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ADAPTIVE CONTROL
–A way to deal with uncertainty

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Abstract

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Abstract. This paper approaches the uncertainty problem from the point of view of adaptive control. The uncertainty is reduced by continuous monitoring of the response of the system to the control actions and appropriate modifications of the control law. It is shown that this approach makes it possible to deal with uncertainties that cannot be handled by high gain robust feedback control.

1. INTRODUCTION

The problem of reducing the consequences of uncertainty has always been a central issue in the field of automatic control. Black's invention of the feedback amplifier was motivated by the desire to make electronic circuits less sensitive to the variability of electronic tubes. The development of modern instrumentation technology has similarly made use of feedback in the form of the force balance principle, to make high quality instruments which are only moderately sensitive to variations in their components.

Feedback by itself has the ability to reduce the sensitivity of a closed loop system to plant uncertainties. Although this was one of the original motivations for introducing feedback, the idea was kept in the background during the intensive development of modern control theory. Lately the problem has received renewed interest. It is now a very active research field and several new schemes for robust control have recently been developed. Such shcemes typically result in constant gain feedback controls, which are insensitive to variations in plant dynamics. The possibilities and limitations of constant gain feedback are treated in Section 2. The purpose is to find out when a constant gain feedback can be designed to overcome uncertainty in process dynamics and when it can not.

An integrator where the sign of the gain is not known is a simple example which can not be handled by constant gain feedback. This example will be used as an illustration throughout the paper.

The main goal of the paper is to approach elimination of uncertainties from the point of view of adaptive control. When the plant uncertainties are such taht they can not be handled by a constant gain robust control law it is natural to try to reduce the uncertainties by

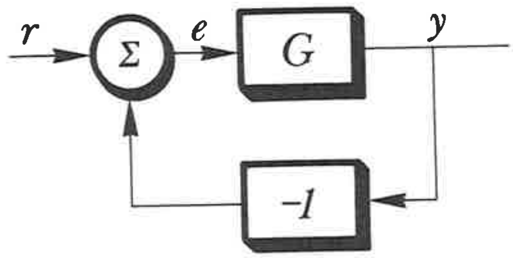

Figure 1. Simple feedback system

experimentation and parameter estimation. Auto-tuning is a simple technique, which has the attractive feature that an appropriate input signal to the process is generated automatically. The method has the additional benefit that parameter estimation and control design are extremely simple to do. This is discussed in Section 3.

Auto-tuning is an intermittent procedure. The regulator has a special tuning mode, which is invoked on the request of an operator or based on some automatic diagnosis. Adaptive control is a method which allows continuous reduction of the uncertainties. An adaptive regulator will continuously monitor the systems response to the control actions and modify the regulator appropriately. The charateristics of such control schemes are discussed in Section 4. Two categories of adaptive control laws, direct and indirect, are discussed in some detail. Some theoretical results on the stability of adaptive control systems are reviewed in Section 5. It is found that the standard assumptions used to prove the stability of direct adaptive control schemes are such that robust high gaiin linear control could equally well be applied. Adaptive controllers are nonlinear feedback systems. There are other types of nonlinear feedback systems, which also can deal with uncertainties. One type is called universal stabilizer. Such a system is briefly discussed in Section 6. Its capability of dealing with an integrator with unknown gain is demonstrated.

Stochastic control theory is a general method of dealing with uncertainties. In Section 7 it is shown how adaptive control laws can be derived from stochastic control theory. The example with the integrator having unknown gain is worked out in some detail.

2. LIMITATIONS OF CONSTANT GAIN FEEDBACK

Conventional feedback can deal with uncertainty in the form of disturbances and modeling errors. Before discussing other techniques for dealing with uncertainty it is useful to understand the possibilities and limitations of constant gain feedback. For this purpose consider the simple feedback system shown in Figure 1. Let G_0 be the nominal loop transfer function. Assume that true loop transfer function is $G = G_0(1+L)$ due to model uncertainties. Notice that $1+L$ is the ratio between the true and nominal transfer functions.

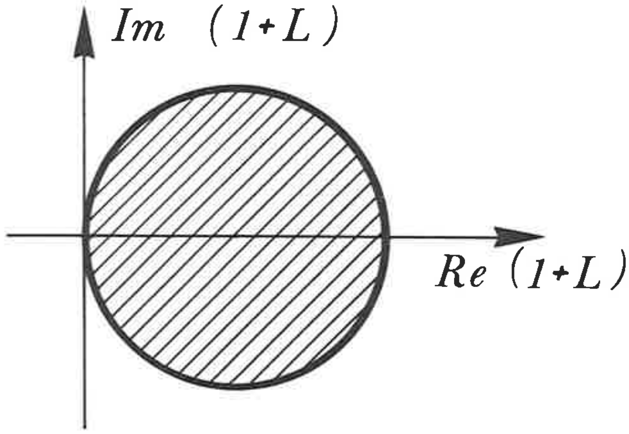

Figure 2. The ratio between the true transfer function G(s) and the nominal transfer function $G_0(s)$ must be in the shaded region for those frequencies where $G_0(i\omega)$ is large.

