LUND UNIVERSITY

Peurbach's Tabulae Eclypsium: A Commentary

Gislén, Lars

Published in:
Journal of Astronomical History and Heritage

DOI:
10.3724/SP.J.1440-2807.2026.01.04

2026

Citation for published version (APA):
Gislén, L. (2026). Peurbach's Tabulae Eclypsium: A Commentary. *Journal of Astronomical History and Heritage*, *29*(1), 63-77. https://doi.org/10.3724/SP.J.1440-2807.2026.01.04

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

Journal of Astronomical History and Heritage, 29(1), 63–77 (2026).

PEURBACH'S *TABULAE ECLYPSIUM*: A COMMENTARY

Lars Gislén
Division of Particle Physics, University of Lund, Dala 7163,
242 97 Hörby, Sweden.
E-mail: LarsG@vasterstad.se

Abstract: The paper is a commentary on the tables in George Peurbach's *Tabulae Eclypsium* and presents their mathematical background with many of the tables analyzed and recalculated. These tables, based on the Alfonsine tradition, made possible more accurate and less tedious calculations of the circumstances of solar and lunar eclipses and were in many respects an improvement over previous similar tables. The paper also analyses Peurbach's model calculation of the 1460 July partial solar eclipse.

Keywords: Peurbach; astronomical tables; syzygy; eclipses; parallax; Sun; Moon.

1 INTRODUCTION

Georg Peurbach (1423–1461) was born Georgius Aunpekh in the Austrian town of Peuerbach near Linz and studied at the University of Vienna under the guidance of Johann von Gmunden (ca. 1380–1442) and later collaborated with his pupil Regiomontanus, Johannes Müller (1436–1476). Peurbach held a position as Court Astrologer to King Ladislas V of Bohemia and Hungary and to Emperor Frederick III.

Peurbach's *Tabulae Eclypsium* (Peurbach, 1514) was not printed until 1514, edited by Georg Tannstätter (1582–1535), humanist, astronomer, astrologer, and medical doctor, although the tables existed as a manuscript probably already in 1459. An early manuscript from about 1480 is extant in the Biblioteka Jagiellońska in Poland (see link in the reference list).

The *Tabulae Eclypsium* is Peurbach's most impressive work and was still used by Tycho Brahe near the end of the sixteenth century. The tables are based on the Paris Alfonsine tables (PAT) but contain new and enlarged tables enabling improved interpolation and thereby more accurate computations. The tables are set up for the meridian of Vienna with a longitudinal time difference of 1hr 20m between Toledo and Vienna. The 1514 edition of *Tabulae Eclypsium* also contains the *Tabula Primi Mobilis* by Regiomontanus, tables intended to be used for computations of right-angle spherical triangles. These tables, however, do not appear in the early manuscript and are not analyzed in this paper. Figure 1 shows the title page of the *Tabulae Eclypsium* with images of a solar and a lunar eclipse. Between the eclipse images is an ominous copyright warning: By imperial edict, no one should print this remarkable work for ten years or have others to print it and sell it after it is printed under penalty of the loss of all copies and fifty gold coins for each one printed or sold.

The layout of the 1514 edition of the *Tabulae Eclypsium* is as follows:

1. Dedications and a foreword by Peurbach with instruction for the use of the tables and model calculations of the partial solar eclipse of July 1460 and the total lunar eclipse in December of the same year.
2. Tables for calculating the mean syzygy, solar mean longitude, lunar anomaly, argument of lunar latitude, and solar apogee.
3. Tables for finding the the time of true syzygy.
4. Tables for the mean motions of the Sun, the Moon arguments and the lunar node.
5. Table for the equation of time.
6. Table for the solar equation.
7. Table for the lunar equations.

Tabulae Eclypſiũ Magiſtri
Georgij Peurbachij.
Tabula Primi mobilis Jo-
annis de Monteregio.

Indices præterea monumẽtoꝝ quæ clariſſimi uiri Studii Viennenſis alumni in Aſtronomia & aliis Mathematicis diſciplinis ſcripta reliquerũt. q̄ ſi lector hæc te oblectauerint: curabimus ut & alia in lucẽ bono Auſpicio aliquando progrediantur.

Poſtremo vt nihil te: quod ſcitu dignum eſt prætereat:
Inuenies ſtudioſe lector ex hoc diligenter impreſſo uolumine mirum quendam & fœcũdiſſimũ pluriũ tabularũ: & pene omniũ inſtrumentoꝝ: puta Aſtrolabii: Sphæræ: Organi Ptolemæi: Metheoroſcopii: Armillarũ: Torqueti: Rectanguli: Quadrantũ: & id genus alioꝝ (quæ recenſere longũ eſſet) uſum & expeditam praxim. Sunt enĩ Theoreumata tabulæ Primi mobilis uniuerſalia; omni prorſus regioni accõmodata; ex ſcientia ſphæricoꝝ triangulorũ tranſumpta. Nouarũ inſuper tabularũ & inſtrumẽtoꝝ inueniendoꝝ area latiſſima pſpicacis ingenii uiris oblata eſt.

Coelum Tabella Fati.

Edicto Imperatorio veti
tum eſt: ne quis in decẽnio
hoc inſigne opus imp̃mat
vel alioꝝ ductu ⁊ impenſis
excuſum venditet: ſub mul
cta amiſſionis omniũ exẽ
plariũ: ⁊ quinquaginta nũ
morũ aureoꝝ pro ſingulis
impreſſis vel venditis.

Figure 1: The title page of the *Tabulae Eclypsium*.

8. Table of the true motion of the Sun.
9. Table of the true motion of the Moon.
10. Parallax tables for climate six and seven.
11. Table for the latitude of the Moon.
12. Tables for eclipse size and duration for the Sun.
13. Tables for eclipse size and duration for the Moon.
14. Table for the semi-diameters of the Sun, the Moon, the umbra and *Variatio umbrae minuenda*.

2 PEURBACH'S PROCEDURE TO FIND THE MEAN SYZYGY

First a target month and year is selected. Then the number of elapsed days from the epoch, noon 1 January CE 1401, to the the beginning of the selected month is calculated. Peurbach uses elapsed years, so 1401 current becomes 1400 elapsed. Peurbach at first uses 40-year periods, then single years and finally months. The 40-year period is chosen to get a whole number of days. In this way Peurbach stepwise computes the change in the Moon's age until the beginning of the target month. The epoch age, 15 days, 0;3,37 hours is then added to give the age of the Moon.[1]

Mathematically the method is as follows. Denote by T_0 the number of days from the epoch to the beginning of the target month. T_0 is then divided by the length of the synodic month M = 29 days, 12;44,3,3 hours. The quotient can be split into an integer n and a fraction f. The integer will be the number of whole synodic month elapsed and the fraction f added to the Moon's age at the epoch will be the age at the beginning of the target month:

$$T_0/M = n + f \tag{1}$$

This age is subtracted from M or $M/2$, if the result is negative from $2M$ or $3M/2$ or with an integer times M for conjunctions and half integer times M for oppositions. The result, the remaining time to syzygy, will be the date and time of the mean syzygy in the selected month.

The epoch for the calculation of the solar longitude, l_S, the lunar anomaly, a_M, and the argument of latitude, a_L, is the first mean New Moon on 14 January 1401, 12;40,26 hours. As counted from this epoch there will be n elapsed synodic months and the required quantities can be calculated:

$$\begin{aligned}\lambda_S &= \lambda_{S,0} + nMv_S\\ \alpha_M &= \alpha_{M,0} + nMv_\alpha\\ \alpha_L &= \alpha_{L,0} + nMv_L\end{aligned} \tag{2}$$

where the quantities with index 0 are the epoch values and v_S, v_α, and v_L are the mean velocities in solar longitude, lunar anomaly, and argument of latitude. Peurbach uses the following numerical Alfonsine values:

$$\begin{aligned}\lambda_{S,0} &= 273;47{,}38°\\ v_S &= 0;59{,}8{,}19{,}37{,}13{,}56°/\text{day}\\ \alpha_{M,0} &= 264;6{,}56°\\ v_\alpha &= 13;3{,}53{,}57{,}30{,}21{,}4{,}13°/\text{day}\\ \alpha_{L,0} &= 83;5{,}54°\\ v_L &= 13;13{,}45{,}39{,}22{,}25\,°/\text{day}\end{aligned} \tag{3}$$

Peurbach's tables give results that are almost identical to the PAT with a 1hr 20m longitude difference from Toledo.

3 PRECESSION AND THE SOLAR APOGEE

In the PAT the epoch is 1 January, CE 1, noon. The longitude of the solar apogee will be increased by precession which is assumed to contain two parts: one linear part and one periodic part, both being a function of the time t in days since the epoch. The periodic part or trepidation is a cyclic function of θ, the longitude of *Octave Sphere,* with a period of 7 000 years. It is given by

$$\theta = \theta_0 + v_\theta t. \quad \theta_0 = 359;12{,}24°.$$
$$v_\theta = 0;0{,}0{,}30{,}24{,}49°/\text{day} \tag{4}$$

The periodic part of the precession, $d,$ is then given by the relation

$$\sin\delta(\theta) = \sin 9° \sin\theta \tag{5}$$

and the total precession is then the sum of the trepidation and a linear term

$$p = v_p t + \delta \tag{6}$$

with $v_p = 0;0{,}0{,}4{,}20{,}41{,}17{,}12°/\text{day}$ corresponding to a period of 49 000 years.

The longitude of the solar apogee of date is then

$$\omega = \omega_0 + p \quad \text{with } w_0 = 71;25{,}23° \tag{7}$$

The table of the longitude of the solar apogee in *Tabulae Eclypsium* starts with the year 1400 and tabulates every five years until 3040. It deviates slightly from the apogee longitudes in the PAT by of the order of some arc minute. The deviation Δ is periodic and can be parametrized by

$$\Delta = 1.3' \sin(3.26\,\theta - 34°) + 1.24' \tag{8}$$

The reason for this small difference is unknown and has negligible effect on the final results (Kremer, 2025).

The mean solar anomaly is the difference of the mean longitude of the Sun and the solar apogee.

4 VELOCITIES

The parameters in this section were determined by least square fitting to Peurbach's tables of the equation of center for the Sun and the Moon and also using the PAT. In this paragraph index S represents the Sun and M the Moon. Quantities with a top bar denote mean values.

- The solar eccentricity taken from the solar equation table: *e* = 2.2689 ≈ 2;16.
- The lunar epicycle radius taken from the lunar equation of argument: *r* = 5.1596 ≈ 5;10.
- The radius of the crank circle (see Figure 2) taken from lunar equation of center: *s* = 10.3262 ≈ 10;20.
- The unit is *R* = 60.

From the PAT we also have:

- The mean solar velocity in longitude $\bar{v}_S = 2;28'\text{hour}$
- The the mean solar velocity in anomaly (argument) is equal to its mean longitude velocity.
- The mean lunar velocity in longitude $\bar{v}_M = 32;56'/\text{hour}.$
- The mean lunar velocity in anomaly $\bar{v}_\alpha = 32;40'/\text{ hour}.$
- The mean lunar velocity in elongation $\bar{v}_\eta =$ 30;29' / hour.

