

0.1 arcsec = 100 au @ 1 kpc

Centroiding precision: $\boxed{\sigma \simeq \dfrac{R}{3(S/N)}}$

$$\left.\begin{aligned} R &= 0.1\ \mathrm{arcsec} \\ S/N &= 500 \end{aligned}\right\} \quad \Rightarrow \quad \sigma \simeq 70\ \mu\mathrm{as}$$

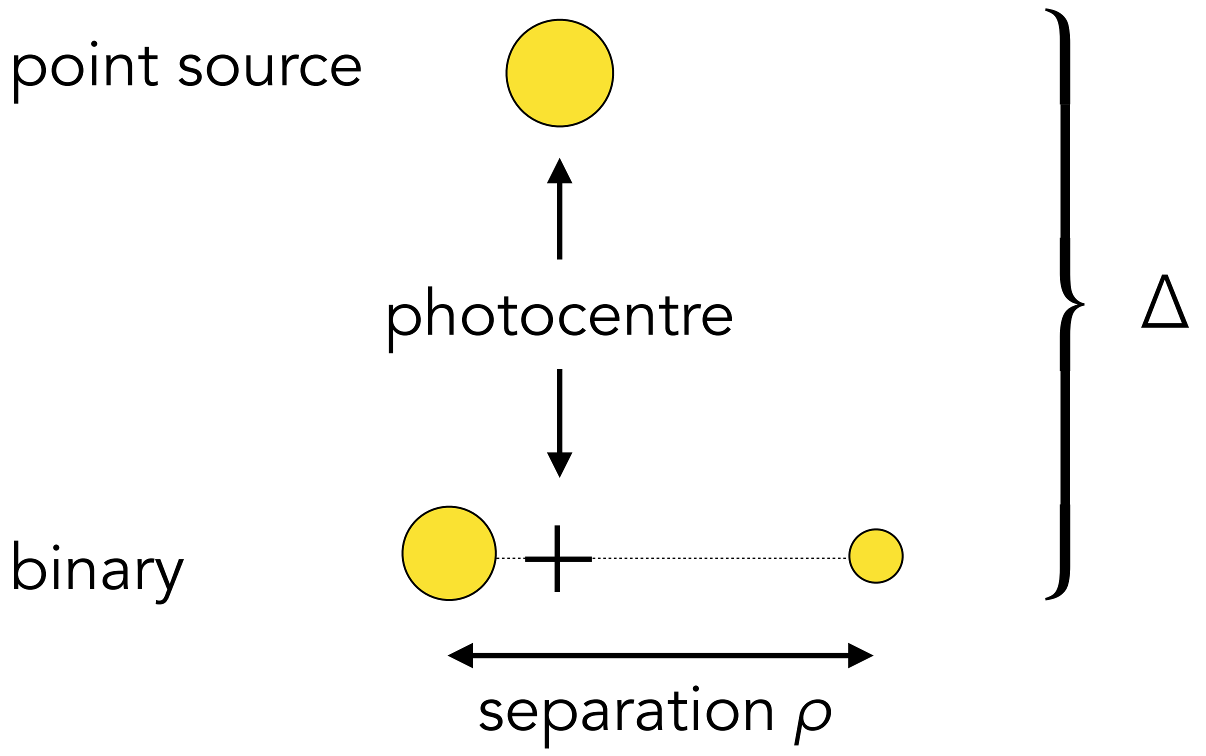
Effective PSF in Gaia (example)

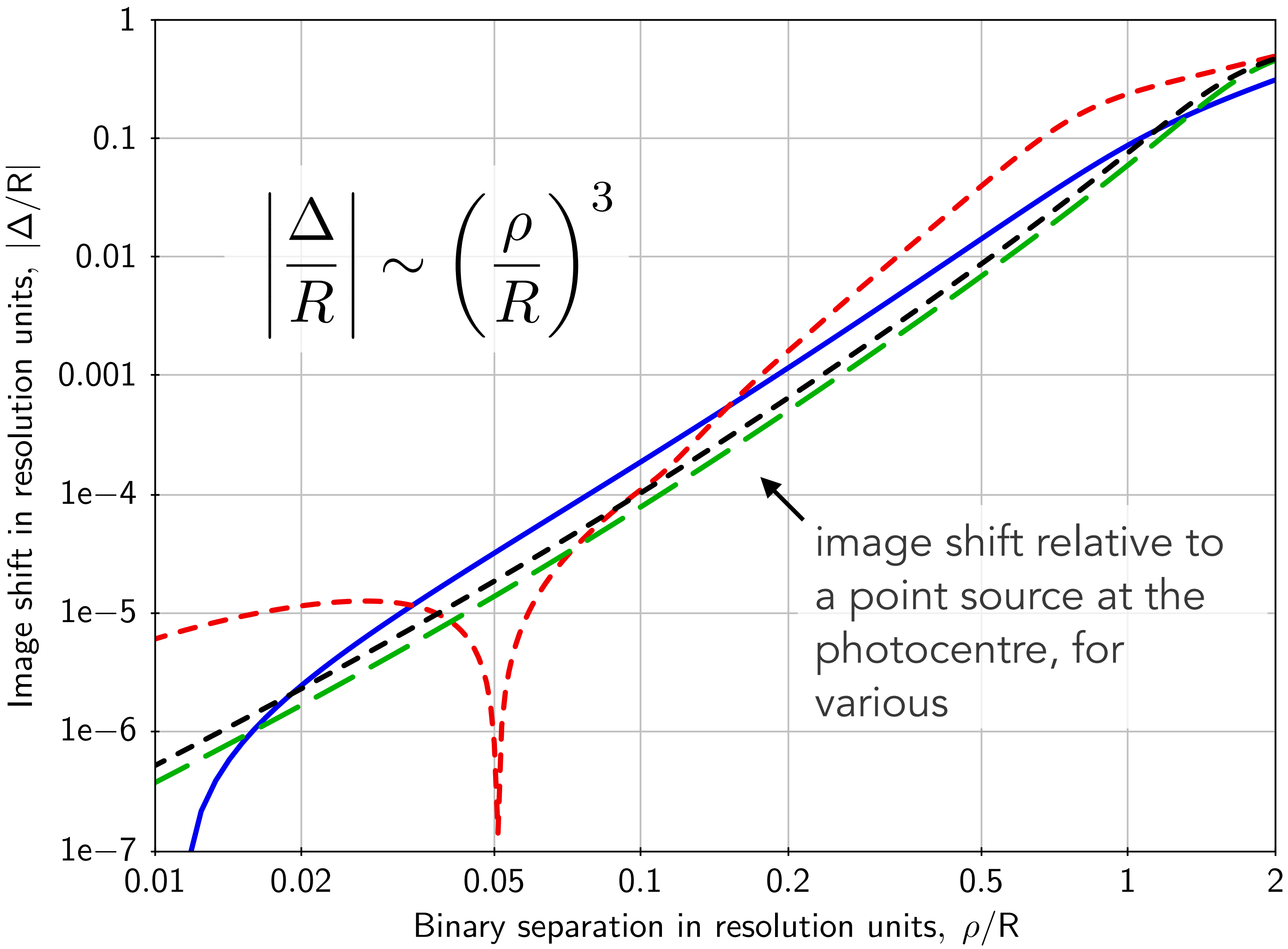


(adapted from
C. Fabricius et al. 2016)