The effect of uncertainties on the stability of the closed loop system will first be discussed. The closed loop poles are the zeros of the equation

$$1 + G_0(s) + G_0(s)L(s) = 0$$

Provided that the nominal system is stable it follows from Rouche's theorem that the uncertainties will not cause instability provided that

$$|L(s)| \leq \left| \frac{1+G_0(s)}{G_0(s)} \right| \tag{2.1}$$

on a contour which encloses the left half plane. The consequences of this inequality will now be discussed. For large loop gains (2.1) reduces to

$$|L(s)| \leq 1$$

This means that the relative uncertainty 1+L must be in the shaded area in Figure 2. It follows from Figure 2 that if the uncertainty in the phase of the open loop system is less than φ in magnitude i.e.

$$|\arg(1+L)| \leq \varphi$$

then the closed loop system is stable provided that the magnitude of the relative uncertainty satisfies

$$0 \leq |1+L| \leq 2 \cos \varphi$$

For these frequencies where the loop gain is high it is thus necessary that the phase uncertainty is less than 90°.

At the crossover frequency ω_c where the loop gain $G_0(i\omega_c)$ has unit magnitude equation (2.1) reduces to

$$|L(i\omega_c)| \leq \sqrt{2(1-\cos\varphi_m)} \qquad (2.2)$$

where φ_m is the phase margin. At higher frequencies where the loop gain is less than one the inequality (2.1) can be approximated by

$$|L(s) \leq \left|\frac{1}{G_0(s)}\right|$$

This means that large uncertainties can be treated where the loop gain is significantly less than one.

Stability is only a necessary requirement. To investigate the effect of uncertainty on the performance of the closed loop system consider the transfer function from the command signal to the output i.e.

$$G_0 = \frac{G}{1+G} = \frac{G_0+G_0L}{1+G_0+G_0L} = \frac{G_0}{1+G_0} \cdot \frac{1+L}{\dfrac{G_0}{1+G_0L}}$$

The error in the closed loop transfer function is thus

$$L_c = \frac{L}{1+G_0+G_0L} \qquad (2.3)$$

This error can be made small either by having a small open loop uncertainty (L) or by having a high loop gain (G_0).

Equations (2.1) and (2.3) give the essence of high gain robust control. The open open loop gain G_0 can be made large for those frequencies where the phase uncertainty is less than 90 degrees. At those frequencies the closed loop transfer function can be made arbitrarily close to the specifications by choosing the gain sufficiently large. For those frequencies where the uncertainty in the phase shift is larger than 90° the total loop gain must be made smaller than one in order to maintain robustness. At the crossover frequency where the loop gain has unit magnitude the allowable phase uncertainty is given by (2.2). The allowable uncertainty depends critically on the phase margin φ_m. Assuming for example that it is desired to have an error in the closed loop transfer function of at most 10% of the crossover frequency. The allowable phase margin is given in Table 1.

Table 1 - Maximum error in the open loop transfer function which give at most 10 % error of the closed loop transfer function at the crossover frequency.

φ_m	10	20	30	45	60
max $\lvert L \rvert$	0.077	0.034	0.052	0.076	0.100

Design techniques which can deal with uncertainty are given in Gutman (1979), Horowitz (1963), Horowitz and Sidi (1973), Leitmann (1980,1983), Kwakernaak (1985), Grübel (1985). A discussion of the multivariable case is given by Doyle and Stein (1981).

It is clear from the discussion above that in order to use robust high gain control it is necessary that the transfer function of the plant has a phase uncertainty less than 90° for some frequencies. Some examples which illustrate the limitations of high gain robust control will now be discussed.

Example 2.1 - Time Delays

Consider a linear plant where the major uncertainty is due to variations in the time delay. Assume that the time delay varies between T_{min} and T_{max}. Furthermore assume that it is required to keep the variations in the phase margin less than 20°. It then follows that the cross-over frequency ω_c must satisfy

$$\omega_c \le \frac{0.35}{T_{max} - T_{min}}$$

The uncertainty in the time delay thus induces an upper bound to the achievable cross-over frequency. □

Example 2.2 - Mechanical Resonances

Mechanical resonances are associated with transfer functions of the type

$$G(s) = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

where the damping normally is very small. The phase of G changes rapidly from 0 to -180° around ω_0. The gain also changes rapidly around ω_0. It increases from one to approximately $1/2\,\xi$ and it increases as ω_0^2/ω^2 with increasing ω. Variations in ξ and ω will thus give substantial phase uncertainty. To achieve robust linear control it is then necessary to make sure that the loop gain is low around ω_0. This is typically achieved by a notch filter. □

Example 2.3 - Integrator Whose Sign is Unknown

An integrator whose sign is not known has either a phase lag of 90° or 270°. Such a system can not be controlled using high gain robust control. □

3. AUTO-TUNING

When the uncertainty is such that robust high gain feedback cannot be applied it is natural to try to reduce the uncertainty by experimentation. Auto-tuning is a methodology for doing this automatically. The principles are straightforward. A model of the process dynamics is determined by making an identification experiment where an input signal is generated and applied to the process. The dynamics of the process is then determined from the results of the experiment. The controller parameters are then obtained from some design procedure. Since the signal generation, the identification and the design can be made in many different ways there are many possible tuning procedures of this kind.