4.1 The Solar True Velocity

The true solar longitude *l* is given by

$$\lambda = \bar{\lambda} - q_S(\alpha) \tag{9}$$

q_S is the equation of center and a function of the solar anomaly α, the solar angular distance from its apogee, and given by the relation

$$\tan q_S = e\sin\alpha/(R + e\cos\alpha) \tag{10}$$

To find the solar velocity we take the time derivative of (10).

The derivative D_S of q_S with respect to α is

$$D_S = e(R\cos\alpha + e)/(R^2 + 2Re\cos\alpha + e^2) \tag{11}$$

The true solar velocity will then be

$$v_S = \bar{v}_S - \bar{v}_\alpha D_S = \bar{v}_S - \bar{v}_S D_S \tag{12}$$

using that the solar velocity in anomaly is equal to its mean velocity. The derivative D_S can be approximated by

$$D_S(\alpha) \approx (q_S(\alpha + 1^\circ) - q_S(\alpha))/1^\circ \tag{13}$$

These formulas reproduce almost exactly Peurbach's solar velocity table.

4.2 The Lunar True Velocity

For the lunar true velocity and using the first lunar model of Ptolemy we have formulas corresponding to those for the Sun but with *e* substituted by *r* and taking into account that the lunar velocity in anomaly is different from the mean velocity in longitude. α is now the lunar anomaly.

$$D_M = r(R\cos\alpha + r)/(R^2 + 2Rr\cos\alpha + e^2) \tag{14}$$

$$v_M = \bar{v}_M - v_\alpha D_M \tag{15}$$

The last term will be the angular velocity distance from the mean velocity. This is the velocity tabulated in *Tabulae Eclypsium*. The function D_M is zero for α = 95° using Peurbach's tables of the lunar equation for anomaly.

However, for the true lunar velocity used implicitly in Peurbach's eclipse tables, Ptolemy's more elaborate final lunar theory with a crank mechanism was used, see Figure 2 (Chabás and Goldstein, 2009: 116–118; Goldstein, 1992; Pedersen, 2010: 226). This mechanism will draw the center of the epicycle, C, back and forth. The observer O sits in the center of the deference circle. The center C of the epicycle moves anti-clockwise along the deferent (blue). The Moon moves clockwise along the epicycle. The center of the epicycle has a fixed distance to the point F that rotates clockwise around the center O with the angle COF being twice the elongation η of the Moon from the Sun. The angle *c* will be the correction to the mean lunar anomaly α. The necessity of using Ptolemy's final model for computing the lunar velocity at syzygy was noted by astronomers in the fourteenth and fifteenth centuries (Chabás and Goldstein, 2009; Goldstein, 1992).

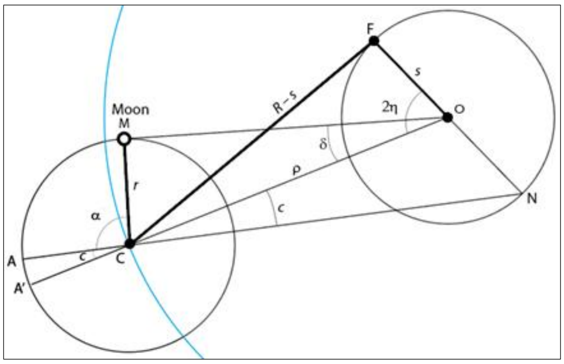


Figure 2: Ptolemy's lunar final model (L. Gislén).

We now derive a modified formula for the lunar true velocity at syzygy in this more elaborate lunar model. The lunar equation of center is a function of the true anomaly *a'* which is given by

$$\alpha' = \alpha + c \tag{16}$$

where *c* is a function of twice the lunar elongation η from the Sun and tabulated in *Tabulae Eclypsium* under the name of *Equatio Centri*. This table is generated by first calculating the distance *r* of the center O of the crank circle from the epicycle center C.

$$\rho = s\cos 2\eta + \sqrt{R^2 - 2Rs + (s\cos 2\eta)^2} \tag{17}$$

This is then used to compute *c* by

$$\tan c = s\sin 2\eta/(\rho + s\cos 2\eta) \tag{18}$$

Note, that at the syzygy (η = 0° or 180°), *c* is zero and $\alpha' = \alpha$, but the derivative of *c* is not zero and will modify the lunar velocity.

Starting from the equation for the true lunar longitude

$$\lambda = \bar{\lambda} - q_M(\alpha) \tag{19}$$

and taking as before the derivative with respect to time we get

$$v_M = \bar{v}_M - \frac{dq_M}{dt} = \bar{v}_M - \frac{dq_M}{d\alpha'}\frac{d\alpha'}{dt} \tag{20}$$

Now using the relations (6.1), (6.3), and (6.5) we get

$$v_M = \bar{v}_M - D_M\frac{d\alpha'}{dt} = \bar{v}_M - D_M\left(\frac{d\alpha}{dt} + \frac{dc}{dt}\right)$$

$$\bar{v}_M - D_M\left(\frac{d\alpha}{dt} + 2\frac{dc}{d(2\eta)}\frac{d\eta}{dt}\right) \tag{21}$$

Using

$\frac{d\alpha}{dt} = \bar{v}_\alpha$ and $\frac{d\eta}{dt} = \bar{v}_\eta$ we get

$$v_M = \bar{v}_M - \left(\bar{v}_\alpha + 2\frac{dc}{d(2\eta)}\bar{v}_\eta\right)D_M \tag{22}$$

The derivative of *c* has the same structure as (3.3) and (4.1) with *r* changed to *s* and *a* to 2η:

$$\frac{dc}{d(2\eta)} = s(R\cos 2\eta + s)/(R^2 + 2Rs\cos 2\eta + s^2) \tag{23}$$

and is to be evaluated for 2η = 0. This results in

$$\frac{dc}{d(2\eta)} = s/(R+s) = Q \approx 0.1468 \tag{24}$$

Inserting this in equation (22) we get

$$v_M = \bar{v}_M - \bar{v}_\alpha(1 + 2Q\bar{v}_\eta/\bar{v}_\alpha)D_M = \bar{v}_M - \bar{v}_\alpha F D_M \tag{25}$$

where the factor $F = 1 + 2Q\bar{v}_\eta/\bar{v}_\alpha$ has the numerical value 1.2740.

If we compare this expression with (15) we see that as the factor *F* > 1 it means that the spread of the lunar velocities from the mean velocity will be greater in the final lunar model than in the first model. Effectively the lunar mean velocity in anomaly is corrected by the factor *F*.

Peurbach would not use the analytical approach above but by the analysis of his other tables it is clear that he used the final lunar model. It is hard to know what methods he used, he could simply have used existing tables based on equivalent models or more probably have used rules given by Regiomontanus (Goldstein, 1992: 8; Regiomontanus, 1496: VI.4) which are essentially equivalent to (25). The factor *Q* can be determined by taking the difference of tabular values of *c* for 0° and 1°, 9′ and divide by 1° resulting in 0.15, very nearly the analytical value of *Q* and gives a value of *F* = 1.28 and $\bar{v}_\alpha F = 41;49'/\mathrm{hour}$. As before the derivative D_M can be approximated by

$$D_M(\alpha) \approx (q_M(\alpha + 1°) - q_M(\alpha))/1° \tag{26}$$

This results in Regiomontanus' rules for the Sun and the Moon:

> Consider the equation of the Sun's motion at a time for which you wish to have the true motion of the Sun in an hour. Note the difference between this equation and the argument of the equation of the next degree greater. From this take the proportional part of 2′ 28″ to 60′ which you subtract from 2′ 28″ if the argument of the sun is less than 93°, add to the same if more, up to as 180° and the true motion of the Sun in an hour will come out ...
>
> Likewise, let it be done on the Moon. Consider the equation of the given argument for the Moon. Also, the equation of the argument is one degree greater. From the difference of these take the proportional part, the proportion of 41′ 49″ to 60′, subtract this from 32′ 56″ if the argument was less than 45° or add if greater than 45° [45° is an error for 95°] to 180°. (Regiomontanus, VI, 4; my English translation).

Table 1: True syzygy lunar velocity in arc minutes per hour.

Anomaly	Velocity (′/hour)	(Goldstein, 1992)
0	29;37	29;37,13
30	29;59	29;59,30
60	31; 1	31; 2,54
90	32;38	32;39,44
120	34;34	34;35,23
150	36;13	36;13,37
180	36;53	36;53,20

Mathematically this can be written:

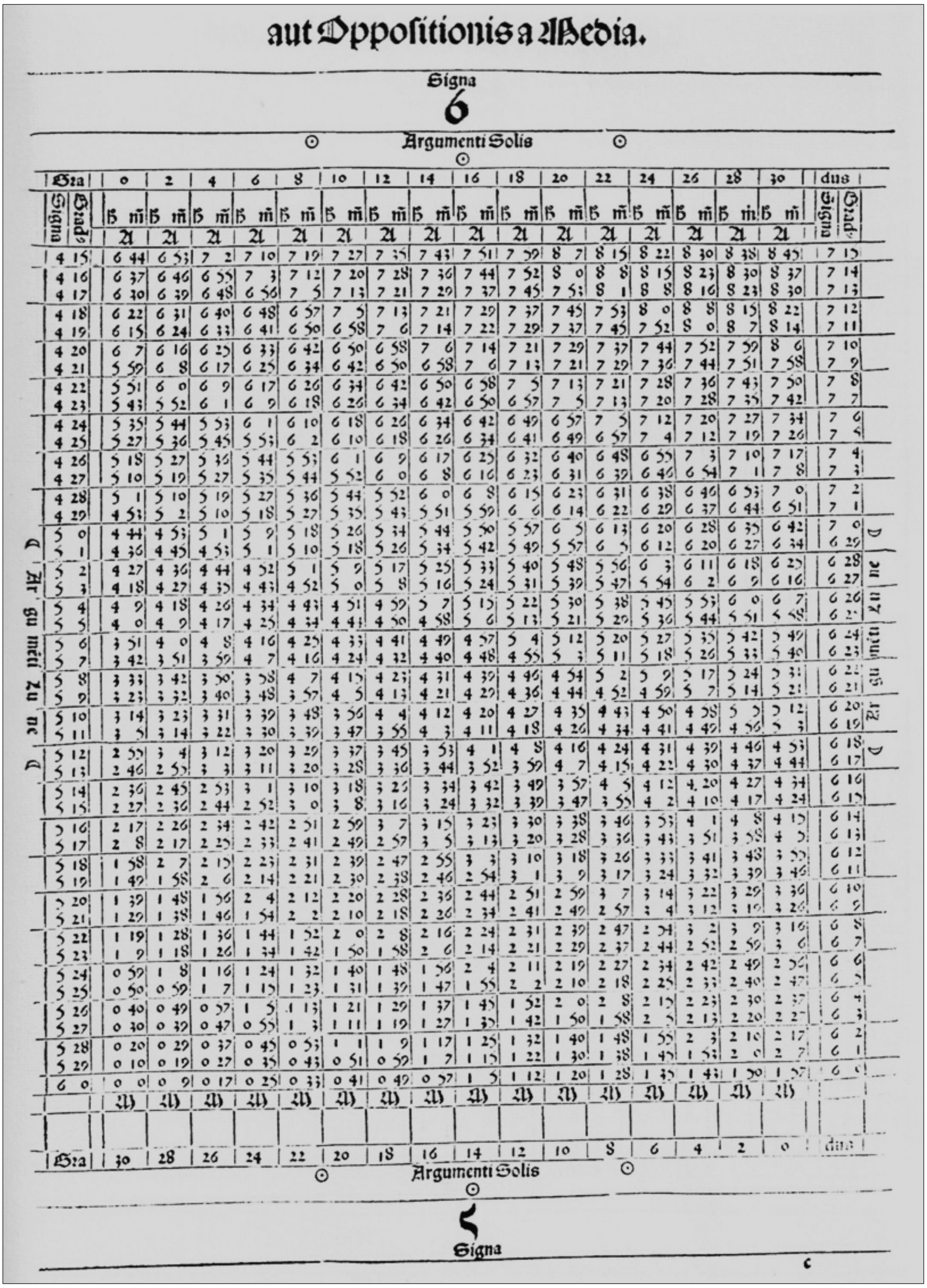
$$v_M = 2;28' - 2;28'D_S \tag{27}$$

$$v_M = 32;56' - 41;49'D_M \tag{28}$$

Goldstein (1992: 12) computes a lunar velocity table using Alfonsine parameters for the mean velocities, *Q* = 0.15, and the relation (28). The result is very close to the analytical result, see Table 1. He also finds an almost identical 14th century tables by John of Genoa and John of Lignères (Goldstein, 1992: Table 3).