Nearly point-like objects

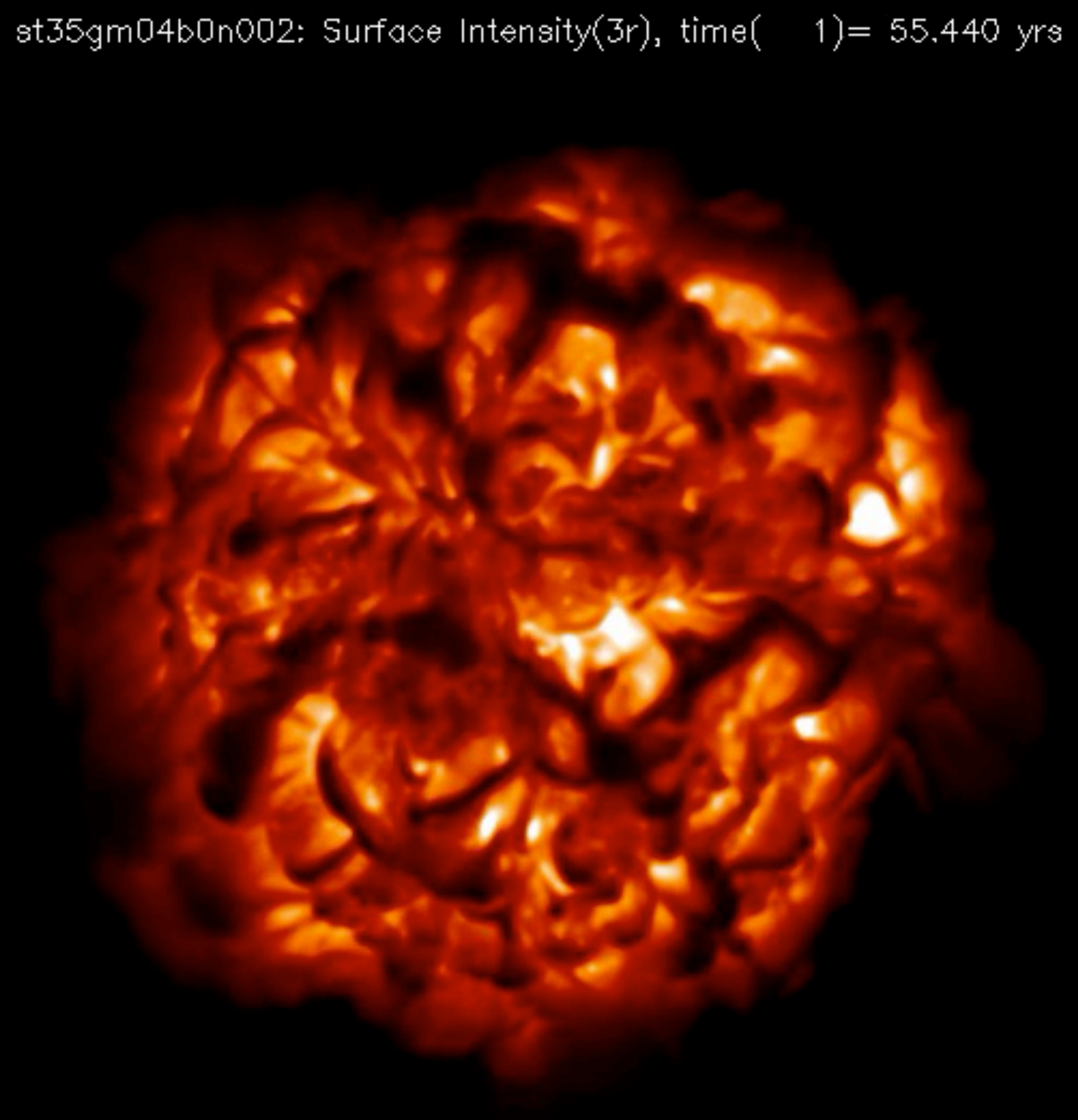
The image of a ***small finite object*** depends only on the PSF and the ***low-order moments*** of the object's flux distribution (total flux, photocentre, ...)





➜ For object size < 0.1–0.2R the structure of the source does not matter, only the photocentre (size < 10–20 au @ 1 kpc)

Supergiants are big stars (> 1 au = 215 $R_\odot$)



~ 5 au

3D radiative hydrodynamic (star-in-a-box) simulation of a 5 $M_\odot$ red supergiant (≈ Betelgeuse) over 12 years

(emergent surface intensity)

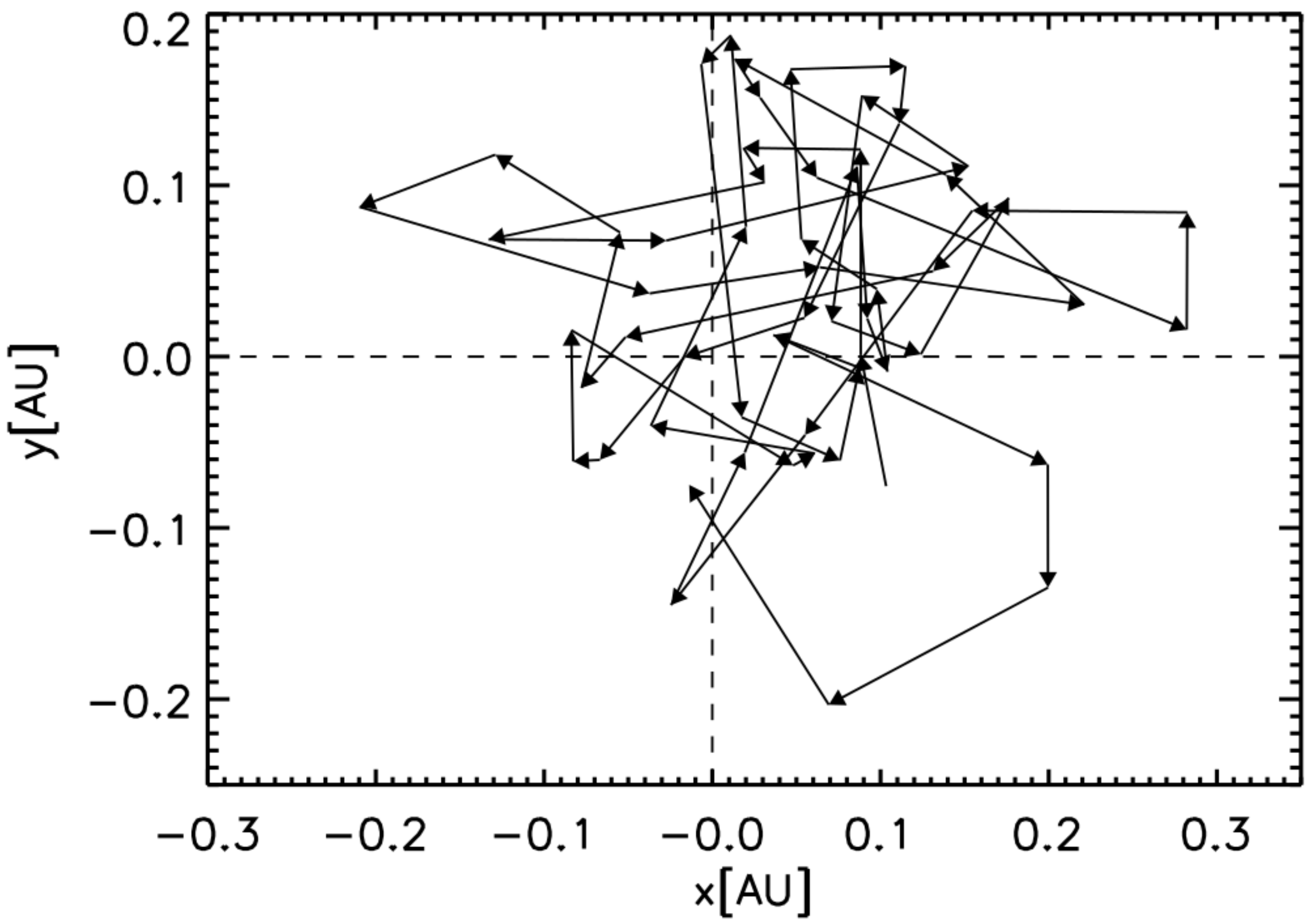
Bernd Freytag at Uppsala University (2020)