Auto-tuning is also useful in another context. There are cases where is is much easier to apply an auto-tuner than to design a robust high gain controller Simple regulators with two or three parameters can be tuned manually if there is not too much interaction between adjustments of different parameters. Manual tuning is, however, not possible for more complex regulators. Traditionally tuning of such regulators have followed the route of modeling or identification and regulator design. This is often a time-consuming and costly procedure which can only be applied to important loops or to systems which are made in large quantities.

Most adaptive techniques can be used to provide automatic tuning. In such applications the adaptation loop is simply switched on and perturbation signals may be added. The adaptive regulator is run until the performance is satisfactory. The adaptation loop is then disconnected and the system is left running with fixed regulator parameters. Below we will discuss a specific auto-tuner which requires very little prior information and also has the interesting property that it generates an appropriate test signal automatically. This is discussed further in Åström and Hägglund (1984a). A nice feature of the technique described below is that an input signal is generated automatically and that the parameter estimation and the control design are very simple. The input signal generated is automatically tuned to the characteristics of the plant. It will have its energy concentrated around the frequencies where the plant has phase lag of 180°.

The Basic Idea

A wide class of process control problems can be described in terms of the intersection of the Nyquist curve of the open loop system with the negative real axis, which is traditionally

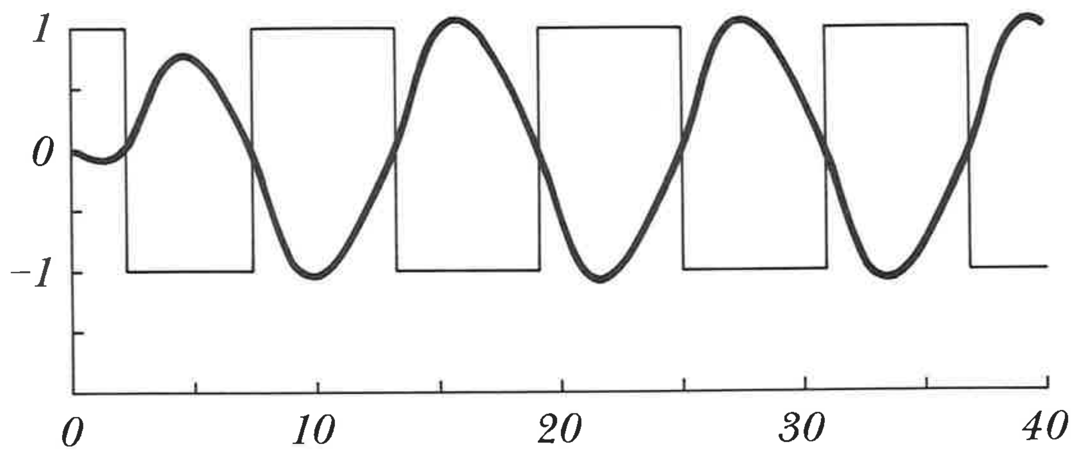

Figure 3. Input and output signals for a linear system under relay control. The system has the transfer function G(s) = 0.5(1-s)/s(s+1)(s+1).

described in terms of the critical gain k_c and the critical period T_c. A method for determining these parameters was described in Ziegler and Nichols (1943). It is done as follows: A proportional regulator is connected to the system. The gain is gradually increased until an oscillation is obtained. The gain when this occurs is the critical gain and the frequency of the oscillation is the critical frequency. It is, however, difficult to perform this experiment in such a way that the amplitude of the oscillation is kept under control.

Relay feedback is an alternative to the manual tuning procedure. If the process is connected to a feedback loop there will be an oscillation as is shown in Figure 3. The period of the oscillation is approximately the critical period. The process gain at the corresponding frequency is approximately given by

$$G_p\left(i\,\frac{2\pi}{T_c}\right) = -\,\frac{a\pi}{4d} \tag{3.1}$$

where d is the relay amplitude and a is the amplitude of the oscillation.

A simple relay control experiment thus gives the desired information about the process. This method has the advantage that it is easy to control the amplitude of the limit cycle by an appropriate choice of the relay amplitude. A simple feedback from the output amplitude to the relay amplitude makes it possible to keep the output amplitude fixed during the experiment. Notice also that an input signal which is almost optimal for the estimation problem is generated automatically. This ensures that the critical point can be determined accurately.

When the critical point on the Nyquist curve is known, it is straightforward to apply the classical Ziegler-Nichols design methods. It is also possible to devise many other design schemes that are based on the knowledge of one point on the Nyquist curve. The procedure can be modified to determine other points on the Nyquist curve. An integrator may be connected in the loop after the relay to obtain the point where the Nyquist curve intersects