5 FINDING THE TRUE SYZYGY

Much work in the Middle Ages was devoted in finding the true syzygy given the time of the mean syzygy. Stöffler (1514), Münster (1536) and Apian (1540) all presented paper instruments intended to reduce the work of finding the true syzygy but resulting in less accuracy. Peurbach presents 48 pages of double entry tables with the anomalies of the Moon and the Sun as arguments (Figure 3) and with a total of 32 400



aut Oppositionis a Media.

Signa
6

☉ Argumenti Solis ☉
☉

☾ Argumenti Lune ☾

Gra		0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	dus	
Signa	Gradꝰ	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	Signa	Gradꝰ
		A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A		
4	15	6 44	6 53	7 2	7 10	7 19	7 27	7 35	7 43	7 51	7 59	8 7	8 15	8 22	8 30	8 38	8 45	7	15
4	16	6 37	6 46	6 55	7 3	7 12	7 20	7 28	7 36	7 44	7 52	8 0	8 8	8 15	8 23	8 30	8 37	7	14
4	17	6 30	6 39	6 48	6 56	7 5	7 13	7 21	7 29	7 37	7 45	7 53	8 1	8 8	8 16	8 23	8 30	7	13
4	18	6 22	6 31	6 40	6 48	6 57	7 5	7 13	7 21	7 29	7 37	7 45	7 53	8 0	8 8	8 15	8 22	7	12
4	19	6 15	6 24	6 33	6 41	6 50	6 58	7 6	7 14	7 22	7 29	7 37	7 45	7 52	8 0	8 7	8 14	7	11
4	20	6 7	6 16	6 25	6 33	6 42	6 50	6 58	7 6	7 14	7 21	7 29	7 37	7 44	7 52	7 59	8 6	7	10
4	21	5 59	6 8	6 17	6 25	6 34	6 42	6 50	6 58	7 6	7 13	7 21	7 29	7 36	7 44	7 51	7 58	7	9
4	22	5 51	6 0	6 9	6 17	6 26	6 34	6 42	6 50	6 58	7 5	7 13	7 21	7 28	7 36	7 43	7 50	7	8
4	23	5 43	5 52	6 1	6 9	6 18	6 26	6 34	6 42	6 50	6 57	7 5	7 13	7 20	7 28	7 35	7 42	7	7
4	24	5 35	5 44	5 53	6 1	6 10	6 18	6 26	6 34	6 42	6 49	6 57	7 5	7 12	7 20	7 27	7 34	7	6
4	25	5 27	5 36	5 45	5 53	6 2	6 10	6 18	6 26	6 34	6 41	6 49	6 57	7 4	7 12	7 19	7 26	7	5
4	26	5 18	5 27	5 35	5 44	5 53	6 1	6 9	6 17	6 25	6 32	6 40	6 48	6 55	7 3	7 10	7 17	7	4
4	27	5 10	5 19	5 27	5 35	5 44	5 52	6 0	6 8	6 16	6 23	6 31	6 39	6 46	6 54	7 1	7 8	7	3
4	28	5 1	5 10	5 19	5 27	5 36	5 44	5 52	6 0	6 8	6 15	6 23	6 31	6 38	6 46	6 53	7 0	7	2
4	29	4 53	5 2	5 10	5 18	5 27	5 35	5 43	5 51	5 59	6 6	6 14	6 22	6 29	6 37	6 44	6 51	7	1
5	0	4 44	4 53	5 1	5 9	5 18	5 26	5 34	5 44	5 50	5 57	6 5	6 13	6 20	6 28	6 35	6 42	7	0
5	1	4 36	4 45	4 53	5 1	5 10	5 18	5 26	5 34	5 42	5 49	5 57	6 5	6 12	6 20	6 27	6 34	6	29
5	2	4 27	4 36	4 44	4 52	5 1	5 9	5 17	5 25	5 33	5 40	5 48	5 56	6 3	6 11	6 18	6 25	6	28
5	3	4 18	4 27	4 35	4 43	4 52	5 0	5 8	5 16	5 24	5 31	5 39	5 47	5 54	6 2	6 9	6 16	6	27
5	4	4 9	4 18	4 26	4 34	4 43	4 51	4 59	5 7	5 15	5 22	5 30	5 38	5 45	5 53	6 0	6 7	6	26
5	5	4 0	4 9	4 17	4 25	4 34	4 42	4 50	4 58	5 6	5 13	5 21	5 29	5 36	5 44	5 51	5 58	6	25
5	6	3 51	4 0	4 8	4 16	4 25	4 33	4 41	4 49	4 57	5 4	5 12	5 20	5 27	5 35	5 42	5 49	6	24
5	7	3 42	3 51	3 59	4 7	4 16	4 24	4 32	4 40	4 48	4 55	5 3	5 11	5 18	5 26	5 33	5 40	6	23
5	8	3 33	3 42	3 50	3 58	4 7	4 15	4 23	4 31	4 39	4 46	4 54	5 2	5 9	5 17	5 24	5 31	6	22
5	9	3 23	3 32	3 40	3 48	3 57	4 5	4 13	4 21	4 29	4 36	4 44	4 52	4 59	5 7	5 14	5 21	6	21
5	10	3 14	3 23	3 31	3 39	3 48	3 56	4 4	4 12	4 20	4 27	4 35	4 43	4 50	4 58	5 5	5 12	6	20
5	11	3 5	3 14	3 22	3 30	3 39	3 47	3 55	4 3	4 11	4 18	4 26	4 34	4 41	4 49	4 56	5 3	6	19
5	12	2 55	3 4	3 12	3 20	3 29	3 37	3 45	3 53	4 1	4 8	4 16	4 24	4 31	4 39	4 46	4 53	6	18
5	13	2 46	2 55	3 3	3 11	3 20	3 28	3 36	3 44	3 52	3 59	4 7	4 15	4 22	4 30	4 37	4 44	6	17
5	14	2 36	2 45	2 53	3 1	3 10	3 18	3 26	3 34	3 42	3 49	3 57	4 5	4 12	4 20	4 27	4 34	6	16
5	15	2 27	2 36	2 44	2 52	3 0	3 8	3 16	3 24	3 32	3 39	3 47	3 55	4 2	4 10	4 17	4 24	6	15
5	16	2 17	2 26	2 34	2 42	2 51	2 59	3 7	3 15	3 23	3 30	3 38	3 46	3 53	4 1	4 8	4 15	6	14
5	17	2 8	2 17	2 25	2 33	2 41	2 49	2 57	3 5	3 13	3 20	3 28	3 36	3 43	3 51	3 58	4 5	6	13
5	18	1 58	2 7	2 15	2 23	2 31	2 39	2 47	2 55	3 3	3 10	3 18	3 26	3 33	3 41	3 48	3 55	6	12
5	19	1 49	1 58	2 6	2 14	2 21	2 30	2 38	2 46	2 54	3 1	3 9	3 17	3 24	3 32	3 39	3 46	6	11
5	20	1 39	1 48	1 56	2 4	2 12	2 20	2 28	2 36	2 44	2 51	2 59	3 7	3 14	3 22	3 29	3 36	6	10
5	21	1 29	1 38	1 46	1 54	2 2	2 10	2 18	2 26	2 34	2 41	2 49	2 57	3 4	3 12	3 19	3 26	6	9
5	22	1 19	1 28	1 36	1 44	1 52	2 0	2 8	2 16	2 24	2 31	2 39	2 47	2 54	3 2	3 9	3 16	6	8
5	23	1 9	1 18	1 26	1 34	1 42	1 50	1 58	2 6	2 14	2 21	2 29	2 37	2 44	2 52	2 59	3 6	6	7
5	24	0 59	1 8	1 16	1 24	1 32	1 40	1 48	1 56	2 4	2 11	2 19	2 27	2 34	2 42	2 49	2 56	6	6
5	25	0 50	0 59	1 7	1 15	1 23	1 31	1 39	1 47	1 55	2 2	2 10	2 18	2 25	2 33	2 40	2 47	6	5
5	26	0 40	0 49	0 57	1 5	1 13	1 21	1 29	1 37	1 45	1 52	2 0	2 8	2 15	2 23	2 30	2 37	6	4
5	27	0 30	0 39	0 47	0 55	1 3	1 11	1 19	1 27	1 35	1 42	1 50	1 58	2 5	2 13	2 20	2 27	6	3
5	28	0 20	0 29	0 37	0 45	0 53	1 1	1 9	1 17	1 25	1 32	1 40	1 48	1 55	2 3	2 10	2 17	6	2
5	29	0 10	0 19	0 27	0 35	0 43	0 51	0 59	1 7	1 15	1 22	1 30	1 38	1 45	1 53	2 0	2 7	6	1
6	0	0 0	0 9	0 17	0 25	0 33	0 41	0 49	0 57	1 5	1 12	1 20	1 28	1 35	1 43	1 50	1 57	6	0
		M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M		
Gra		30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	dus	

☉ Argumenti Solis ☉
☉

5
Signa

c

Figure 3: A page from the true syzygy table (after Peurbach, 1514.

entries.