www.astro.uu.se/~bf/publications/2015_10_24_Uppsala_Astronomdagarna/Talk/

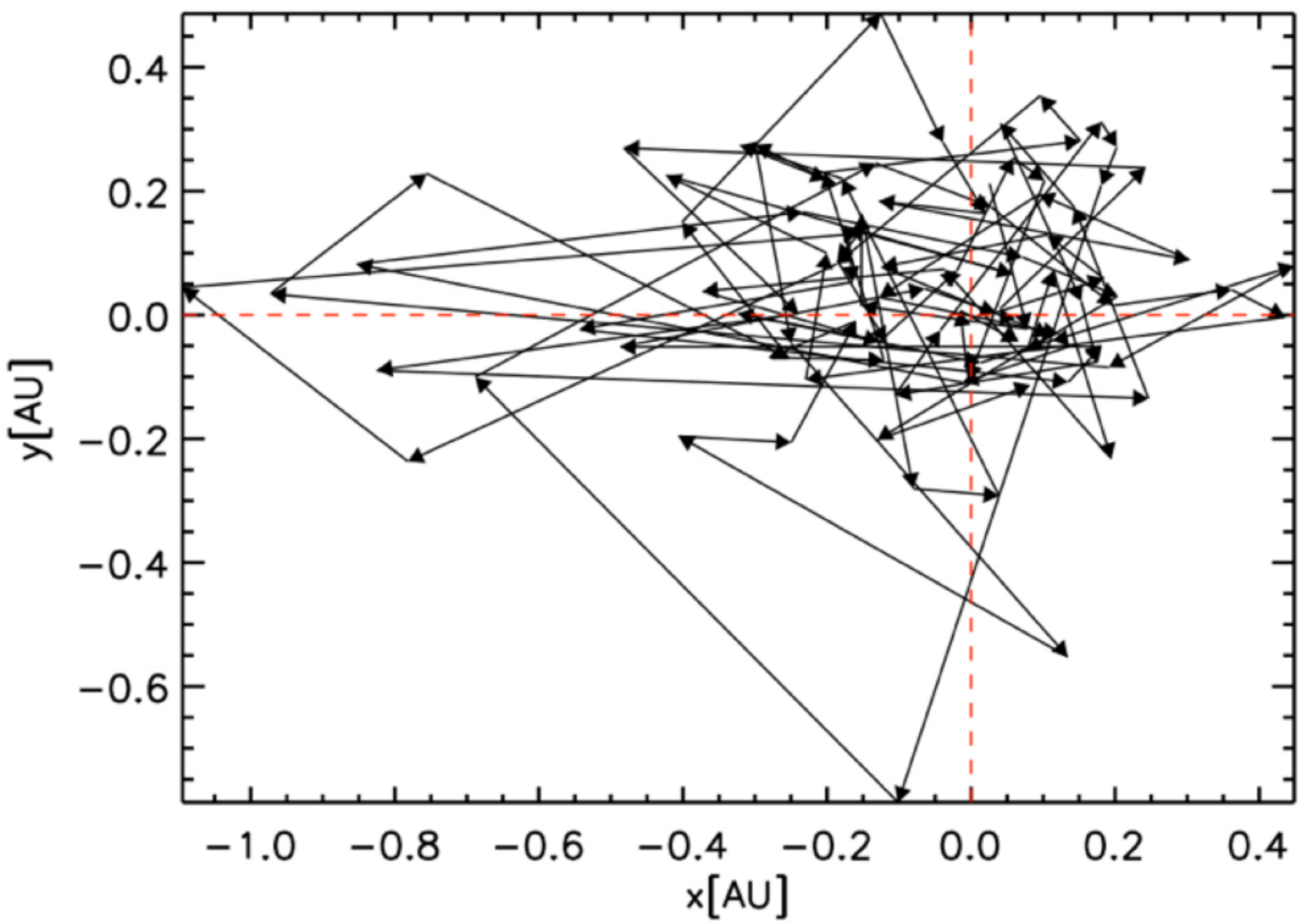
The "random walk" of the photocentre

Photocentric positions from star-in-a-box simulations (snapshots ~23 days apart)

Red supergiant (R ~ 4 au) over 5 years



AGB star (R = 1.87 au) over 30 years



(Chiavassa et al. 2011, 2018)

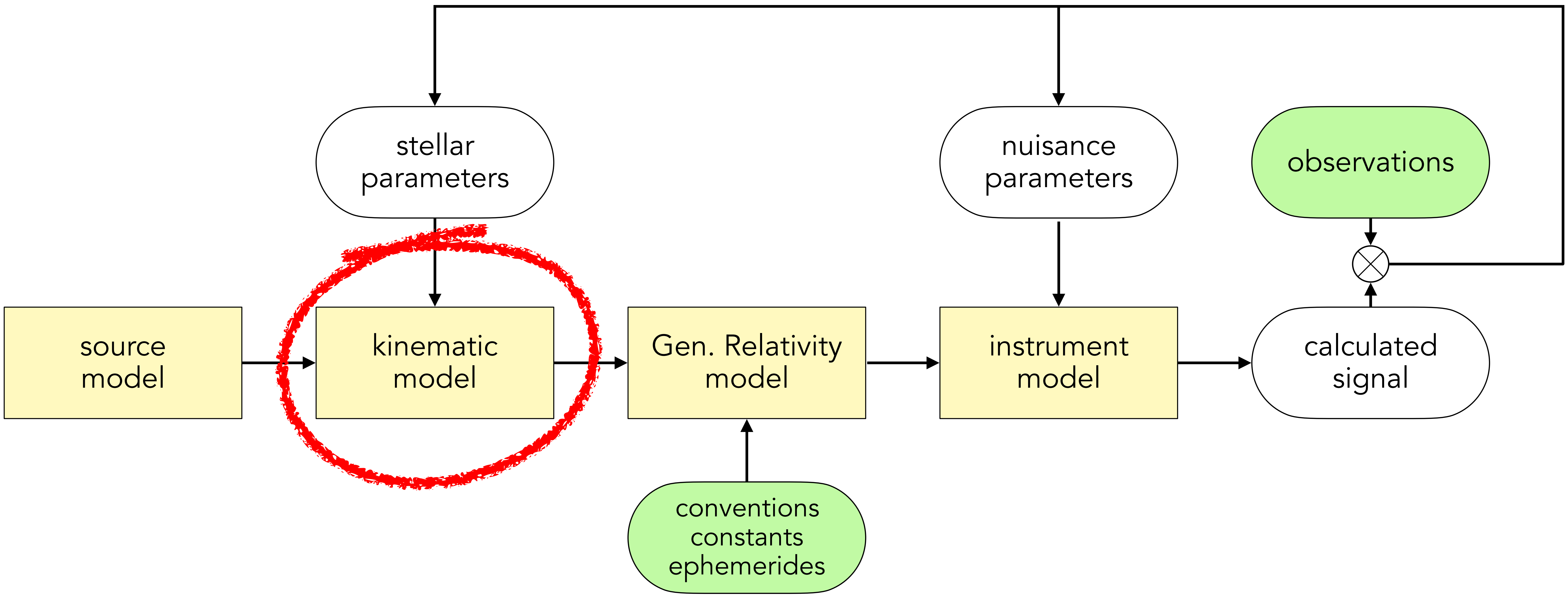
Parallax of the Mira variable BX Cam

Catalogue	Parallax [mas]	Months	N_{vis}
Gaia DR2	4.13 ± 0.25	22	11
Gaia EDR3	1.76 ± 0.10	34	30
VERA VLBI[a]	1.73 ± 0.03	34	

