Rank Reduction with Convex Constraints

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2017

Less is sometimes more

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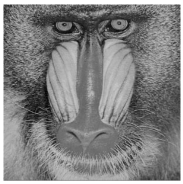
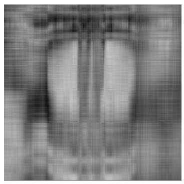
Popular science summary of the PhD Thesis *Rank Reduction with Convex Constraints*, February 2016. The thesis can be downloaded from: `http://www.control.lth.se/publications`

The law of parsimony (William of Ockham, 1287 – 1347) says that *"Among competing hypotheses, the one with the fewest assumptions should be selected"*. Even though we are not aware of it, this principle is constantly applied by everyone. In fact, reducing things to their simple abstract features is one of the great skills of the human brain. For example, in order to recognize an object as a vehicle, we are only required to remember a few features such as wheels and size. While the human brain has developed this skill over thousands of years, scientists and engineers are trying to utilize the same strategies in modern technological products, such as self-deriving cars, smart phones or vacuum cleaning robots. As easy as it may seem from a human perspective, as challenging it is to find features that can be understood by computing units.

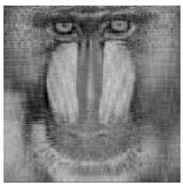
A common approach in addressing this problem is to apply heuristics that approximate large sets of measured data by the sum of their dominant components. This is similar to what is done in data compression, such as image compression in modern cameras. The idea behind image compression is to remove parts from an image file that hardly contribute to what can be seen on a screen. As a result, the reduced pictures require significantly less space on a data storing device. This, in turn, makes it cheaper to process these pictures for purposes such as handwriting or face recognition. This thesis focuses to a large extent on the mathematical analysis and development of such heuristics in order to obtain optimal approximations of data sets.



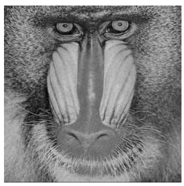
Original image



5 components



15 components



100 components

Visualization of neglecting components in image compression with 298 components in the original image.

The other part of the thesis deals with parsimony in mathematical models. In many areas, mathematical models are used to substitute experimental trials or obtain predictions such as the weather forecast. The number of components (assumptions) in a mathematical model are represented by the number of necessary equations. Since each additional equation increases the computational effort in simulations, scientist and engineers desire to keep the total number of equations as small as possible. The systematic way of removing equations that have negligible influence is called *model reduction*. Traditional model reduction techniques, however, neglect to preserve physical meaningful constraints such as the fact that the amount of precipitation can only be measured in positive quantities. Therefore, simulations of the reduced models may lack interpretability. This thesis modifies one of these traditional methods to guarantee the preservation of positive quantities.

In summary, both parts of this thesis essentially try to extract the dominant components that are contained in mathematical models or data sets. Based on the success of these methods, this helps to save computational resources as well as to gain deeper insights into large data sets and mathematical models.