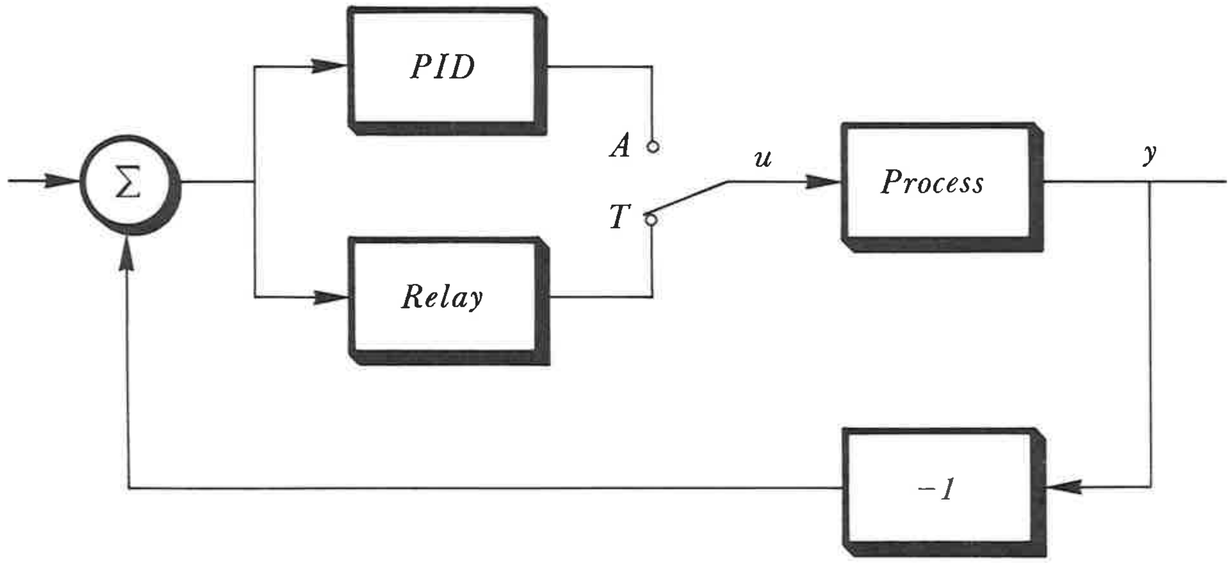

Figure 4. Block diagram of an auto-tuner. The system operates as a relay controller in the tuning mode (T) and as an ordinary PID regulator in the automatic control mode (A).

the negative imaginary axis. New design methods, which are based on such experiments, are described in Åström and Hägglund (1984b).

Methods for automatic determination of the frequency and the amplitude of the oscillation will be given to complete the description of the estimation method. The period of an oscillation can be determined by measuring the times between zero-crossings. The amplitude may be determined by measuring the peak-to-peak values of the output. These estimation methods are easy to implement because they are based on counting and comparisons only. More elaborate estimation schemes like least squares estimation and extended Kalman filtering may also be used to determine the amplitude and the frequency of the limit cycle oscillation. Simulations and experiments on industrial processes have indicated that little is gained in practice by using more sophisticated methods for determining the amplitude and the period.

A block diagram of a control system with auto-tuning is shown in Figure 4. The system can operate in two modes. In the tuning mode a relay feedback is generated as was discussed above. When a stable limit cycle is established its amplitude and period are determined as described above and the system is then switched to the automatic control mode where a conventional PID control law is used.

Practical Aspects

There are several practical problems which must be solved in order to implement an auto-tuner. It is e.g. necessary to account for measurement noise, level adjustment, saturation of actuators and automatic adjustment of the amplitude of the oscillation. It may be advantageous to use other nonlinearities than the pure relay. A relay with hysteresis gives a system which is less sensitive to measurement noise.

Measurement noise may give errors in detection of peaks and zero crossings. A hysteresis in the relay is a simple way to reduce the influence of measurement noise. Filtering is another possibility. The estimation schemes based on least squares and extended Kalman filtering can be made less sensitive to noise. Simple detection of peaks and zero crossings in combination with an hysteresis in the relay has worked very well in practice. See e.g. Åström (1982).

The process output may be far from the desired equilibrium condition when the regulator is switched on. In such cases it would be desirable to have the system reach its equilibrium automatically. For a process with finite low-frequency gain there is no guarantee that the desired steady state will be achieved with relay control unless the relay amplitude is sufficiently large. To guarantee that the output actually reaches the reference value, it may be necessary to introduce manual or automatic reset.

It is also desirable to adjust the relay amplitude automatically. A reasonable approach is to require that the oscillation is a given percentage of the admissible swing in the output signal.

Auto-Tuning with Learning

Auto-tuning is a simple way to reduce uncertainty by experimentation. In many cases the characteristics of a process may depend on the operating conditions. If it is possible to measure some variable which correlates well with the changing process dynamics it is possible to obtain a system with interesting characteristics by combining the auto-tuner with a table look-up function. When the operating condition changes a new tuning is performed on demand from the operator. The resulting parameters are stored in a table together with the variable which characterizes the operating condition. When the process has been operated over a range covering the operating conditions the regulator parameters can be obtained from the table. A new tuning is then required only when other conditions change. A system of this type is semi-automatic because the decision to tune rests with the operator. The system will, however, continue to reduce the plant uncertainty.