[a] Matsuno et al. PASJ 72, 56 (2020)

N_{vis} = number of visibility periods (≈ independent snapshots)

Forward modelling (basic Hipparcos, Gaia)



Kinematic model (beyond the Solar System)

Uniform space velocity relative to the Solar System Barycentre (SSB):

$$\boldsymbol{b}(t) = \boldsymbol{b}(t_0) + (t - t_0)\boldsymbol{v} \qquad\qquad (t_0 = \text{reference epoch})$$

The model has six free parameters:

- the components of $\boldsymbol{b}(t_0)$ and $\boldsymbol{v}$ in the Barycentric Celestial Reference Frame
- or (more conveniently) the astrometric parameters

$$\left.\begin{array}{ll} \alpha_0,\ \delta_0 & = \text{direction from SSB at the reference epoch} \\ \varpi & = \text{parallax} \\ \mu_{\alpha*},\ \mu_\delta & = \text{proper motion along RA and Dec} \end{array}\right\} \text{5-param. model}$$

$$\begin{array}{ll} v_R & = \text{radial velocity (from spectroscopy, or neglected)} \end{array}$$

Approximations in the 5-parameter model

- Perspective acceleration is negligible except for nearby, high-proper motion stars
  - $v_R = 0$ can usually be assumed when a RV is not available

- Galactic orbits are curved
  - local effects of tidal accelerations are ~1 to 3 µas cy^{-2} independent of distance (Butkevich & Lindegren 2014)

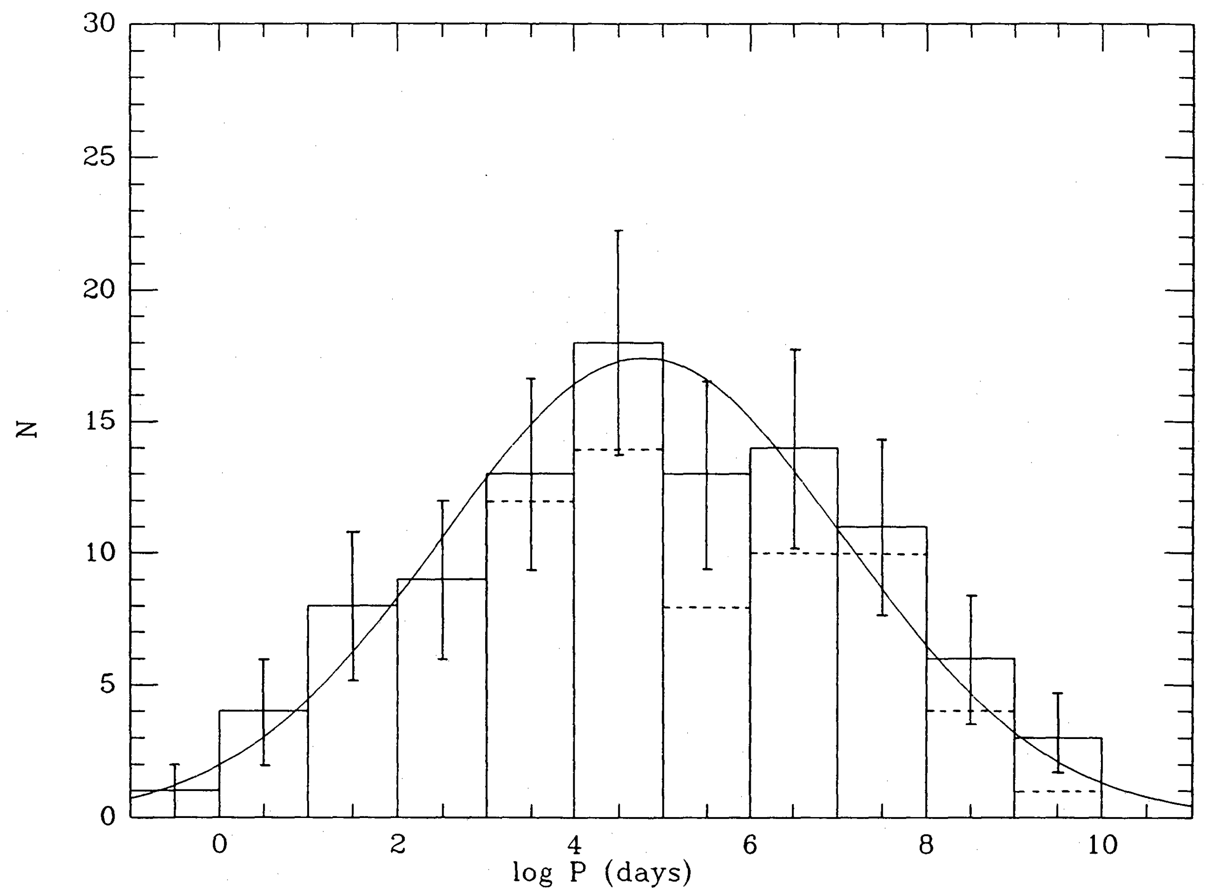
- Light-time effects are ignored
  - negligible for the modelling but not always for the interpretation of observations, e.g.

$$v_T^{(\mathrm{true})} = \mu \times \left(\frac{A}{\varpi}\right) \times (1 + v_R/c)$$

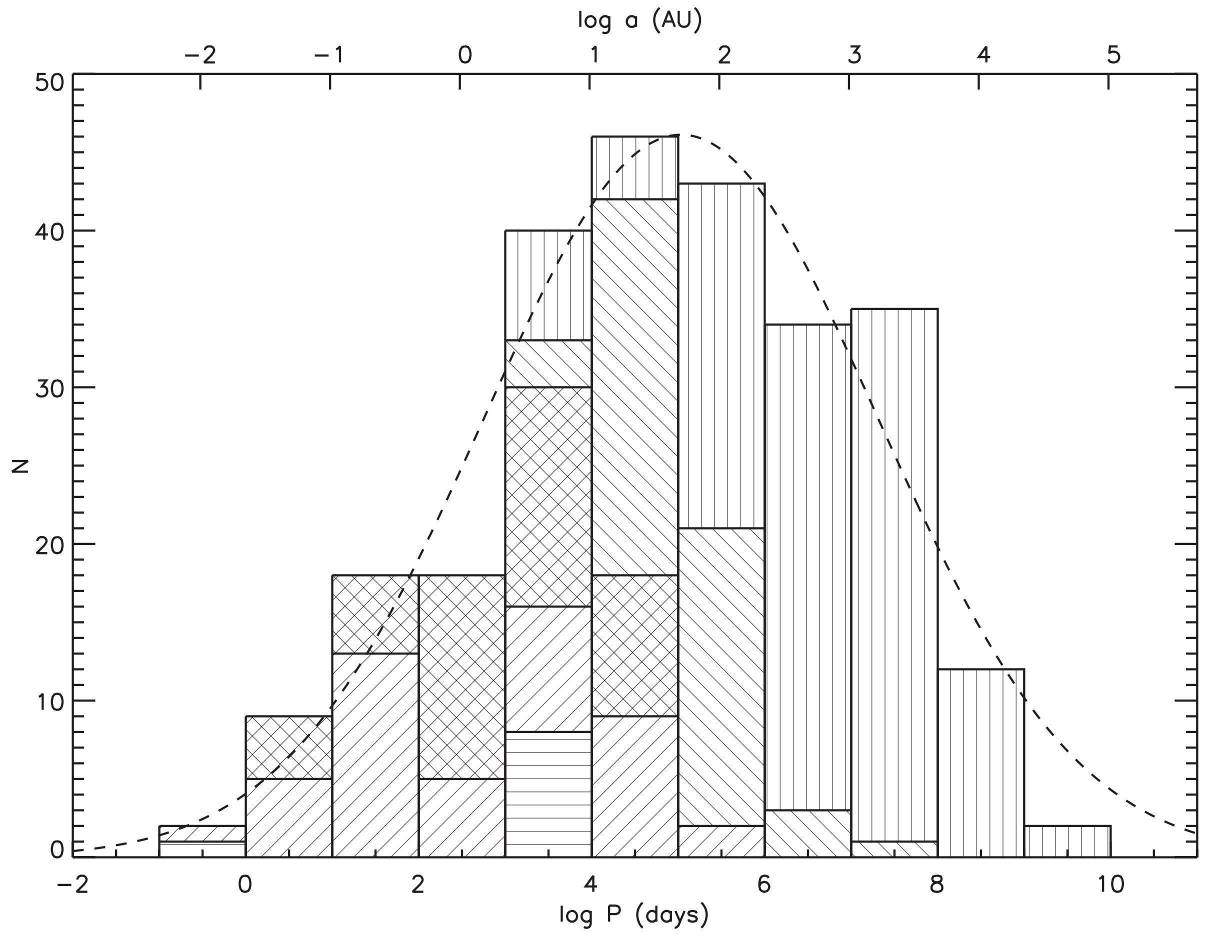
- For gravitationally bound systems (e.g. binaries) the model applies to the centre of mass
  - ***potentially huge deviations for the individual components or the photocentre***