As shown by Kremer (2021) Peurbach's tables are based on the *Tabulae permanentes* by John of Murs but with the solar argument expanded, probably by linear interpolation, from 6 to 2 degrees and the intervals of lunar arguments from 6 to 1 degrees. The algorithm used for finding the time difference ΔT between true and mean syzygy is described by the equations

$$\Delta T = \left(q_M(\bar{\alpha}_M) - q_S(\bar{\alpha}_S)\right) / \left(v_M(\bar{\alpha}_{corr}) - v_S(\bar{\alpha}_S)\right) \qquad (29)$$

with $\bar{\alpha}_{corr} = \bar{\alpha}_M + \frac{13\left(q_M(\bar{\alpha}_M) - q_S(\bar{\alpha}_S)\right)}{24}$

where q_M and q_S are the PAT lunar and solar equations and v_M and v_S the lunar and solar velocities of John of Murs, essentially taken from John of Genoa's tables (Goldstein, 1992)

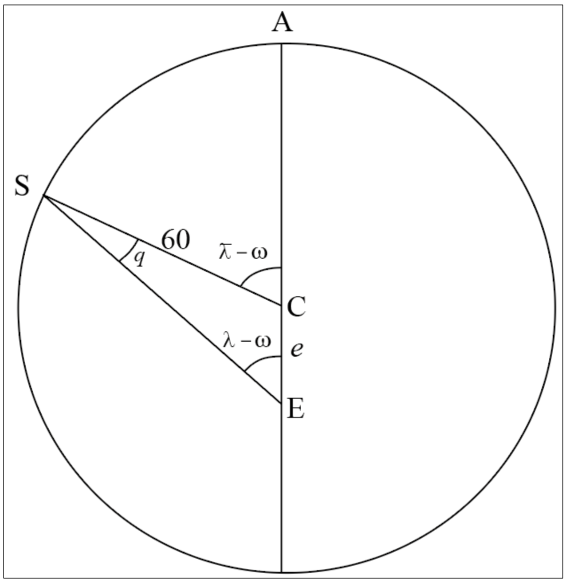


Figure 4: The solar equation (L. Gislén).

and $\bar{\alpha}_M$ and $\bar{\alpha}_S$ are the mean anomalies of the Moon and the Sun at the time of mean syzygy. A recalculation using this formula reproduces Peurbach's tables very accurately with some deviations of the order of one minute.

6 THE EQUATION OF TIME

Before proceeding with the eclipse calculation, the true syzygy time has to be corrected by the equation of time. The equation of time is the difference between apparent solar time, the time shown by a sundial and the linear time used for astronomical computations. It is given in a table, *Tabula Equationis dierum novissime constituta presupponens Augem solis in principio Cancri et declinationem Almeonis,* in minutes and seconds for every solar degree from 0° to 360° divided into columns of 30° for the ecliptic signs. It is to be entered with the true solar longitude.

The formula for calculating the equation of time as a function of the true solar longitude is (after van Dalen,1993: 99):

$$E_h(\lambda) = \frac{1}{D}(\lambda + q(\lambda) - a(\lambda) + C) \qquad (30)$$

D is a constant to convert from angle to time and *D* has the value 0;15 = 0.25. $q(\lambda)$ is the solar equation with the true solar longitude as argument and given by

$$\sin q(\lambda) = \frac{e}{60}\sin(\lambda - \omega) \qquad (31)$$

where *e* is the solar eccentricity and ω is the longitude of the solar apogee. $a(\lambda)$ is the right ascension function and depends on the obliquity of the ecliptic. *C* is a constant to make the equation of time additive. Equation (31) follows directly by applying the trigonometric sine theorem to the triangle CES in Figure 4. A is the solar apogee, S the Sun and E the position of the observer. The angle ACS is the mean anomaly, the angle AES the true anomaly.

A best parameter fit to Peurbach's table of the equation of time using van Dalen's Gauss–Newton method gives the obliquity of the ecliptic as 23;34° (*declinationem Almenonis*), 90° (*Augem solis in principio Cancer*) for the solar apogee, confirming what is written in the head of the table, and 2;16 for the solar eccentricity. The value for the obliquity is used in the subsequent analysis.

7 SOLAR AND LUNAR EQUATIONS

These tables are recalculations by Peurbach of the tables in the PAT. The table for the solar and lunar equations are given for arguments in steps of 10 arc minutes and differ in details from the corresponding tables in the PAT being more consistent with the analytical theory. Peurbach's solar equation table agrees apart from some typographical errors with the theoretical equation (10) with *e* = 2;16, 7 with an accuracy of less than 1 arc second. The lunar model parameters are given in Section 4.

8 PARALLAX

8.1 Theory

For a solar eclipse it is necessary to take into account parallax, the fact that the apparent positions of the Sun and the Moon are influenced by the observer not being in the center of the Earth. The lunar and solar parallaxes, P_M and P_S, are functions of the zenith distance *z* and the distances d_S and d_M of the Sun and the Moon from the center of the Earth and is by Figure 5 given by the equations

$$\sin P_M = \frac{r}{d_M}\sin z \qquad \sin P_S = \frac{r}{d_S}\sin z \qquad (32)$$

The effective parallax, *P,* is the difference of the lunar and solar parallaxes. As the parallax angles are small, it is a good approximation to write

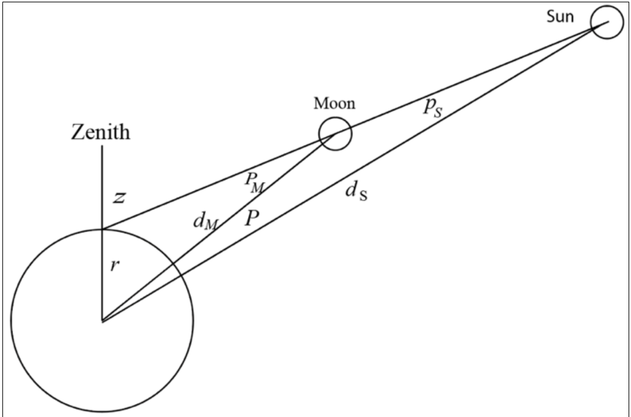


Figure 5 (left): Lunar and solar parallax (after Kennedy, 1956).

Figure 6 (right): Parallax in longitude and latitude (L. Gislén).

$$P \approx \left(\frac{r}{d_M} - \frac{r}{d_S}\right) \sin z \qquad (33)$$

This parallax is always directed toward the horizon. The horizontal parallax, P_0, the parallax when z = 90°, depends on the distances of the Moon and and the Sun. The variation with the distance of the Sun is small and its mean distance is used. For the Moon the distance depends on the anomaly or the angular distance of the Moon from its apogee. In the Alfonsine model used by Peurbach, the ratio between the maximum and minimum syzygy distances of the Moon is given by

$$(R + r)/(R - r) \approx 1.188 \qquad (34)$$

with R = 60 and r = 5;10. The general formula for the distance r at syzygy is a function of the anomaly a:

$$\rho = \sqrt{R^2 + 2Rr\cos\alpha + r^2} \qquad (35)$$

The horizontal parallax used by Peurbach can be derived from his parallax tables as is shown in the next Section.

The parallax is projected into two components, the parallax in longitude, P_λ, along the ecliptic that will change the time of the central eclipse and the perpendicular parallax in latitude, P_β that will affect the lunar latitude. The projections are determined by an angle γ, see Figure 6.

In Figure 6, z is the zenith distance of the Moon, M. N is the nonagesimal, the highest point of the ecliptic which is located 90° away along the ecliptic from the ascendant A, the rising sign which is the crossing between the ecliptic and the Eastern horizon. The zenith distance of the nonagesimal is z_N. At the nonagesimal the vertical great circle from the zenith through the nonagesimal is perpendicular to the ecliptic. γ is the angle between the ecliptic and a vertical great circle from the zenith through the Moon.

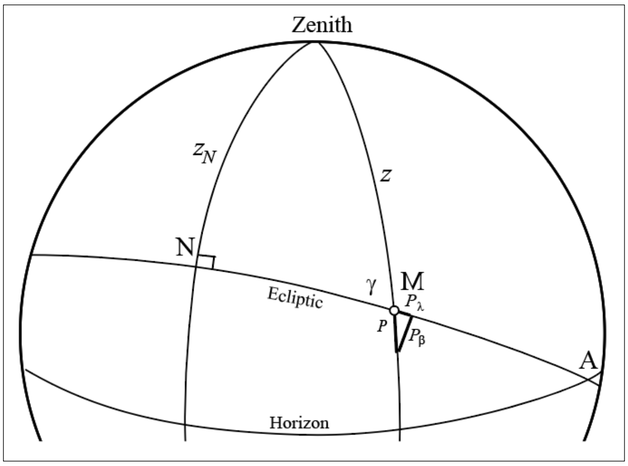
The longitude of the ascendant, λ_A, is a function of the solar ecliptic longitude, the observer's geographical latitude, and the hour angle of the Sun. It can be calculated analytically (Meeus, 1998: 99) but Peurbach would have used contemporary tables (Chabás and Goldstein, 2009: 107).

Once this longitude is known the longitude of the nonagesimal, λ_N, is given by

$$\lambda_N = \lambda_A - 90° \qquad (36)$$

The angle γ can then now be computed using the sine theorem of spherical trigonometry

$$\sin\gamma = \sin z_N \,/\, \sin z \qquad (37)$$



An alternative equivalent relation was known already to Menelaus of Alexandria (ca. 70–140 CE), and used by Ptolemy (Pedersen, 2010, 76).

Zenith distances z can be computed from the formula

$$\cos z = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos H \qquad (38)$$

where φ is the geographical latitude of the observer, δ the declination and H the hour angle of the planet. The hour angle is the time from noon in angular measure where 15° corresponds to one hour, negative before noon and positive after. The declination can be computed from the longitude λ by the formula

$$\sin\delta = \sin\varepsilon \sin\lambda \qquad (39)$$

where ε = 23;34° is the obliquity of the ecliptic.

The zenith distance of the nonagesimal, z_N, can be computed using the longitude λ_N of the nonagesimal and taking into account the different hour angle of the nonagesimal. This angle can be computed from the difference in right ascensions α and α_N of the Moon/Sun and the nonagesimal. The right ascensions, α and α_N, are computed from the respective longitudes by the formula

$$\tan a = \cos\varepsilon \tan\lambda \qquad (40)$$

where e is the obliquity of the ecliptic.

Tables of declination and right ascension are common in astronomical tables of the time (Chabás and Goldstein, 2012: 23, 26).

The hour angle H_N of the nonagesimal is then computed from the solar hour angle and the difference between the solar and nonagesimal right ascensions:

$$H_N = H + \alpha - \alpha_N \qquad (41)$$

With these formulas the angle γ can be calculated and then the component parallaxes

$$P_\lambda = P\cos\gamma \qquad (42)$$
$$P_\beta = P\sin\gamma \qquad (43)$$

Here the small spherical triangle in Figure 6 is considered as plane which considering the small angles is a good approximation. The longitudinal parallax is always directed away from the nonagesimal.

Finally, the true velocity of the Moon relative to the Sun, the elongation velocity needs to be known to convert the longitudinal parallax into time unit. This velocity is, as shown in the previous Section, a function of the anomaly of the Moon, the variation with the solar anomaly is very small and can be neglected.

Using this mathematical approach it is now possible to recalculate the two components of parallax using as input the geographical latitude of the observer, the ecliptic longitude of the Moon/Sun, the anomaly of the Moon, and the hour angle of the Moon/Sun. The angular parallax in longitude is then divided by the true elongation velocity to generate the parallax correction at the time of he eclipse. This correction is algebraically added to the original hour angle and the calculation is repeated several times until the correction does not change. Such an iteration process is documented by Al-Battānī (Nallino, 1903: 277–278). By this iteration a transcendental equation of the type

$$x = f(\alpha + x) \qquad (44)$$

is solved numerically. A similar type of equation and procedure appears in Indian and Southeast Asian ancient astronomy and is analyzed by Neugebauer (1962: 121) and Kennedy (1956: 51).