4. ADAPTIVE CONTROL

Adaptive control is another way to deal with uncertainties. A block-diagram of a typical adaptive regulator is shown in Figure 5. The system can be thought of as composed of two loops. The inner loop consists of the process and an ordinary linear feedback regulator. The parameters of the regulator are adjusted by the outer loop, which is composed of a recursive parameter estimator and a design calculation. To obtain good estimates it may also be

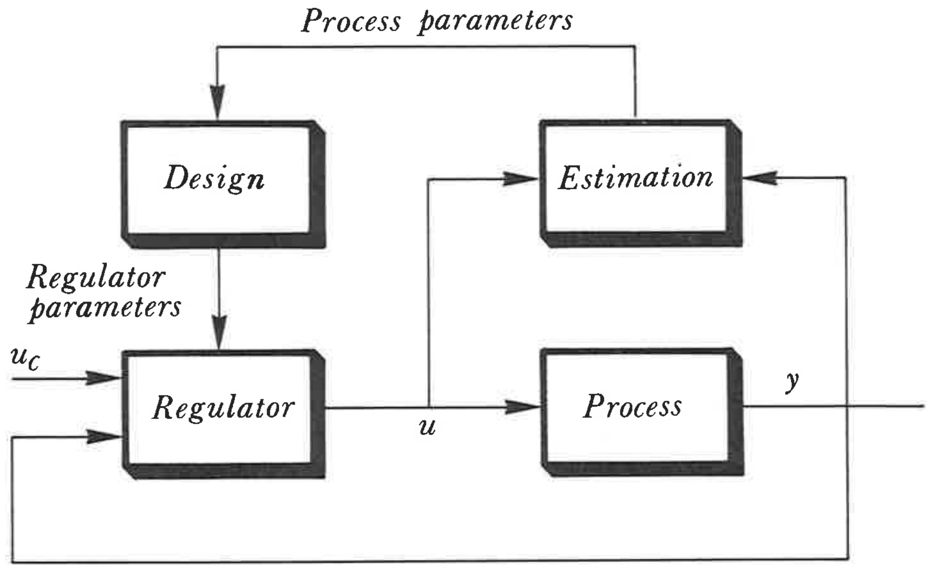

Figure 5. Block diagram of an adaptive regulator

necessary to introduce perturbation signals. This function is omitted from the figure for simplicity. Notice that the system may be reviewed as an automation of process modeling and design where the process model and the control design is updated at each sampling period.

The block labeled "regulator design" in Figure 5 represents an on-line solution to a design problem for a system with known parameters. This underlying design problem can be solved in many different ways. Design methods based on on phase- and amplitude margins, pole-placement, minimum variance control, linear quadratic gaussian control and other optimization methods have been considered, see Åström (1983). Robust design techniques can of course also be used.

The adaptive regulator also contains a recursive parameter estimator. Many different estimation schemes have been used, for example stochastic approximation, least squares, extended and generalized least squares, instrumental variables, extended Kalman filtering and the maximum likelihood method.

The adaptive regulator shown in Figure 5 is called indirect or explicit because the regulator parameters are updated indirectly via estimation of an explicit process model. It is sometimes possible to reparameterize the process so that it can be expressed in terms of the regulator parameters. This gives a significant simplification of the algorithm because the design calculations are eliminated. In terms of Figure 5 the block labelled design calculations disappears and the regulator parameters are updated directly. The scheme is then called a direct scheme. Direct and indirect adaptive regulators have different properties which is illustrated by an example.

Example 3.1

Consider the discrete time system described by

$$y(t+1) + \underline{ay(t)} = \underline{bu(t)} + e(t+1) + \underline{ce(t)} \qquad t=\ldots-1,0,1,\ldots \tag{4.1}$$

where $\{e(t)\}$ is a sequence of zero-mean uncorrelated random variables. If the parameters a, b and c are known the proportional feedback

$$u(t) = -\theta y(t) = \frac{a-c}{b}\, y(t) \tag{4.2}$$

minimizes the variance of the output. The output then becomes

$$y(t) = e(t) \tag{4.3}$$

This can be concluded from the following argument. Consider the situation at time t. The variable $e(t+1)$ is independent of $y(t)$, $u(t)$ and $e(t)$. The output $y(t)$ is known and the signal $u(t)$ is at our disposal. The variable $e(t)$ can be computed from past inputs and outputs. Choosing the variable $u(t)$ so that the terms underlined in equation (4.1) vanishes thus makes the variance of $y(t+1)$ as small as possible. This gives (4.2) and (4.3). For further details, see Åström (1970).

Since the process (4.1) is characterized by <u>three</u> parameters a straightforward explicit self-tuner would require estimation of three parameters. Estimation of the parameter c is also a nonlinear problem. Notice, however, that the feedback law is characterized by <u>one</u> parameter only. A self-tuner which estimates this parameter can be obtained based on the model

$$y(t+1) = \theta y(t) + u(t) \tag{4.4}$$

The least squares estimate of the parameter θ in this model is given by

$$\theta(t) = \frac{\sum_{k=1}^{t} y(k)\,[y(k+1) - u(k)]}{\sum_{k=1}^{t} y^2(k)} \tag{4.5}$$

and the control law is then given by

$$u(t) = -\theta(t)y(t) \tag{4.6}$$

The self-tuning regulator given by (4.5) and (4.6) has some remarkable properties which can be seen heuristically as follows. Equation (4.5) can be written as

$$\frac{1}{t}\sum_{k=1}^{t} y(t+1)y(t) = \frac{1}{t}\sum_{k=1}^{t} [\theta(t)y^2(k) - u(k)y(k)] = \frac{1}{t}\sum_{k=1}^{t} [\theta(t) - \theta(k)]y^2(k)$$

Assuming that y is mean square bounded and that the estimate $\theta(t)$ converges as $t \to \infty$ we get

$$\lim \frac{1}{t}\sum_{k=1}^{t} y(k+1)y(k) = 0 \tag{4.7}$$

The adaptive algorithm (4.5), (4.6) thus attempts to adjust the parameter θ so that the correlation of the output at lag one is zero. If the system to be controlled is actually governed by (4.1) it follows from (4.3) that the estimate will converge to the minimum variance control law under the given assumption. This is somewhat surprising because the structure of (4.4) which was the basis of the adaptive regulator is not compatible with the true system (4.1). More details are given in Åström and Wittenmark (1973,1985) □

Indirect Adaptive Control

An advantage of indirect adaptive control is that many different design methods can be used. The key issue in analysis of the indirect schemes is to show that the parameter estimates converge. This will in general require that the model structure used is appropriate and that the input signal is persistently exciting. To ensure this it may be necessary to introduce perturbation signals. Provided that proper excitation is provided there are no difficulties in controlling an integrator whose gain may have different sign.