Why is the 5-parameter model useful even for binaries?

~50% of local G dwarfs have companions with a very wide, log-normal distribution of periods

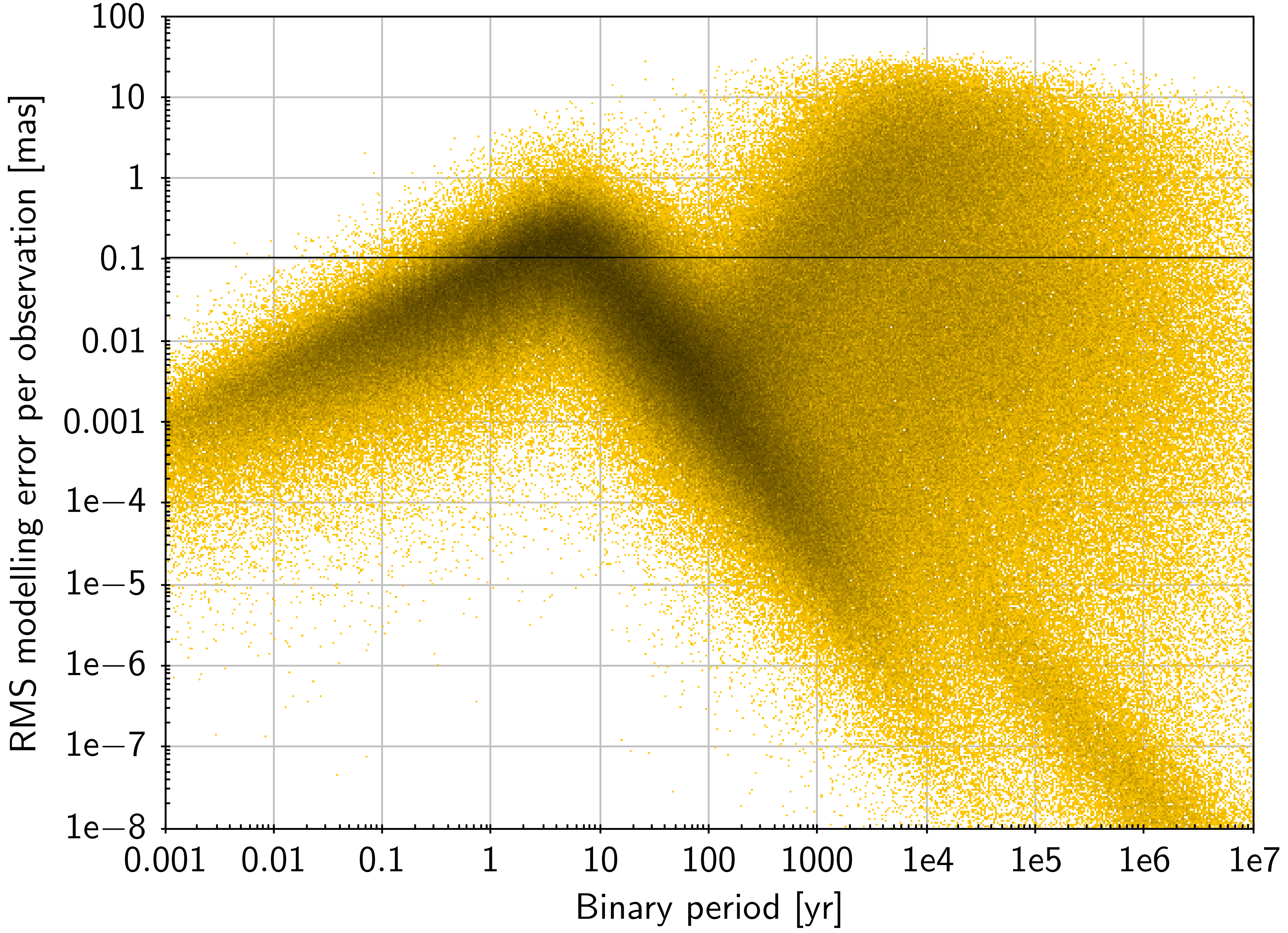


Duquennoy & Mayor (1991):
median P = 180 yr, sigma = 2.3 dex



Raghavan et al. (2010):
median P = 300 yr, sigma = 2.3 dex

Modelling errors for binaries when the 5p model is used

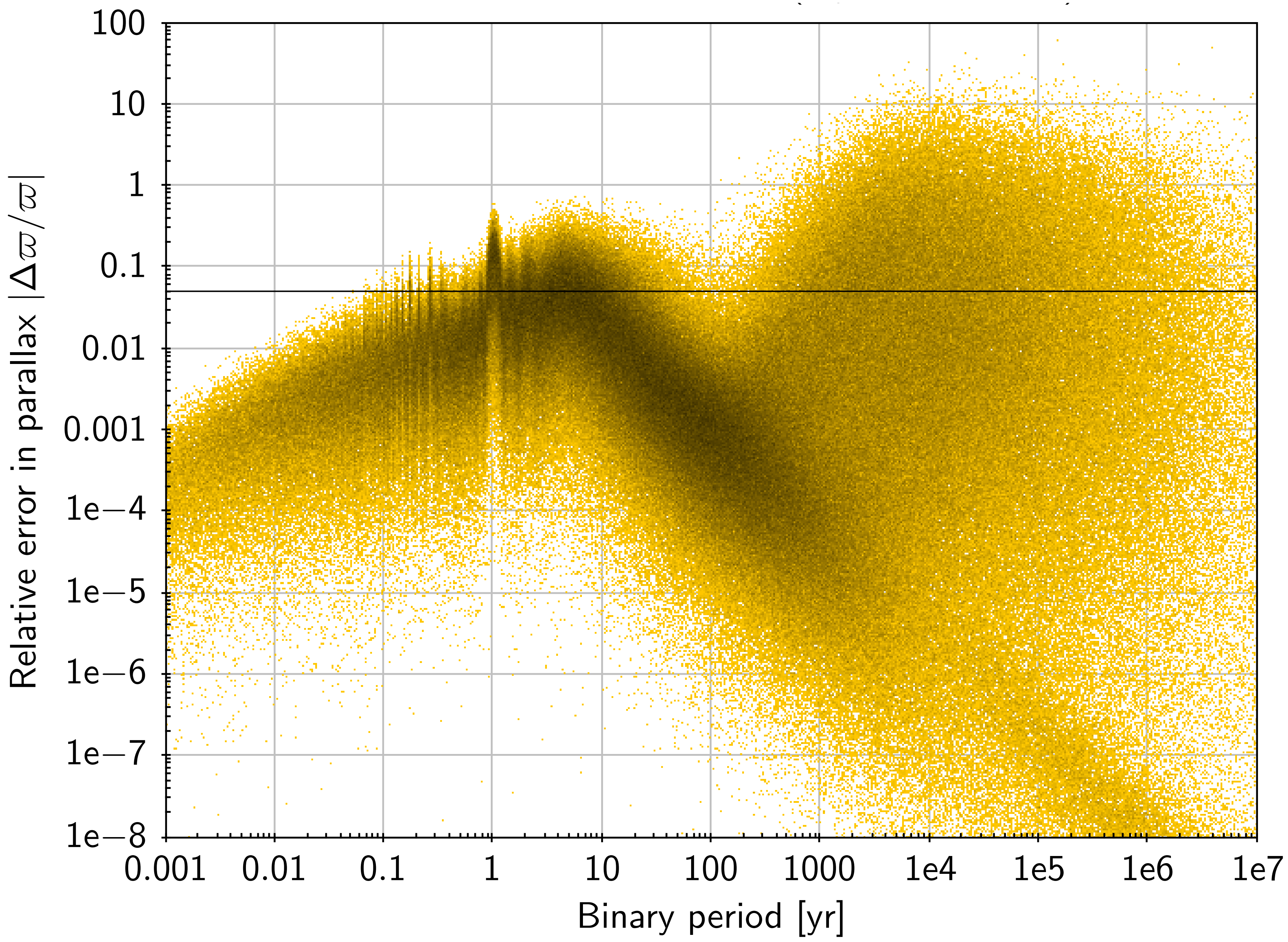


22% have RMS modelling error > 0.1 mas (~random noise level)

Simulated Gaia observations of 1 M 15th mag stars over 5 yr, assuming that they ***all*** have companions as in the Duquennoy & Mayor study