8.2 Parallax Tables

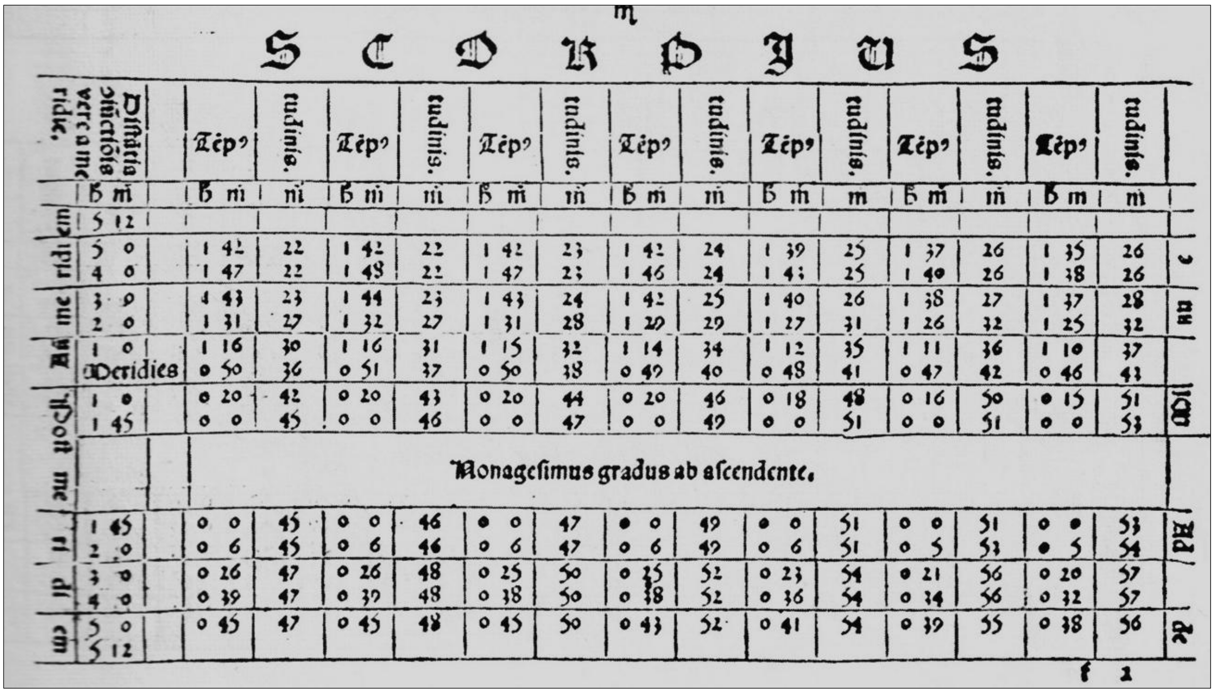
These tables are given by Peurbach for *Clima Sextum* and *Clima Septimum*. Each climate is divided into twelve tables corresponding to the twelve signs of the ecliptic longitude of the Sun and headed by the names of these signs. Each of the parallax tables has entries for the anomaly of the Moon in whole signs and takes care of influence of the different distances and velocities of the Moon. Each lunar anomaly entry has three columns. The first column is the input for the hour angles before and after noon and headed *Distantiae Coniunctionis*. These times have before been corrected by the equation of time. The second column is headed by *Tempus* and shows the longitudinal parallax in hours and minutes to arrive at the the corrected time of the eclipse. The third column, *Diversitate Latitudines,* contains the parallax in lunar latitude in arc minutes and is always South (negative). There is an empty horizontal row around the middle of the table for the hour angle when the longitude of the nonagesimal is equal to the longitude of the Moon. For times earlier than this time the longitudinal parallax is negative, for times later it is positive. At the nonagesimal the parallax is perpendicular to the ecliptic and the longitudinal parallax is zero. Noon is also marked specially (*Meridies*). *Clima Septimum* has a fourth column, *Motus Lunae,* that shows how far the Moon moves during the time shown in the second column.

There is a symmetry in the tables that can be used to spot errors and simplify calculations. The pairs Aries/Libra, Taurus/Virgo, Gemini/Leo, Scorpio/Pisces, and Sagittarius/Aquarius just have the sign of the hour angle switched. Also, for Cancer and Capricorn the data are symmetric under a switch of sign. Figure 7 shows the table for Scorpio in the sixth climate.

Similar tables are found in the Alfonsine Tables of 1483 and Al-Battānī but only taking account of the the dependence on the varying lunar anomaly by having tables for far and near positions of the Moon in the epicycle corresponding to anomalies of 0° and 180° respectively.

A way to extract the horizontal parallax from Peurbach's parallax tables is to use that the zenith distance of the nonagesimal, when the Sun is in Capricorn at noon, is equal to the sum of the geographical latitude and the declination of the nonagesimal, and that the parallax is then purely in lunar latitude. Assuming that Peurbach used the classical criteria that climate six has a maximal daylength of 15;30 hours and climate seven of 16;0 hours when the Sun is in Cancer, we can calculate that these climates correspond to geographical latitudes of 45;24° and 48;54° respectively. For climate six when the Sun and the nonagesimal is in Capricorn, Peurbach's tables give a maximum parallax in lunar latitude of 58 arc minutes and for climate seven it is 59 arc minutes. Using the geographical latitudes above we can compute the horizontal parallax to be about 62 arc minutes and the minimum parallax as 52 arc minutes. These are the values used in my recalculations.

With the theory in Section 8.1, a very satisfactory fit can be made to Peurbach's parallax tables for lunar anomalies 0° and 180°, the ones that were recalculated. There are some typographical errors in the tables, in the tables of Taurus for climate six the longitude parallax values for the hours before noon are clearly wrong and in the table of Aries for climate seven and lunar anomaly 180°, the latitude column is displaced one step vertically. These errors are the same as in the early manuscript. In the tables for climate seven in the printed 1514 edition, the order of the ecliptic signs is changed, beginning with Cancer and the tables for Aries and Pisces have their names switched and some data are missing. The residues be-



♏

Diſtātia ɔiūctiōis vera a meridie.	S		C		D		R		P		J		U		S	
	Tēp⁹	tudinis.	Tēp⁹	tudinis.	Tēp⁹	tudinis.	Tēp⁹	tudinis.	Tēp⁹	tudinis.	Tēp⁹	tudinis.	Tēp⁹	tudinis.
h m	h m	m	h m	m	h m	m	h m	m	h m	m	h m	m	h m	m
5 12														
5 0	1 42	22	1 42	22	1 42	23	1 42	24	1 39	25	1 37	26	1 35	26
4 0	1 47	22	1 48	22	1 47	23	1 46	24	1 43	25	1 40	26	1 38	26
3 0	1 43	23	1 44	23	1 43	24	1 42	25	1 40	26	1 38	27	1 37	28
2 0	1 31	27	1 32	27	1 31	28	1 29	29	1 27	31	1 26	32	1 25	32
1 0	1 16	30	1 16	31	1 15	32	1 14	34	1 12	35	1 11	36	1 10	37
Meridies	0 50	36	0 51	37	0 50	38	0 49	40	0 48	41	0 47	42	0 46	43
1 0	0 20	42	0 20	43	0 20	44	0 20	46	0 18	48	0 16	50	0 15	51
1 45	0 0	45	0 0	46	0 0	47	0 0	49	0 0	51	0 0	51	0 0	53
	Nonageſimus gradus ab aſcendente.													
1 45	0 0	45	0 0	46	0 0	47	0 0	49	0 0	51	0 0	51	0 0	53
2 0	0 6	45	0 6	46	0 6	47	0 6	49	0 6	51	0 5	53	0 5	54
3 0	0 26	47	0 26	48	0 25	50	0 35	52	0 23	54	0 21	56	0 20	57
4 0	0 39	47	0 39	48	0 38	50	0 38	52	0 36	54	0 34	56	0 32	57
5 0	0 45	47	0 45	48	0 45	50	0 43	52	0 41	54	0 39	55	0 38	56
5 12														

f 2

Figure 7: Parallax table for Scorpio, climate six. *Tabulae Eclypsium* 1514.

Table 2: Residues. SD is the standard deviation.

Climate No.	Climate 6				Climate 7			
Detals	Longitude/time minute		Latitude/arc minute		Longitude/time minute		Latitude/arc minute	
	Average		SD		Average		SD	
Ari 0	0.6	2.6	–1.4	0.5	–0.3	1.7	–0.1	0.9
Ari 180	0.4	2.3	–1.4	0.4	–1.0	0.9	0.2	0.7
Tau 0	3.9	7.1	–0.2	1.1	0.2	1.9	–0.5	0.5
Tau 180	3.4	6.3	–0.5	1.1	0.0	1.5	–0.6	0.6
Gemi 0	0.1	1.4	–0.9	0.8	0.0	1.4	–0.8	0.9
Gemi 180	0–3	1.4	–0.9	0.5	0.0	1.1	1.0	1.0
Can 0	1.7	2.4	–0.8	0.9	–0.4	0.9	–1.0	0.4
Can 180	1.9	2.3	–0.8	1.5	–0.4	1.1	–1.1	0.4
Sco 0	0.1	1.6	–1.2	0.8	0.7	2.1	–1.4	0.6
Sco 180	–1.6	2.8	–1.1	0.8	0.6	1.7	–1.5	0.5
Sag 0	0.4	1.9	1.2	1.4	0.0	1.6	–1.1	0.3
Sag 180	–1.0	1.6	–1.4	0.8	–1.0	1.2	–1.0	0.6
Cap 0	–0.4	2.7	–2.0	0.8	0.9	1.5	–0.9	0.5
Cap 180	–0.4	2.2	–1.0	0.6	0.3	0.5	–1.1	0.7

tween Peurbach and the analytical values are in general quite small, for the parallax time in longitude about 2 time minutes and for the parallax in latitude it is about 1 arc minute.

Table 2 shows residues of the tables for lunar argument 0 and 180. The errors in Taurus for climate six have larger residues (shown in red).

Peurbach gives values of the parallax for lunar anomalies being a multiple of 30°. This is certainly an improvement relative to earlier less complete tables which could introduce discrepancies in the parallax time of about five minutes and in the latitude parallax of up to 10′. The PAT gives parallax tables for all the climates from one to seven, but only climates five to seven would be useful in Europe.