Direct Adaptive Control

The direct adaptive control schemes may work well even if the model structure used is not correct as wa shown in Example 3.1. The direct schemes will, however, require other assumptions. Assume e.g. that the process to be controlled can be described by

$$A(q)y(t) = B(q)u(t) + v(t) \tag{4.8}$$

where u is the input, y is the output, v is a disturbance and A9q) and B(q) are polynomials in the forward shift operator. Stability of adaptive control of (4.8) have been given by Egardt (1979), Fuchs (1979), Goodwin et al. (1980), Gawthrop (1980), de Larminat (1979), Morse (1980), and Narendra et al. (1980). So far the stability proofs are available only for some simple algorithms. The following assumptions are crucial:

(A1) the relative degree d = deg A - deg B is known,

(A2) the sign of the leading coefficient b_0 of the polynomial B(q) is known, (A3) the estimated model is at least of the same order as the process,

(A4) the polynomial B has all zeros inside the unit disc.

The assumption A1 means that the time delay is known with a precision, which corresponds to a sampling period. This is not unreasonable. For continuous time systems the assumption means that the Together with assumption (A2) it also means that the phase is known at high frequencies. If this is the case, it is possible to design a robust high gain regulator for the problem, see Horowitz (1963), Horowitz and Sidi (1973), Leitmann (1979) and Gutman (1979). For many systems like flexible aircraft, electromechanical servos and flexible robots, the main difficulty in control is the uncertainty of the dynamics at high frequencies, see Stein (1980).

Assumption A3 is very restrictive, since it implies that the estimated model must be at least as complex as the true system, which may be nonlinear with distributed parameters. Almost all control systems are in fact designed based on strongly simplified models. High frequency dynamics are often neglected in the simplified models.

Assumption A4 is also crucial. It arises from the necessity to have a model, which is linear in the parameters in the direct schemes.

5. ROBUST ADAPTIVE CONTROL

For a long time the research on stability of adaptive control systems focussed on proofs of global stability for all values of the adaptation gain. The results obtained under such premises are naturally quite restrictive. To get some insight into this consider a continuous time system described by

$$y = G(p)u \tag{5.1}$$

where u is the input, y is the output, G is the transfer function of the system and p = d/dt is the differential operator. Consider also the model reference adaptive control law given by

$$\begin{aligned} u &= \theta^T \varphi \\ \frac{d\theta}{dt} &= - k\varphi e \\ e &= y - y_m \end{aligned} \tag{5.2}$$

where y_m is the desired model output, e the error and θ a vector of adjustable parameters. The components of the vector φ are functions of the command signal. In a simple case, where the regulator is a combination of a proportional feedback and a proportional feedforward, φ becomes

$$\varphi = [r \ -y]^T$$

where r is the reference signal.

It follows from (5.1) and (5.2) that

$$\frac{d\theta}{dt} + k\varphi[G(p)\varphi^T\theta] = k\varphi y_m \tag{5.3}$$

This equation gives insight into the behavior of the system. Assume that the adaptation loop is much slower than the process dynamics. The parameters then change much slower than the regression vector φ and the term $G(p)\varphi^T\theta$ in (3.3) can then be approximated by its average i.e.

$$G(p)\varphi^T\theta \approx \overline{[G(p)\varphi^T(\theta)]}\theta \tag{5.4}$$

where "———" denotes time averages. Notice that the regression vector φ depends on the parameters. The following approximation to (5.3) is obtained

$$\frac{d\theta}{dt} + k\overline{\varphi(\theta)[G(p)\varphi^T(\theta)]}\theta \approx k\varphi y_m \tag{5.5}$$

This is the normal situation because the adaptive algorithm is motivated by the fact the parameters change slower than the other variables in the system under this assumption. Notice, howerer, that it is not easy to guarantee that the parameters change slowly by choosing k sufficiently small.

Equation (5.4) is stable if $k\varphi[G(p)\varphi^T]$ is positive. This is true e.g. if G is strictly positive real and if the input signal is persistently exciting. However, if the transfer function G(s) is strictly positive real it is also possible to design a robust high gain feedback for the system. We thus arrive at the paradox that the assumption required to show stability of the adaptive system will allow the design of a robust feedback. The assumption that G(s) is strictly positive real is, however, not necessary as is shown by the following example.