See also Penoyre et al. (2020), 301.05

Parallax errors for binaries when the 5p model is used

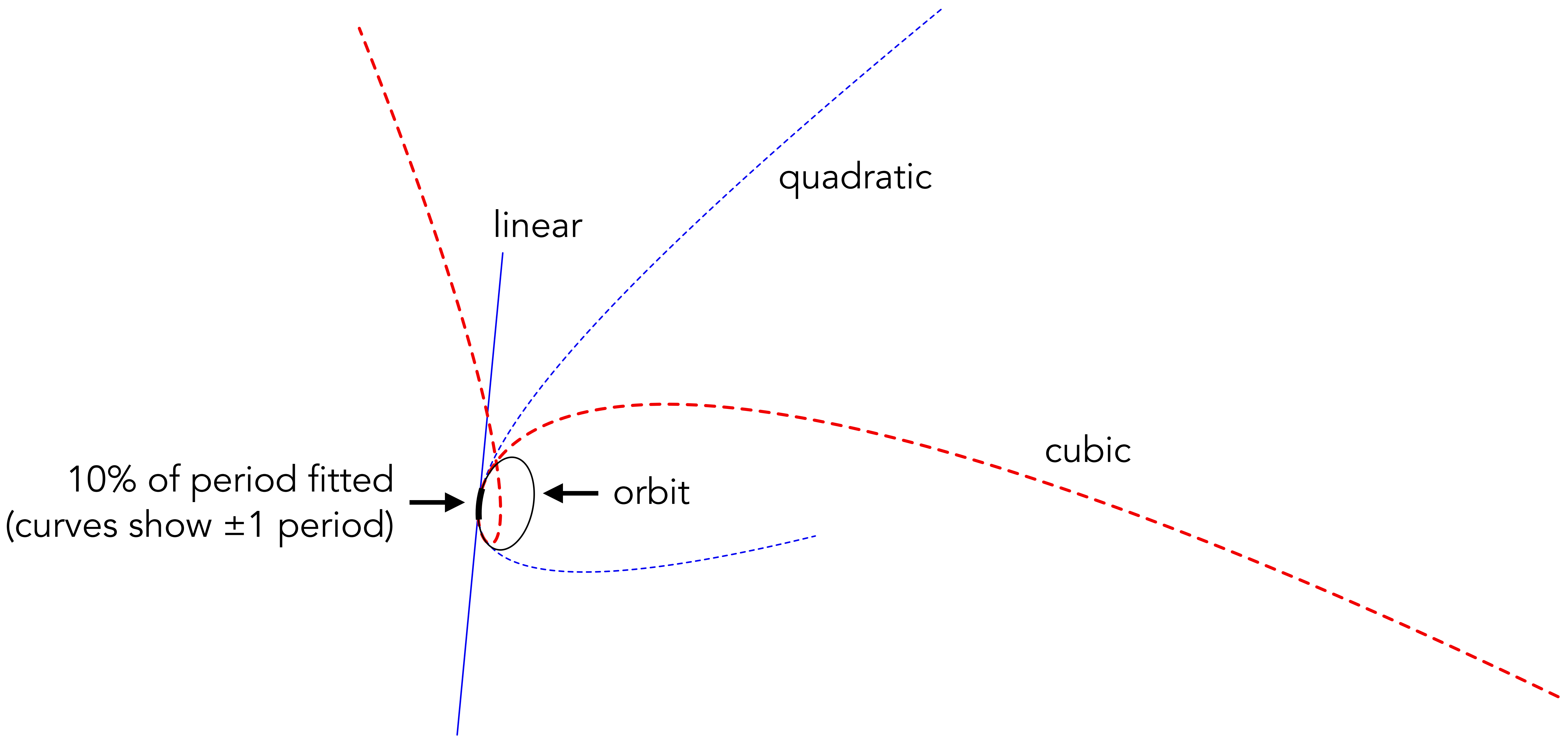


14% have relative parallax error > 5%

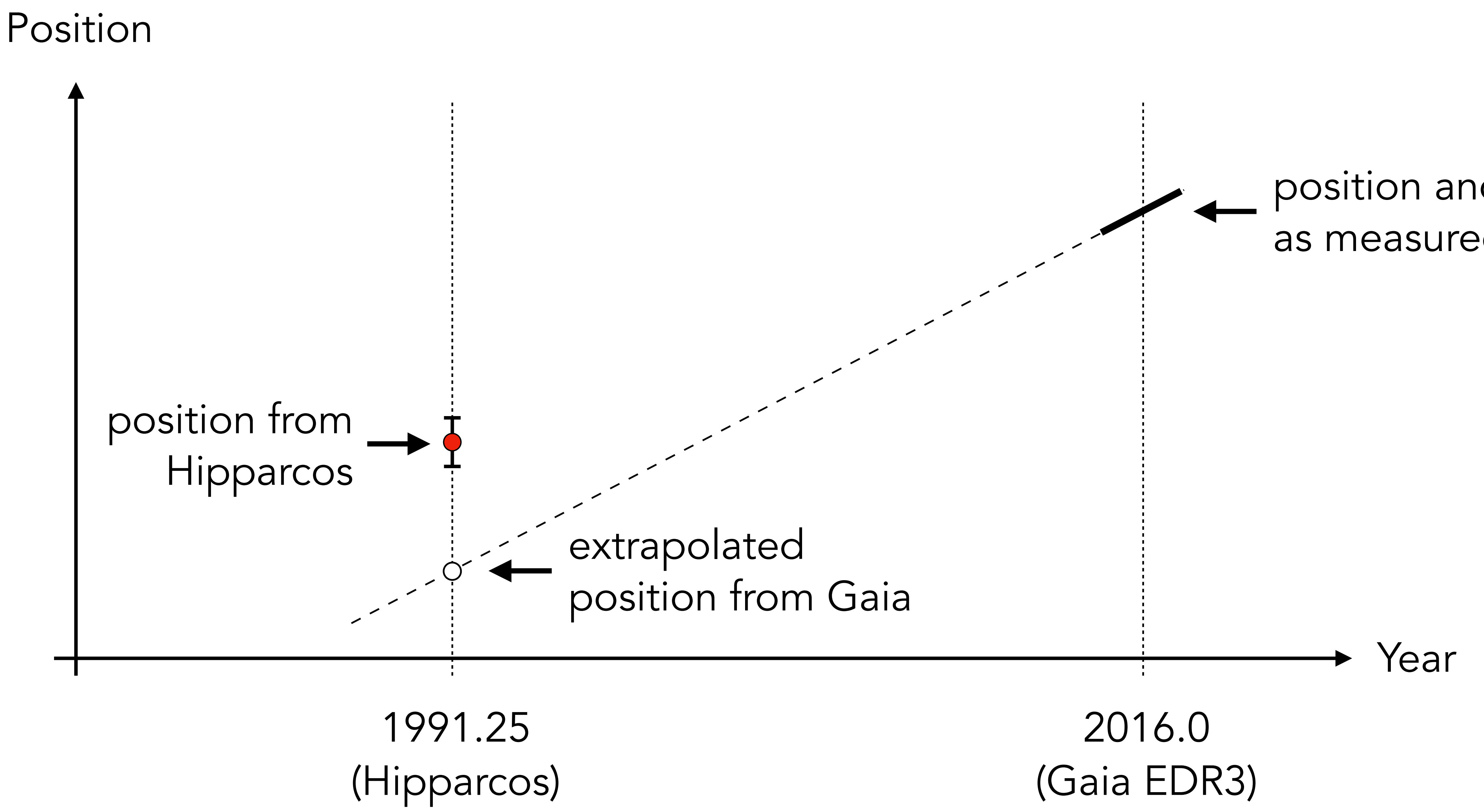