9 TABLES FOR THE LATITUDE OF THE MOON

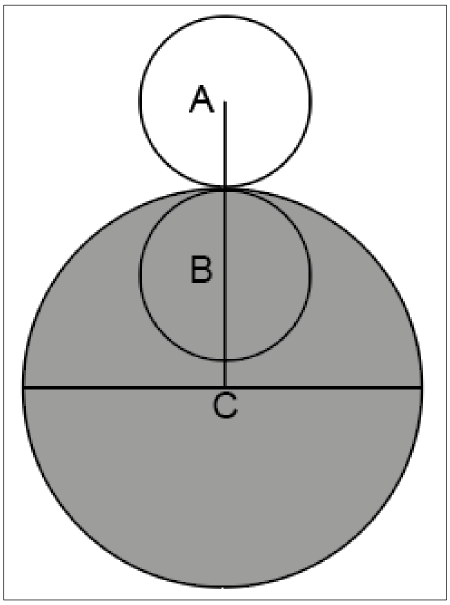
Peurbach’s table has entries for every 10 arc minute from 0° to 21° of the lunar argument of latitude, α_L, with this interval covering those latitudes that are possible in an eclipse. The latitude β is closely approximated by

$$\sin\beta = \sin 5° \sin\alpha_L \qquad (45)$$

10 ECLIPSE SIZE AND DURATION

10.1 Eclipse Geometry

In this section index U denotes the shadow and and P the object being shadowed. In a solar



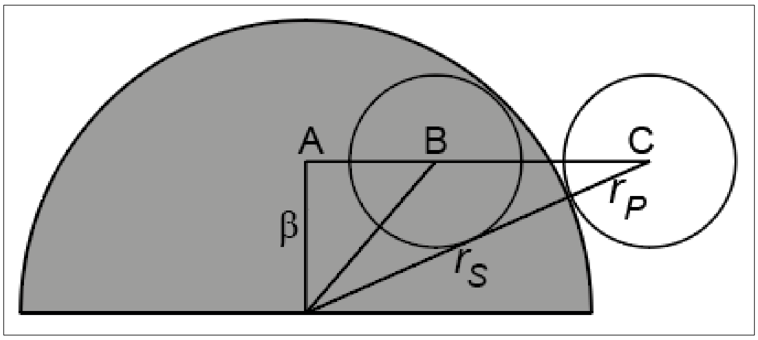


Figure 8 (left): With a latitude BC, the eclipse size will be 12. With AC it is zero (Lars Gislén).

Figure 9 (above): The angular distance AB converted to time corresponds to *Mora*, BC to *Casus*, and the angle AC to *Motus Lunae* (Lars Gislén).

eclipse, U is the Moon, and P the Sun. In a lunar eclipse U is the umbra, the shadow of the Earth, and P the Moon. The semi-diameters are denoted by *r*.

From Figures 8 and 9 we derive the following quantities:

The eclipse size, *Puncta,* is a linear function of the latitude *b*.

$$p = 6(r_U + r_P - \beta)/r_P \qquad (46)$$

It is normalized such that when $\beta = r_U - r_P$ the eclipse object is just totally obscured and the eclipse size is 12. Note that for a total eclipse, the size can be larger than 12.

The half duration of total obscurity, *Mora.* This is not computed for a solar eclipse and is zero for a partial eclipse

$$m = \sqrt{(r_U - r_P)^2 - \beta^2}/\Delta v, \qquad \beta \le r_U - r_P \qquad (47)$$

Δv is the difference between the true lunar and solar angular velocity, see sections 5 and 6.

The half duration of of the entire eclipse.

$$t = \sqrt{(r_U + r_P)^2 - \beta^2}/\Delta v \, \beta \le r_U + r_P \qquad (48)$$

The half duration of partial phase, *Casus*. For a solar eclipse *m* is assumed to be zero.

$$c = \begin{cases} t - m, & \beta \le r_U - r_P \\ t, \; r_U - r_P < \beta \le r_U + r_P \end{cases} \qquad (49)$$

The angle travelled by the Moon during half of the entire eclipse, *Motus Lunae*

$$M = tv_{Moon} \qquad (50)$$

10.2 Tables for Puncta, Casus, Mora, and Motus Lunae

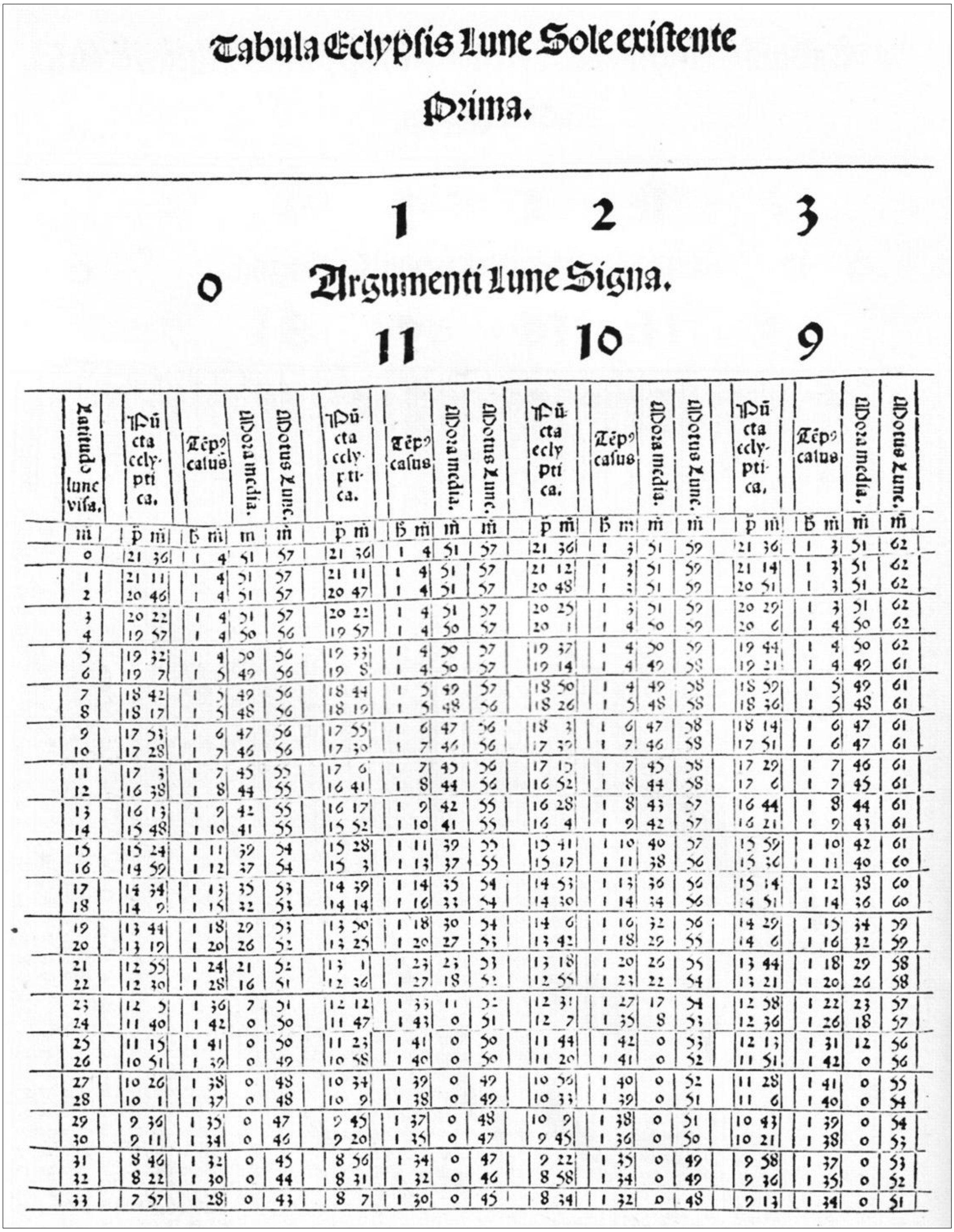
Using the formulas in the previous sections and Peurbach's table of semi-diameters of the Sun, the Moon and the umbra (see Section 11), the tables for the determination of eclipse size and duration were recalculated. There are three tables for each of solar and lunar eclipses called *Prima*, *Secunda*, and *Tertia*, standing for solar anomalies of 0°, 90°, and 180°. Each of these tables have entries of lunar anomaly being multiples of 30°. For solar eclipses the quantities tabulated as a function of the lunar latitude are the size of the eclipse (*Puncta*), the half duration of the eclipse (*Causa*) and the movement of the Moon during half of the eclipse (*Motus Lunae*). For a lunar eclipse also the time of the partial phase (*Mora*) is tabulated. The residues are in general very small and indicate only rounding errors in the tables, see Table 3, which shows the average and standard deviations of the total residues for the smallest unit, 1/60 for *Puncta*, one time minute for *Casus* and *Mora*, and one arc minute for *Motus Lunae*. Figure 10 shows part of the table Prima for the Moon.

Peurbach's tables were an improvement as compared with earlier tables, enabling interpolation for different solar and lunar distances. The biggest improvement, about five minutes in duration time, would be for small eclipses.

Peter Apian may have used Peurbach's tables when he constructed his instruments for lunar eclipses in his *Astronomicum Caesareum* (Gislén, 2016), although this is a speculation.

Table 3: Residues of smallest units.

	Average	Stand. Deviation
Sun Prima	0.00	0.37
Sun Secunda	0.01	0.32
Sun Tertia	0.08	0.38
Moon Prima	0.00	0.47
Moon Secunda	−0.03	0.43
Moon Tertia	0.00	0.55



Tabula Eclypſis Lune Sole exiſtente

Prima.

