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ON A STABILIZING PROPERTY OF ADAPTIVE REGULATORS

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ABSTRACT

An objective with adaptive control of stochastic systems is to obtain in some sense optimal control of the process. For practical applications it is important that the controller is not too sensitive with respect to assumptions concerning the process. For example it is desired that the stability of the closed loop system can be guaranteed. In this paper a stabilizing property is considered for a certain class of adaptive regulators, the so called self-tuning regulators. The investigated controllers are based on simultaneous recursive parameter estimation and control. It is shown that, under fairly weak conditions, these controllers have a stabilizing property in the sense that the output-signal is bounded.

1. INTRODUCTION

The main objective with adaptive controllers is to get a possibility to control unknown processes. It is thus desirable to make as few assumptions as possible concerning the controlled process. Usually when the adaptive controllers are designed it is assumed that the process is of a certain type.

Assumptions can be made concerning the order and the time delay of the system. Also assumptions must be made concerning the disturbances acting on the system. For practical applications it is important that these assumptions easily can be verified. One feature of an adaptive controller that is desirable is that the stability of the closed loop system can be guaranteed.

The stability analysis of a system controlled by an adaptive regulator is far from trivial. The closed loop system is non-linear and timevariant. Even if the properties of the different parts of the controller, i.e. the estimation and the control routines, are well known there might be difficulties when they are connected into a closed loop system.

For stability analysis a common tool is linearization around the desired solution. This is not a suitable tool for stochastic systems since there is a non-zero probability that the system will be forced outside the area where the linearization is valid. Also it is the behaviour far from the optimal solution that might be particularly interesting, since this gives a feeling for the transient properties of the controller.

The problem of stability for adaptive controllers is not easy to solve even for deterministic systems. One example is the adaptive controller based on Narendra's adaptive observer. In this case the convergence of the controller can be shown under the assumption that the output of the system is bounded, i.e. that the closed loop system is stable. This is, however, a condition that is difficult to show. The problem is so far solved for special types of processes [1]. An other example is the model reference adaptive controllers where fairly strong assumptions must be made on the controlled system.

This paper discusses a stabilizing property of a certain class of adaptive regulators, the self-tuning regulators [2],[3]. These controllers are based on recursive parameter identification and minimum variance control. It will be shown that the input and the output of the process are bounded in the mean square sense under weak conditions. In Section 2 the class of regulators is presented and heuristic arguments are given for the stabilizing property. In Section 3 the results are given and a proof is outlined. The properties are illustrated with a simple example in Section 4. Conclusions and references are given in Sections 5 and 6 respectively.

2. ADAPTIVE REGULATORS BASED ON RECURSIVE IDENTIFICATION

A fairly straightforward approach to adaptive control is to combine a recursive identification method with a regulator of a certain structure. The parameters of the regulator are then determined based on the current system parameter estimates, provided by the identification scheme. This approach has been described in [3], and various special cases have been discussed earlier. A certain version, which will be described in more detail below, has been successfully applied to several industrial processes.

The overall stability properties of adaptive regulators based on recursive identification can be heuristically discussed as follows. If something goes wrong and the feedback yields an unstable system, then the output and input signals increase rapidly. As the signals increase, more information about the true system becomes available, and the system parameter estimates converge rapidly to their true values. If now the feedback law, based on the true parameter values, yields a stable closed loop system (a very reasonable assumption) then the unstable behaviour of the system is quickly stopped after a "burst" in the output signal.

However, this discussion is purely heuristic and has some shortcomings. The most important feature is that if only one of the system's modes becomes unstable, only information about this one increases rapidly. Therefore only a certain combination of the system parameter estimates converges rapidly to its true value. This may not be sufficient to ensure that the closed loop system is stabilized or that the unstable mode changes significantly to reveal other modes of the systems. A more complete analysis of the structure of the regulator versus the identification method is therefore required.

In the next section we shall perform such an analysis for the following adaptive regulator. The identification method is chosen to be recursive least squares [4] and the regulator is the minimum variance controller, see e.g. [5].

More formally, we have a model

$$y(t+k) + \hat{a}_1 y(t-1)+\ldots+ \hat{a}_{\hat{n}} y(t-n) = \hat{b}_1 u(t-1)+\ldots+ \hat{b}_{\hat{m}} u(t-\hat{m}) \qquad (1)$$

where y is the output and u is the input. The time delay in the process is k, and it is assumed to be known. The parameters $\hat{a}_i$ $\hat{b}_i$ are estimated using the LS criterion, i.e.