Simulated Gaia observations of 1 M 15th mag stars over 5 yr, assuming that they ***all*** have companions as in the Duquennoy & Mayor study

See also Penoyre et al. (2020), 301.05

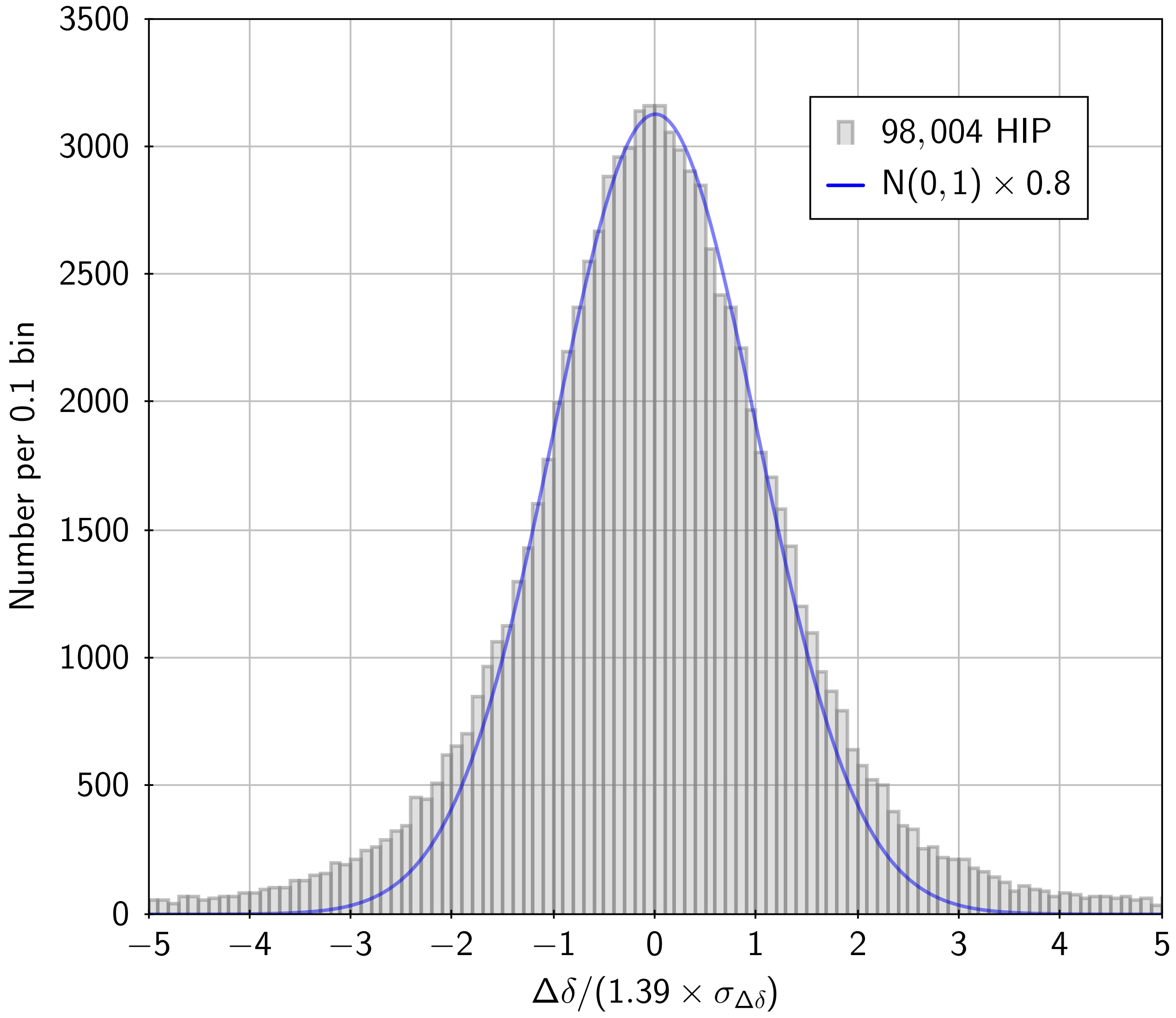
More accurate models for orbital motion (over limited time)