<u>Example 5.1</u>

Consider a system where only a feedforward gain is adjusted and let the command signal be a sum of sinusoids i.e.

$$r(t) = \sum_{k=1}^{n} a_k \sin(\omega_k t)$$

Using the model reference algorithm given by (5.2) the parameter estimates satisfy

$$\frac{d\theta}{dt} = kr[1 - \theta]G(p)r$$

Assuming that the gain is small and using averages we find that the estimates are approximately given by

$$\frac{d\bar{\theta}}{dt} = ka[1 - \bar{\theta}] \tag{5.6}$$

where

$$a = \frac{1}{2}\sum_{k=1}^{n} a_k^2 \cos[\arg G(i\omega_k)] \tag{5.7}$$

The equation (5.6) is stable if a is positive. Consider first the case of a single sinusoidal, $n = 1$, the equation is then unstable if the frequency of the command signal is chosen so that $G(i\omega_n)$ has a phase-shift larger than 90°. If the input contains several frequencies it is necessary that the dominating contribution to (5.7) comes from frequencies where the phase of $G(i\omega)$ is less than 90°. □

6. UNIVERSAL STABILIZERS

Adaptive control systems are nonlinear systems with a special structure. They are often designed based on the idea of automating modeling and design. It is natural to ask if there are other types of nonlinear controls which also can deal with uncertainties in the process model. A special class of systems were generated as attempts of solving the following problem which was proposed by Morse (1983). Consider the system

$$\frac{dy}{dt} = ay + bu$$

where a and b are unknown constants. Find a feedback law of the form

$$u = f(\theta, y)$$

$$\frac{d\theta}{dt} = g(\theta, y)$$

which stabilizes the system for all a and b. Morse conjectured that there are no rational f and g which stabilize the system. Morse's conjecture was proven by Nussbaum (1983) who also showed that there exist nonrational f and g which stabilize the system, e.g. the following functions

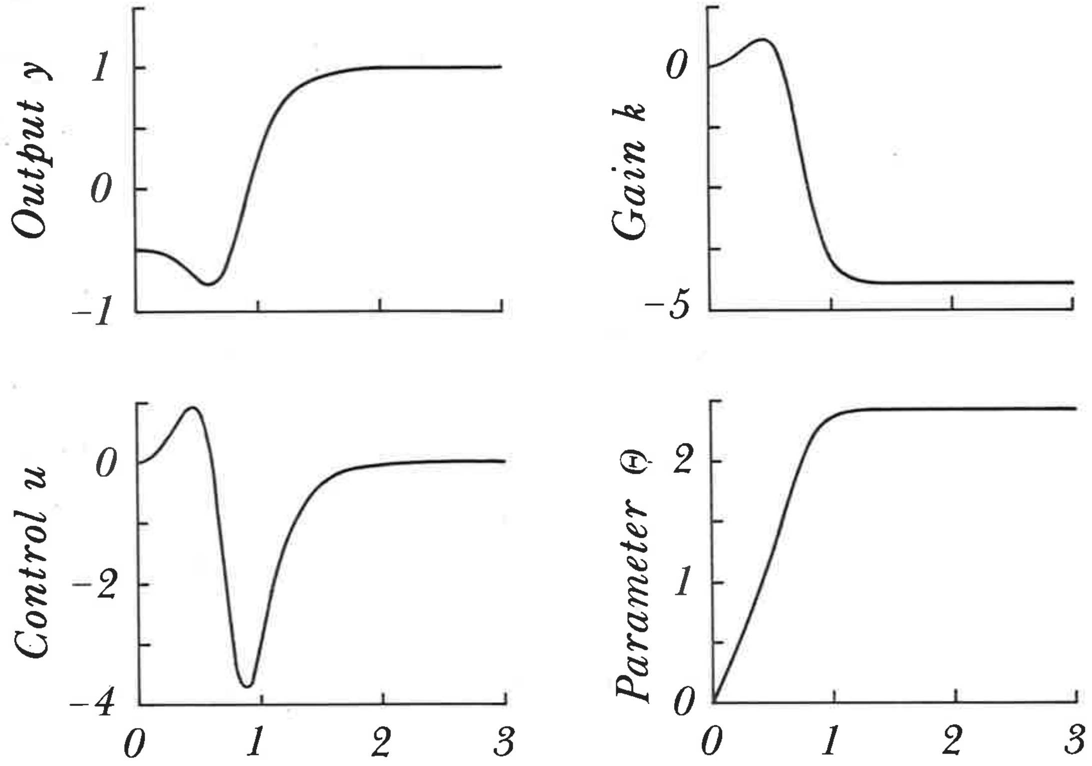

Figure 6. Simulation of an integrator with Nussbaum's control law.

$$f(\theta, y) = y\theta^2 \cos\theta$$

$$g(\theta, y) = y^2$$

This correspond to proportional feedback with the gain

$$k = \theta^2 \cos\theta$$

Figure 6 shows a simulation of this control law applied to an integrator with unknown gain. Notice that the regulator is initialized so that the gain has the wrong sign. In spite of this the regulator recovers and changes the gain appropriately. Nussbaum's regulator is of considerable principal interest because it shows that the assumption A2 is not necessary. The control law is, however, not necessarily a good control law in a practical situation because it may generate quite violent control actions. The initial conditions for the simulation shown in Figure 6 were chosen quite carefully.