$$V_N = \sum_{i}^{N-k} \left(y(t+k)+ \hat{a}_1 y(t-1)+\ldots+ \hat{a}_{\hat{n}} y(t-n)- \hat{b}_1 u(t-1)-\ldots-\hat{b}_{\hat{m}} u(t-\hat{m})\right)^2 \qquad (2)$$

is minimized w.r.t. $\hat{a}_i$ and $\hat{b}_i$. Let the minimizing parameter at step N be denoted by $\hat{a}_i(N)$, $\hat{b}_i(N)$. It is well known how these are found recursively, e.g. [4]. The control is then chosen as

$$u(t) = \frac{1}{\hat{b}_1(t)} [\, \hat{a}_1(t)\, y(t)+\ldots+ \hat{a}_n(t)\, y(t-\hat{n}+1) - \hat{b}_2(t)\, u(t-1)+$$

$$+\ldots+ \hat{b}_{\hat{m}}(t)\, u(t-\hat{m}+1)] \tag{3}$$

This resulting adaptive, or self-tuning regulator is described in more detail in [2] and [3].

3. A STABILIZING PROPERTY

In this section we shall prove that the self-tuning regulator (1)-(3) possesses an overall stability property regardless of disturbances in (1). The proof is based on the heuristic discussion in Section 2, complemented with a proper analysis of how the unstable modes change. Due to the limited space here, only an outline of the proof can be given; the full proof can be found in [6].

<u>Theorem</u>: Suppose that the true process can be described by

$$y(t+k)+a_1 y(t-1)+\ldots+a_n y(t-n) = b_1 u(t-1)+\ldots+b_m u(t-m)+v(t+k) \tag{4}$$

where $v(\cdot)$ is some disturbance such that

$$\frac{1}{N} \sum_1^N v(t)^2 < C_1 \tag{5}$$

Let the self-tuning regulator (1)-(3) be applied to (4) with $\hat{n} \geq n$, $\hat{m} \geq m$. Then the overall system is stable in the sense that

$$r_N = \frac{1}{N} \sum_1^N y(t)^2 < C_2 \tag{6}$$

If the system (4) is minimum phase, then also

$$\frac{1}{N} \sum_1^N u(t)^2 < C_3 \tag{7}$$

The constants C_i are independent of N, but may depend on the realization of the (possibly stochastic) sequence v.

<u>Proof</u>: Introduce

$$\varphi(t) = [\, y(t)\ldots,\ y(t-\hat{n}+1),\ u(t),\ldots\ u(t-\hat{m}+1)]^T$$

$$\theta_o = [\, a_1 \ldots a_n,\ 0\ldots0;\ b_1 \ldots b_m\ 0\ldots0\,]^T$$

$$\hat{\theta} = [\ \hat{a}_1 \ldots \hat{a}_n \ldots \hat{a}_{\hat{n}},\ \hat{b}_1 \ldots \hat{b}_m \ldots \hat{b}_{\hat{m}}\ ]^T$$

$$\hat{\theta}(t) = [\ \hat{a}_1(t),\ldots,\ \hat{a}_{\hat{n}}(t),\ \hat{b}_1(t),\ldots,\ \hat{b}_{\hat{m}}(t)]^T$$

$$\tilde{\theta} = \hat{\theta} - \theta_o$$

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta_o$$

Then (4) can be written

$$y(t+k+1) = \theta_o^T\ \varphi(t) + v(t+k+1)$$

and (3) is

$$\theta(t)^T\ \varphi(t) = 0$$

Hence

$$y(t+k+1) = \tilde{\theta}(t)^T\ \varphi(t) + v(t+k+1) \tag{8}$$

Let

$$R(t) = \frac{1}{t} \sum_{s=1}^{t} \varphi(s-k-1)\ \varphi(s-k-1)^T$$

Then the LS criterion (2) can be written

$$V_t(\tilde{\theta}) = \tilde{\theta}^T R(t) \tilde{\theta} + 2\,\frac{1}{t} \sum_{s=1}^{t} \tilde{\theta}^T \varphi(s-k-1)\ v(s)t + \frac{1}{t} \sum_{s=1}^{t} v(s)^2,$$

According to (5)

$$V_t(0) \le C_1$$

Therefore,

$$V_t(\theta(t)) \le C_1$$

and using Schwarz' inequality this implies that

$$\tilde{\theta}(t)^T\ R(t)\ \tilde{\theta}(t) \le 4C_1$$

or

$$\tilde{\theta}(t)^T\ \frac{R(t)}{r_t}\ \tilde{\theta}(t) \le 4C_1/r_t \tag{9}$$