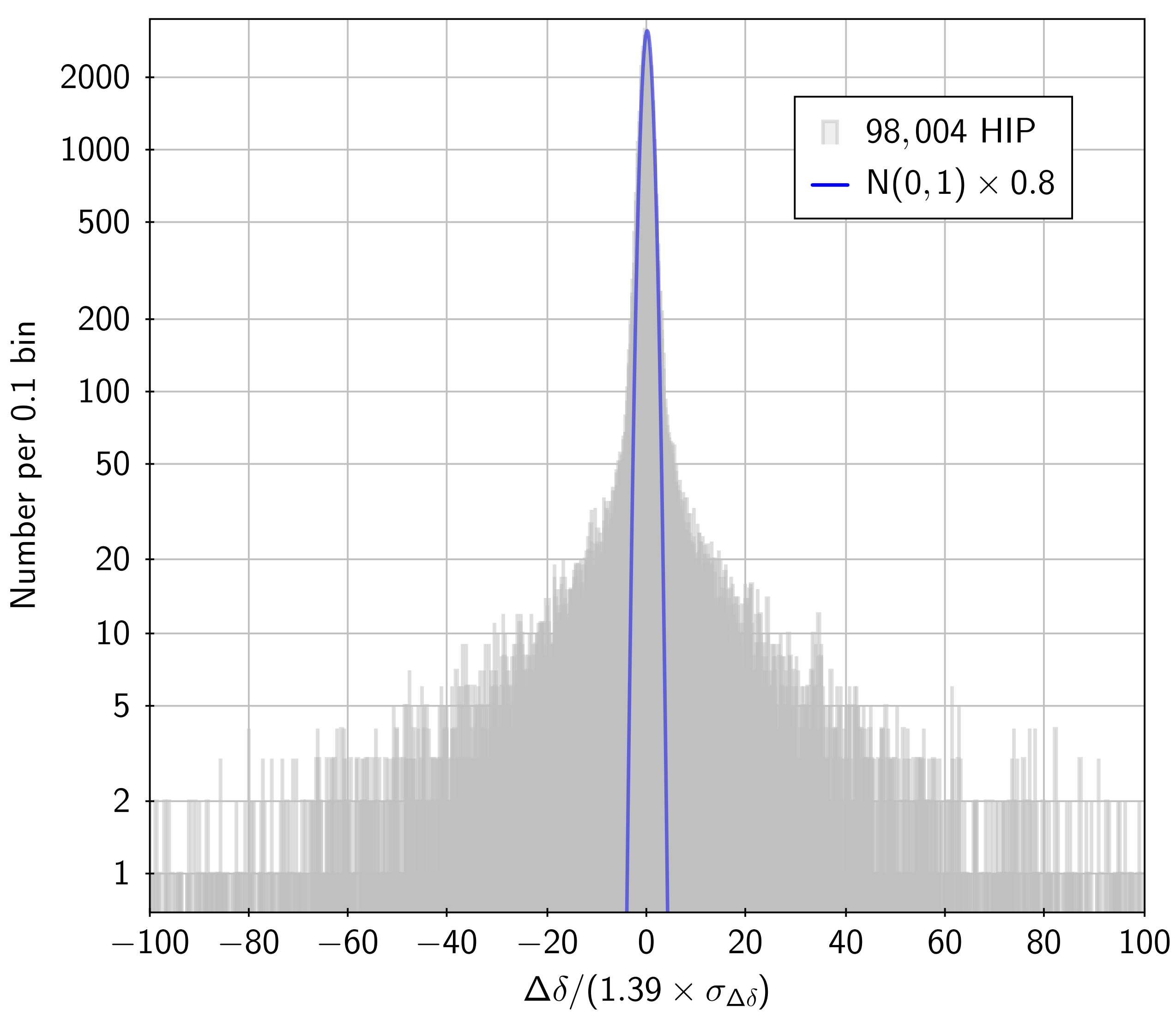


A direct test of the 5p model: Hipparcos vs Gaia



Position differences Hip–Gaia (only declination shown)

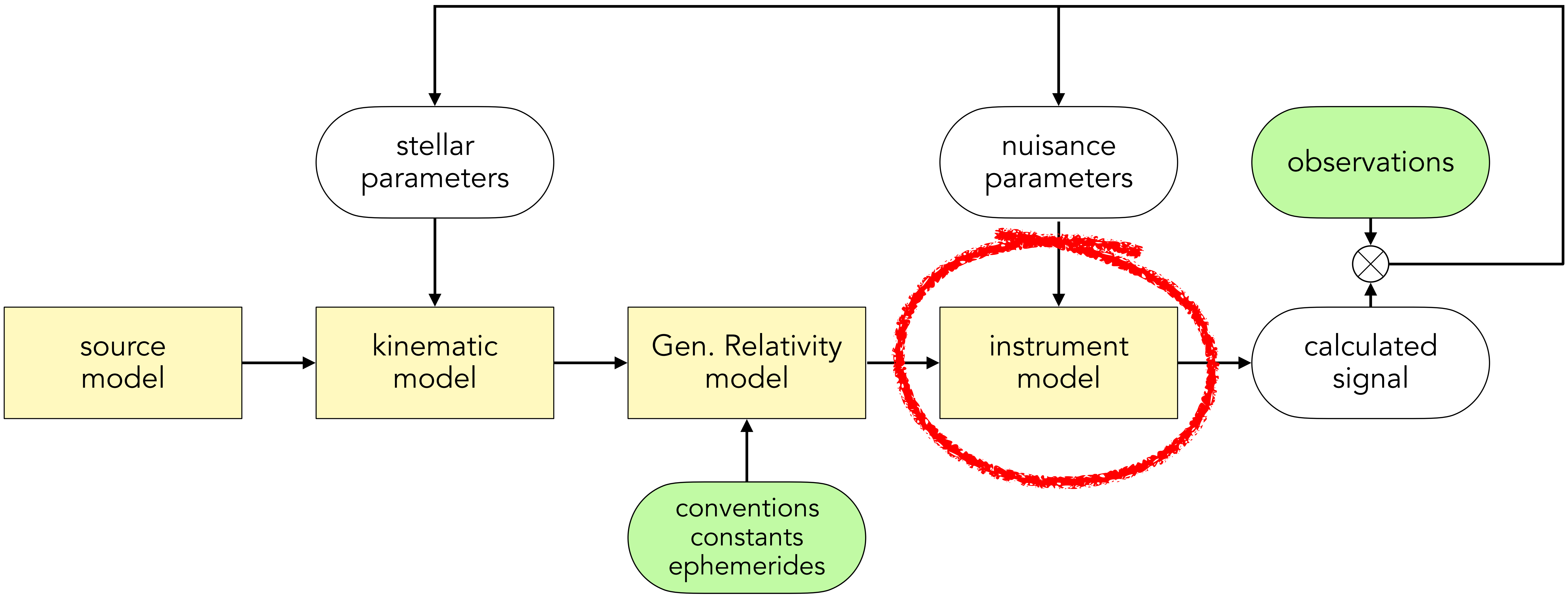




80% of the stars follow roughly the expected distribution, 20% are in the wide tails

(acceleration or Δμ stars - potential for discovering substellar companions; Brandt 2018 and others)

Forward modelling (basic Hipparcos, Gaia)



Approximations in the 5-parameter model

- Perspective acceleration is negligible except for nearby, high-proper motion stars

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} = -2\mu\varpi v_R/A \quad \Rightarrow \quad \Delta\mathrm{pos} = -(\mu\varpi v_R/A) \times \Delta t^2$$

$$\Rightarrow \quad v_R = -\frac{A}{\mu\varpi\Delta t^2} \times \Delta\mathrm{pos}$$

If the positions [...] are accurately observed for sufficiently long periods of time [...] we shall be in position to determine the radial velocities of these stars independently of the spectroscope and with an excellent degree of precision.

F. Schlesinger, Astron. J. 30, 137 (1917)

Attainable pre-Gaia precision was limited to several km/s (van de Kamp, Gatewood & Russell, Dravins et al.)

Astrometric radial velocities from Hip – Gaia EDR3 (selected results)

Name	Sp	Spectroscopic RV (SIMBAD), km/s	Astrometric RV km/s
Barnard's star	M4	−110.4	−110.9 ± 0.4
Kapteyn's star	M1	+245.2	+244.1 ± 0.7
Proxima Centauri	M5.5	−22.4	−23.2 ± 0.7
Groombridge 1830	K1	−98.0	−98.6 ± 1.0
61 Cyg AB	K5, K7	−65.1	−65.5 ± 2.3
LAWD 37	DQ	–	+28.0 ± 4.9
Van Maanen 2	DZ	+263	−14.0 ± 7.1

$M_B/M_A = 0.76 \pm 0.05$

(Lindegren & Dravins, submitted to A&A, arXiv:2105.09014)

Summary

- ✓ The basic astrometric processing of Gaia observations relies on some very simple models of the objects and their kinematics (point source, uniform space motion)
- ✓ These models are useful because they are simple, robust, and ***accurate enough*** for a majority (~80%) of the objects
- ✓ For parallax measurement, unresolved structures of size $\gtrsim$ 1 au are problematic (supergiants, binaries for a certain range of periods)
- ✓ For many of the sources where the simple models do ***not*** work well, more sophisticated analysis and follow-up observations will provide a wealth of new, exciting information!