1 2 3

0 Argumenti Lune Signa.

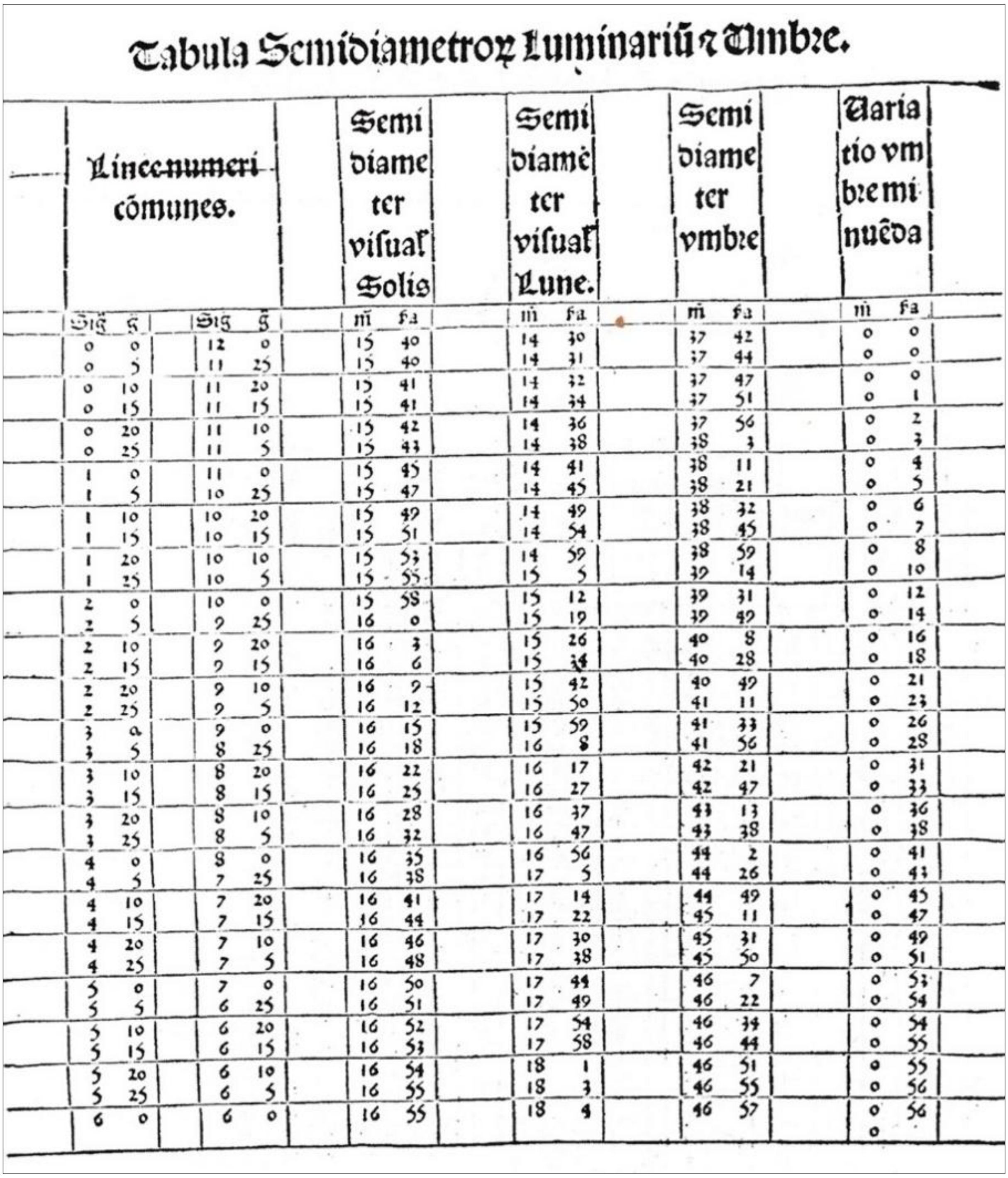
11 10 9

Latitudo lune viſa.	Pūcta eclyptica.	Tēpꝰ caſus	Mora media.	Motus Lune.	Pūcta eclyptica.	Tēpꝰ caſus	Mora media.	Motus Lune.	Pūcta eclyptica.	Tēpꝰ caſus	Mora media.	Motus Lune.	Pūcta eclyptica.	Tēpꝰ caſus	Mora media.	Motus Lune.
m	p m	h m	m	m	p m	h m	m	m	p m	h m	m	m	p m	h m	m	m
0	21 36	1 4	51	57	21 36	1 4	51	57	21 36	1 3	51	59	21 36	1 3	51	62
1	21 11	1 4	51	57	21 11	1 4	51	57	21 12	1 3	51	59	21 14	1 3	51	62
2	20 46	1 4	51	57	20 47	1 4	51	57	20 48	1 3	51	59	20 51	1 3	51	62
3	20 22	1 4	51	57	20 22	1 4	51	57	20 25	1 3	51	59	20 29	1 3	51	62
4	19 57	1 4	50	56	19 57	1 4	50	57	20 1	1 4	50	59	20 6	1 4	50	62
5	19 32	1 4	50	56	19 33	1 4	50	57	19 37	1 4	50	59	19 44	1 4	50	62
6	19 7	1 5	49	56	19 8	1 4	50	57	19 14	1 4	49	58	19 21	1 4	49	61
7	18 42	1 5	49	56	18 44	1 5	49	57	18 50	1 4	49	58	18 59	1 5	49	61
8	18 17	1 5	48	56	18 19	1 5	48	56	18 26	1 5	48	58	18 36	1 5	48	61
9	17 53	1 6	47	56	17 55	1 6	47	56	18 3	1 6	47	58	18 14	1 6	47	61
10	17 28	1 7	46	56	17 30	1 7	46	56	17 39	1 7	46	58	17 51	1 6	47	61
11	17 3	1 7	45	55	17 6	1 7	45	56	17 15	1 7	45	58	17 29	1 7	46	61
12	16 38	1 8	44	55	16 41	1 8	44	56	16 52	1 8	44	58	17 6	1 7	45	61
13	16 13	1 9	42	55	16 17	1 9	42	55	16 28	1 8	43	57	16 44	1 8	44	61
14	15 48	1 10	41	55	15 52	1 10	41	55	16 4	1 9	42	57	16 21	1 9	43	61
15	15 24	1 11	39	54	15 28	1 11	39	55	15 41	1 10	40	57	15 59	1 10	42	61
16	14 59	1 12	37	54	15 3	1 13	37	55	15 17	1 11	38	56	15 36	1 11	40	60
17	14 34	1 13	35	53	14 39	1 14	35	54	14 53	1 13	36	56	15 14	1 12	38	60
18	14 9	1 15	32	53	14 14	1 16	33	54	14 30	1 14	34	56	14 51	1 14	36	60
19	13 44	1 18	29	53	13 50	1 18	30	54	14 6	1 16	32	56	14 29	1 15	34	59
20	13 19	1 20	26	52	13 25	1 20	27	53	13 42	1 18	29	55	14 6	1 16	32	59
21	12 55	1 24	21	52	13 1	1 23	23	53	13 18	1 20	26	55	13 44	1 18	29	58
22	12 30	1 28	16	51	12 36	1 27	18	52	12 55	1 23	22	54	13 21	1 20	26	58
23	12 5	1 36	7	51	12 12	1 33	11	52	12 31	1 27	17	54	12 58	1 22	23	57
24	11 40	1 42	0	50	11 47	1 43	0	51	12 7	1 35	8	53	12 36	1 26	18	57
25	11 15	1 41	0	50	11 23	1 41	0	50	11 44	1 42	0	53	12 13	1 31	12	56
26	10 51	1 39	0	49	10 58	1 40	0	50	11 20	1 41	0	52	11 51	1 42	0	56
27	10 26	1 38	0	48	10 34	1 39	0	49	10 56	1 40	0	52	11 28	1 41	0	55
28	10 1	1 37	0	48	10 9	1 38	0	49	10 33	1 39	0	51	11 6	1 40	0	54
29	9 36	1 35	0	47	9 45	1 37	0	48	10 9	1 38	0	51	10 43	1 39	0	54
30	9 11	1 34	0	46	9 20	1 35	0	47	9 45	1 36	0	50	10 21	1 38	0	53
31	8 46	1 32	0	45	8 56	1 34	0	47	9 22	1 35	0	49	9 58	1 37	0	53
32	8 22	1 30	0	44	8 31	1 32	0	46	8 58	1 34	0	49	9 36	1 35	0	52
33	7 57	1 28	0	43	8 7	1 30	0	45	8 34	1 32	0	48	9 13	1 34	0	51

Figure 10: Part of an eclipse table for the Moon (after Puerbach, 1514).

11 SEMI-DIAMETERS

These are tables for the semi-diameters of the Sun, the Moon, the umbra, and a correction to the umbra called *Variatio umbrae minuenda* due to the varying distance of the Sun (see Figure 11). There are no similar tables in the original PAT although the 1524 edition has identical tables as a later addition. Peurbach's tables also differ from corresponding tables in for example Al-Khwārizmī (Gislén, 2025) and are based on tables given by John of Gmunden (Goldstein and Chabás, 2024).



Tabula Semidiametroꝝ Luminariū ꝛ Umbꝛe.

Lineenumeri cōmunes.				Semi diameter viſual Solis		Semi diamē ter viſual Lune.		Semi diame ter vmbꝛe		Variatio vmbꝛe minuēda	
Sig	G	Sig	G	m̄	ſa	m̄	ſa	m̄	ſa	m̄	ſa
0	0	12	0	15	40	14	30	37	42	0	0
0	5	11	25	15	40	14	31	37	44	0	0
0	10	11	20	15	41	14	32	37	47	0	0
0	15	11	15	15	41	14	34	37	51	0	1
0	20	11	10	15	42	14	36	37	56	0	2
0	25	11	5	15	43	14	38	38	3	0	3
1	0	11	0	15	45	14	41	38	11	0	4
1	5	10	25	15	47	14	45	38	21	0	5
1	10	10	20	15	49	14	49	38	32	0	6
1	15	10	15	15	51	14	54	38	45	0	7
1	20	10	10	15	53	14	59	38	59	0	8
1	25	10	5	15	55	15	5	39	14	0	10
2	0	10	0	15	58	15	12	39	31	0	12
2	5	9	25	16	0	15	19	39	49	0	14
2	10	9	20	16	3	15	26	40	8	0	16
2	15	9	15	16	6	15	34	40	28	0	18
2	20	9	10	16	9	15	42	40	49	0	21
2	25	9	5	16	12	15	50	41	11	0	23
3	a	9	0	16	15	15	59	41	33	0	26
3	5	8	25	16	18	16	8	41	56	0	28
3	10	8	20	16	22	16	17	42	21	0	31
3	15	8	15	16	25	16	27	42	47	0	33
3	20	8	10	16	28	16	37	43	13	0	36
3	25	8	5	16	32	16	47	43	38	0	38
4	0	8	0	16	35	16	56	44	2	0	41
4	5	7	25	16	38	17	5	44	26	0	43
4	10	7	20	16	41	17	14	44	49	0	45
4	15	7	15	16	44	17	22	45	11	0	47
4	20	7	10	16	46	17	30	45	31	0	49
4	25	7	5	16	48	17	38	45	50	0	51
5	0	7	0	16	50	17	44	46	7	0	53
5	5	6	25	16	51	17	49	46	22	0	54
5	10	6	20	16	52	17	54	46	34	0	54
5	15	6	15	16	53	17	58	46	44	0	55
5	20	6	10	16	54	18	1	46	51	0	55
5	25	6	5	16	55	18	3	46	55	0	56
6	0	6	0	16	55	18	4	46	57	0 0	56

Figure 11: Semi-diameters (after Puerbach, 1514).

12 PEURBACH'S CALCULATION OF THE 1460 JULY SOLAR ECLIPSE

In conclusion, details of the calculation of the solar eclipse on 18 July in *Tabulae Eclypsium* are shown as an illustration of the work involved even with Peurbach's rather user-friendly tables. Time is measured from noon. Table 4 shows the calculation of the mean syzygy time.

The tables for the mean longitudes give:

- Mean eclipse time: 1460 July 18 1;1,51 hours
- Mean longitude of the Sun/Moon: 125;26,48°
- Mean anomaly of the Moon: 211;7,4°
- Mean argument of latitude: 7;14,32°
- Longitude of solar apogee: 90; 48,53°
- Mean anomaly of the Sun: 34,37,55°

Using the anomalies of the Sun and the Moon, the correction to true syzygy, interpolated from the double-entry tables of conjunction/

opposition, is –6;59 hours and the corrected eclipse time is 17 July, 18;3 hours rounded to minutes.

The longitudes and anomalies of the Sun and the Moon are then corrected for this change in time using their respective mean velocities:

- Corrected mean longitude of the Sun: 125;9,36°
- Corrected mean longitude of the Moon: 121;36,48°
- Corrected mean argument of latitude: 3;23,34° (not in text);
- Corrected mean anomaly of the Moon: 207;19,2°
- Corrected mean anomaly of the Sun: 34;20,43°

Using the tables of equations, the true solar and lunar longitudes are computed. The solar anomaly table gives a solar equation of –1;11, 4° and a true solar longitude: 123;58,32°.

The elongation of the mean Moon from the mean Sun is 356;27,12°. Doubled this to 352;54,24° and entering the the table for the lunar equation of center gives –1;3,0° resulting in a true lunar anomaly of 206;16°. This as argument in the table of lunar argument generates a lunar equation of +2;21,45° and a true lunar longitude of 123;58,33°.