If now (6) does not hold so r_t tends to infinity along a subsequence, then r_s is arbitrarily large and increasing for $t \le s \le t'$ for some sufficiently large t. This implies via (9) that $\tilde{\theta}(s)$ is arbitrarily close to the null space of $R(t)/r_t$ for $t \le s \le t'$. Since r_s is increasing, y(s) is "large" for a number of s=t,...,t'. In view of (8) this means that $\tilde{\theta}(s)^T \varphi(s)$ also is "large". Since $\tilde{\theta}(s)$ is arbitrarily close to the null space of $R(t)/r_t$, $\varphi(s)$ cannot belong to the range space of $R(t)/r_t$. (Since the matrix is symmetric, the null space and the range space are orthogonal.) Hence R(t') gets a significant contribution from matrices with range space not belonging to the range space of $R(t)/r_t$. In other words, the rank of $R(t')/r_{t'}$ is higher than that of $R(t)/r_t$. Repeating the argument at mos $\hat{n}+\hat{m}$ times, it follows that $R(t')/r_{t'}$ has full rank, yielding the only possible choice in (9) $\tilde{\theta}(s)=0$ (i.e. the true parameters). This gives $y(s)=v(s)$, which contradicts the assumption that r_s increases for $t \le s \le t'$ at an arbitrarily high level.

If the system is minimum phase, the inverse system is stable. If the input to the inverse system, y, satisfies (6), then the output, u, must satisfy (7).

□

This theorem shows a fairly important feature of the self-tuning regulator (1)-(3). Under quite weak conditions, the most important one being that the time delay has to be known, the regulator is capable of stabilizing any system in the sense (6). This stabilization takes place regardless of the character of the disturbances (as long as (5) holds) and regardless of or whether the estimate $\hat{\theta}(t)$ converges or not.

4. EXAMPLE

In order to illustrate the stabilizing property of the discussed self-tuning regulator we consider to following process

$$y(t) + a(t)y(t-1) = u(t-1) + e(t) \qquad e(t) \in N(0,1) \tag{10}$$

where a(t) is a timevarying parameter. The parameter is altered between -1 and 1 every 150th step of time. The model is assumed to have the structure

$$y(t) + \hat{a}y(t-1) = u(t-1)$$

It is assumed that the parameter $\hat{b}_1$ in (1) is fixed to its true value. In [6] the stability is shown for this case if the parameter $\hat{b}_1$ is fixed to a value which is sufficiently close to the correct value. It can be advantegous to fix $\hat{b}_1$ for reasons of identifiability. In this case the control law is characterized by one parameter and it is thus sufficient to identify one parameter.

The control law is

$$u(t) = \hat{a}y(t) \tag{11}$$

The closed loop system is stable when

$$a - 1 < \hat{a} < a + 1$$

Thus if the estimated parameter has the same sign as the true parameter then the closed loop system will be unstable when the parameter a changes sign. The magnitude of the output will then increase and the estimated parameter will rapidly be changed in order to stabilize the system. In the simulation a weighting factor is introduced in (2), that accomplishes an exponential discounting of old data. With such a factor it is easier for the system to follow timevarying parameters, see e.g. [3]. The effect of old data is decreased to about 10% of the initial weight after about $2/(1-\lambda)$ steps of time where λ is the weighting factor.

Figure 1 shows the estimated and the true parameter when $\lambda = 0.98$. This means that about 100 old values are used in the identification and the effect of a change in a should be almost forgotten before a new change occures. Figure 2 shows the output signal. The instability is seen at t = 150, 300 and 450. It is seen that the system is stabilized rapidly. In this example the system is unstable during 5-6 steps of time each time the parameter a changes sign.

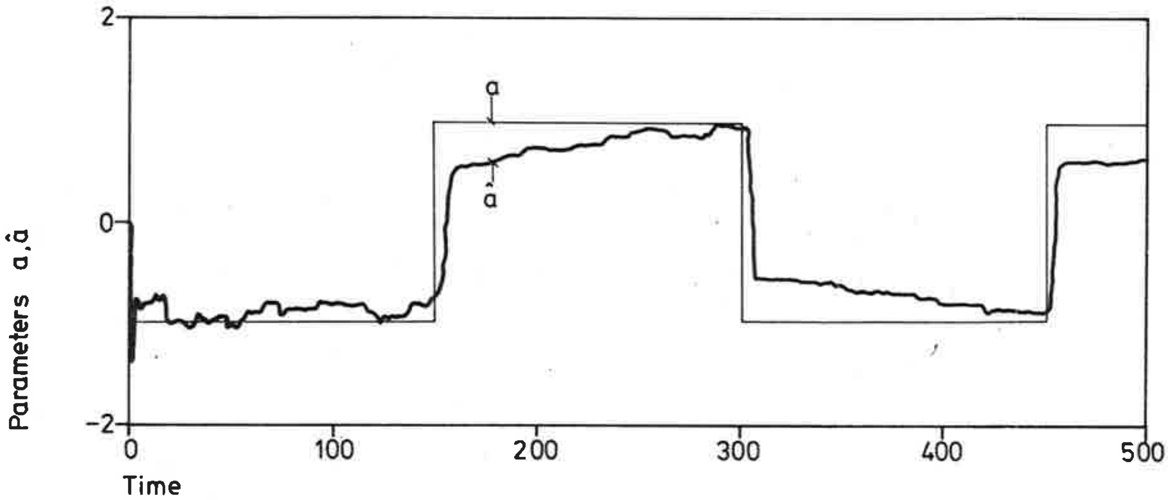

Figure 1. The estimated and the true parameter values when the process (10) is controlled by the control law (11).