The closeness of the computed solar and lunar true longitudes is a verification of the correctness of the calculation procedure. The lunar node (retrograde) is in –241;46,47°. The complement is 118;13,13° and subtracted from the true lunar longitude gives the true argument of latitude 5;45,20°. The longitude of the node is introduced without any explanation but the value is exactly the one in the PAT. The lunar equation is added to the corrected argument of latitude gives 5;45,19°.

The true solar longitude gives an equation of time of 11;52 minutes of time which is rounded to 12 minutes. Thus, the equated time of the eclipse will be 17 July 18;15 hours. This is 5;45 hours before noon.

The next step is to correct for parallax. Vienna is assumed to be in climate seven and the Sun is in the beginning of sign 4 or Leo, the lunar anomaly is close to sign 7. Interpolating between the the parallax tables, Peurbach gets a correction in time of –1;26 hours and in lunar latitude a correction of –37 arc minutes. This results in the apparent eclipse time of 18 July 16;49 hours. The correction in time will decrease the argument of latitude to 4;53,44° and the table for lunar latitude then gives a lunar latitude of 26 arc minutes North which by the parallax –37 arc minutes is corrected to 11 arc minutes South.

Entering the solar eclipse table Prima (closest to the solar anomaly) with this latitude and a lunar anomaly of about 7 signs gives an eclipse size of 8;35 and a casus (half-duration of the eclipse) of 55 minutes. The beginning of the eclipse will be 15;54 hours after noon and the end 17;44 hours after noon. A modern calculation gives the local apparent central eclipse time 17;08 hours after noon, an eclipse size of 8;41, and a half-duration of 50 minutes. The eclipse will start about half an hour before sunrise, 35 minutes according to Peurbach.

During the half duration of 55 minutes the Moon moves 33 arc minutes. This will affect the argument of latitude by ±33 arc minutes and generate a ±3 arc minutes change in the apparent latitude of the Moon and the apparent latitude at the beginning of the eclipse will be 14 arc minutes and at the end 8 arc minutes (South).

The semi-diameter of the Moon using Peurbach's tables with the lunar anomaly as argument is 17;48 arc minutes. Using the anomaly of the Sun the corresponding solar semi-diameter is 15;47 arc minutes. The sum of the semi-diameters is 33:35 arc minutes. With these quantities and the lunar latitudes given, a sketch of the solar eclipse is drawn in Figure 12.

13 NOTES

1. The format a; b, c ... means a + b/60 + c/60/60 ...

Table 4: Mean syzygy calculation.

	Lunar Age			
	Day	Hour	Min	Sec
1440	07	08	38	27
19	28	20	12	06
1459	36	04	50	33
June	04	19	35	42
Age	41	00	26	15
Periods	59	01	28	06
Subtract Age	41	00	26	15
New Moon Date	18	01	01	51

14 ACKNOWLEDGEMENTS

I am grateful to Richard L. Kremer for pointing out the origin of Peurbach's tables for the true syzygy.

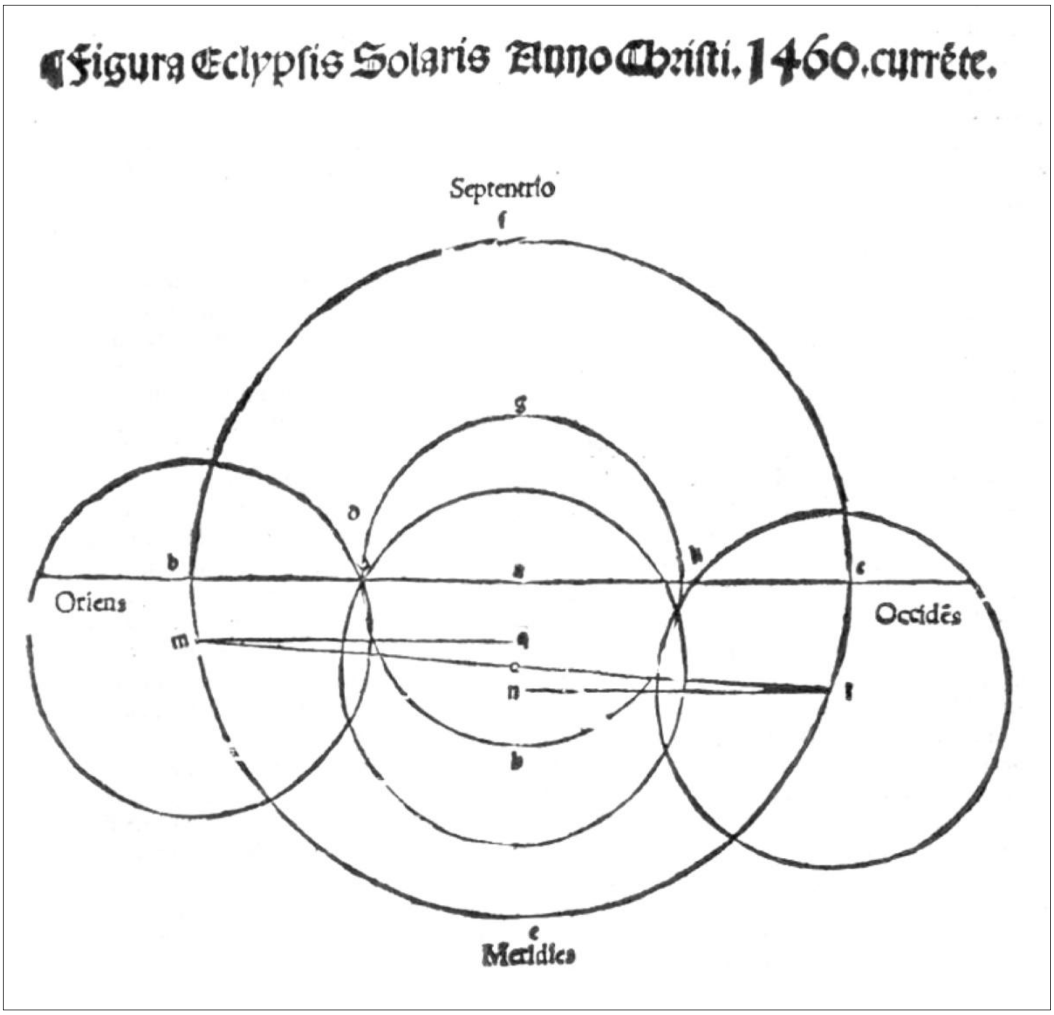


Figure 12: Peurbach's sketch of the solar eclipse of July 1460 (after Peurbach, 1514).

15 REFERENCES

Alfonsine Tables, 1483, Venice.
Apian, P., 1540. *Astronomicum Caesareum*. Ingoldstadt.
Chabás, J., and Goldstein, B.R., 1992. Nicolaus de Heybech and his table for finding true syzygy. *Historia Mathematica*, 19, 265–298.
Chabás, J., and Goldstein, J.R., 2009. *The Astronomical Tables of Giovanni Bianchini*. Leiden, Brill.
Chabás, J., and Goldstein, J R., 2012. *A Survey of European Astronomical Tables in the Late Middle Ages*. Leiden, Brill.
Gislén, L., 2016. A lunar eclipse volvelle in Petrus Apianus' *Astronomicum Caesareum*. *Journal of Astronomical History and Heritage*, 19(3), 247–254.
Gislén, L., 2025. Solar, lunar and umbral sizes in the Al-Khwārizmī and Peurbach eclipse tables. *Journal of Astronomical History and Heritage*, 28(3), 771–777.
Goldstein, B.R., 1992. Lunar velocity in the Ptolemaic tradition. In Harman, P.M., and Shapiro, A.E. (eds.), *The Investigation of Difficult Things*. Cambridge, Cambridge University Press. Pp. 3–17.
Goldstein, B.R., and Chabás, J., 2024. Tables of the radii of the Sun, the Moon, and the shadow, from John of Gmunden to Longomontanus. *Archive for the History of Exact Sciences*, 78, 67–86.
Kennedy, E.S., 1956. Parallax theory in Islamic astronomy. *Isis*, 47, 33–53.
Kremer, R.L., 2011. Experimenting with paper instruments in fifteenth and sixteenth century astronomy: computing syzygies with isotemporal lines and salt dishes. *Journal for the History of Astronomy*, 42, 223–258.
Meeus, J., 1998. *Astronomical Algorithms*. Richmond, Willman-Bell.
Münster, S., 1536. *Organum Uuranicum*. Basel.
Nallino, 1903. *Al-Battānī sive Albatenii Opus Astronomicum*. Milano, Publicazioni del Reale Osservatorio di Brera.

Neugebauer, O., 1962. *The Astronomical Tables of al- Khwārzimī*. Historisk-filosofiske Skrifter udgivet af Det Kongelige Danske Videnskabernes Selskab Bind 4, nr 2.

Pedersen, O., and Jones, A., 2010. *A Survey of the Almagest*. New York, Springer.

Peurbach, G., ca. 1480. *Tabulae Eclypsium*. MS, University of Jagiellonian, Kraków, Biblioteka Jagiellońska, No. 607 (https://archive.org/details/jbc.bj.uj.edu.pl.NDIGORP031281/page/n15/mode/2up).

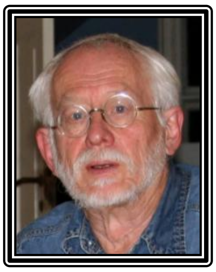
Peurbach, G., 1514. *Tabulae Eclypsium*. Wien (https://archive.org/details/bub_gb_SRYkk0MwYvQC).

Regiomontanus, 1496. *Epytoma Joannis de Monte Regio in Almagestum Ptolemei*. Venice. (https://archive.org/details/EpytomaIoannisd00Regi).

van Dalen, B., 1993. Ancient and Mediaeval Astronomical Tables (ISBN 90-393-0167-0).

Stöffler, J., 1514. *Tabulae Astronomicae Verarum Mediarum Coniunctunum & Oppositonum Solis & Lunae Exactissima*. Tubingae.

Dr. Lars Gislén was born in Lund (Sweden) in 1938, and received a PhD in High Energy Particle Physics from the University of Lund in 1972. He worked in 1970 and 1971 as a researcher at the Laboratoire de Physique Théorique in Orsay (France) with models of high energy particle scattering. He has also done research on atmospheric optics and with physical modelling of biological systems and evolution.



He has worked as an Assistant Professor (University Lector) at the Department of Theoretical Physics at the University of Lund, where he gave courses on classical mechanics, electrodynamics, statistical mechanics, relativity theory, particle physics, cosmology, solid state physics and system theory. For more than twenty years he was a delegation leader and mentor for the Swedish team in the International Physics Olympiad and the International Young Physicists’ Tournament.

Lars retired in 2003, and since then his interests have focused on Medieval European astronomy and on the astronomy and calendars of India and Southeast Asia. He has published more than 35 research papers in this field. He has also made public several spreadsheet tools implementing a number of astronomical models from Ptolemy to Kepler as well as computer tools for the calendars of India and Southeast Asia. He is a member of the IAU./Users/lars/Desktop/IMG_1068.jpeg.