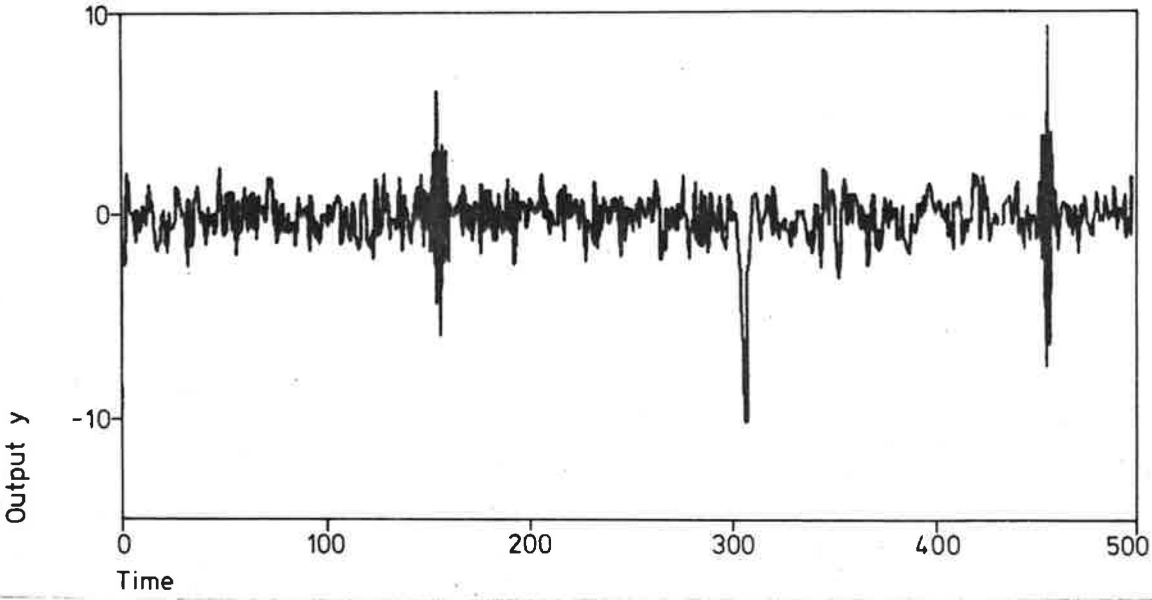

Figure 2. The output signal when controlling the process (10). At t = = 150, 300 and 450 the closed loop system is unstable for 5-6 steps of time before the estimator has changed the estimated parameter.

5. CONCLUDING REMARKS

An ability to stabilize any system is perhaps the most important feature of an adaptive controller. If the overall stability of the controller is sensitive to certain assumptions that are difficult to verify a priori, then the controller is useless in practice. If the dynamics of the process is subject to changes that make it impossible to stabilize it with a constant regulator, then an adaptive stabilizing regulator may be sufficient, even if it does not behave optimally from other points of view.

We have here shown that the self-tuning regulator (1)-(3) possesses such a stabilizing capability. This regulator has been analysed previously in [2], [6]. Possible convergence points and actual convergence (w.p. 1) to the optimal regulator have been studied under varying assumptions on $v(\cdot)$. It has been shown that if $v(\cdot)$ is a moving average of order k or less, then we have desired convergence, and this can take place also for more general disturbances (but not necessarily).

The remarkable feature of the present result is that it holds almost regardless of the characteristics of $v(\cdot)$. No stochastic assumptions or assumptions on zero mean etc are introduced. It is required that the time delay, k, is known, but this is usually not difficult to accomplish, since it is often easy to estimate k from basic transport delays etc.

References

1. Narendra, K S: Private communication.
2. Åström, K J, Wittenmark, B: On self-tuning regulators. Automatica 9, 185-199, 1973.
3. Åström, K J, Borisson, U, Ljung, L, Wittenmark, B: Theory and applications of adaptive regulators based on recursive parameter estimation. Preprint 6th IFAC World Congress, Boston 1975.
4. Åström, K J, Eykhoff, P: System identification - A survey. Automatica 7, 123-162, 1971.
5. Åström, K J: Introduction to stochastic control theory. Academic Press, 1970.
6. Ljung, L, Wittenmark, B: Asymptotic properties of self-tuning regulators. Report 7404, Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1974.