7. DUAL CONTROL THEORY

Uncertainties can also be captured using nonlinear stochastic control theory. The system and its environment are then described by a stochastic model. To do so the parameters are introduced as state variables and the parameter uncertainty is modeled by stochastic models. An unknown constant is thus modeled by the differential equation

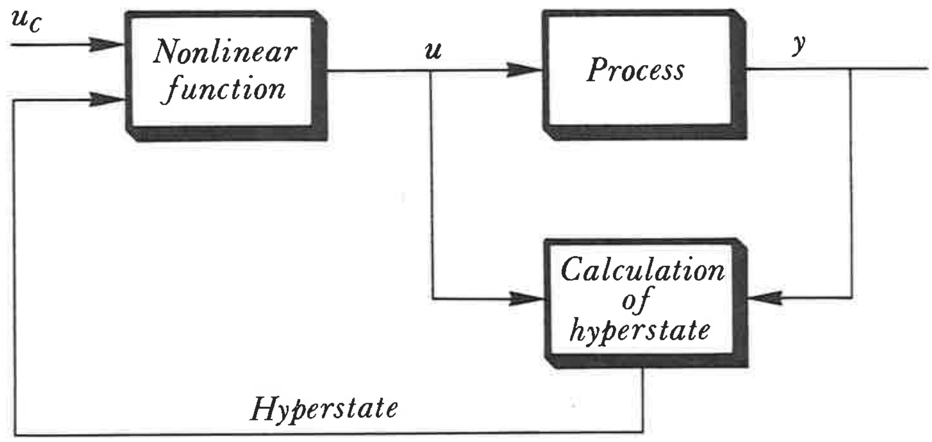

Figure 7. Block diagram of an adaptive regulator obtained from stochastic control theory.

$$\frac{d\theta}{dt} = 0$$

with an initial distribution that reflects the parameter uncertainty. Parameter drift is modeled by adding random variables to the right hand sides of the equations. A criterion is formulated as to minimize the expected value of a loss function, which is a scalar function of states and controls.

The problem of finding a control, which minimizes the expected loss function, is difficult. Under the assumption that a solution exists, a functional equation for the optimal loss function can be derived using dynamic programming, see Bellman (1957,1961). The functional equation, which is called the Bellman equation, is a generalization of the Hamilton-Jacobi equation in classical variational calculus. It can be solved numerically only in very simple cases. The structure of the optimal regulator obtained is shown in Figure 7. The controller can be thought of as composed of two parts: a nonlinear estimator and a feedback regulator. The estimator generates the conditional probability distribution of the state from the measurements. This distribution is called the hyperstate of the problem. The feedback regulator is a nonlinear function, which maps the hyperstate into the space of control variables. This function can be computed off-line. The hyperstate must, however, be updated on-line. The structural simplicity of the solution is obtained at the price of introducing the hyperstate, which is a quantity of very high dimension. Updating of the hyperstate requires in general solution of a complicated nonlinear filtering problem. Notice that there is no distinction between the parameters and the other state variables in Figure 7. This means that the regulator can handle very rapid parameter variations.

The optimal control law has interesting properties which have been found by solving a number of specific problems. The control attempts to drive the output to its desired value, but it will also introduce perturbations (probing) when the parameters are uncertain. This improves the quality of the estimates and the future controls. The optimal control gives the correct balance between maintaining good control and small estimation errors. The name dual control was coined by Feldbaum (1965) to express this property. Optimal stochastic control theory also offers other possibilities to obtain sophisticated adaptive algorithms, see Saridis (1977).

It is interesting to compare the regulator in Figure 7 with the self-tuning regulator in Figure 5. In the adaptive regulator the states are separated into two groups, the ordinary state variables of the underlying constant parameter model and the parameters which are assumed to vary slowly. In the optimal stochastic regulator there is no such distinction. There is no feedback from the variance of the estimate in the adaptive regulator although this information is available in the estimator. In the optimal stochastic regulator there is feedback from the conditional distribution of parameters and states. The design calculations in the adaptive regulator is made in the same way as if the parameters were known exactly. Finally there are no attempts in the adaptive regulator to introduce the estimates when they are uncertain. In the optimal stochastic regulator the control law is calculated based on the hyperstate which takes full account of uncertainties. This also introduces perturbations when estimates are poor. The comparison indicates that it may be useful to add parameter uncertainties and probing to the adaptive regulator. A simple example illustrates the dual control law and some approximations.

Example 7.1

Consider a discrete time version of the integrator with unknown gain

$$y(t+1) = y(t) + bu(t) + e(t), \qquad (7.1)$$

where u is the control, y the output and e normal $(0,\sigma_e)$ white noise. Let the criterion be to minimize the mean square deviation of the output y. This is a special case of the system in Example 3.1 with a = 1 and c = 0. When the parameters are known the optimal control law is given by (3.2) i.e.

$$u(t) = -\frac{y(t)}{b} \qquad (7.2)$$

If the parameter b is assumed to be a random variable with a Gaussian prior distribution, the conditional distribution of b, given inputs and outputs up to time t, is Gaussian with mean $\hat{b}(t)$ and standard deviation $\sigma(t)$. The hyperstate is then characterized by the triple $(y(t),\hat{b}(t),\sigma(t))$. The equations for updating the hyperstate are the same as the ordinary Kalman filtering equations, see Åström (1970) and (1978).

Introduce the loss function

$$V_N = \sigma_e^{-2} \min_u E\left\{ \sum_{k=t+1}^{t+N} y^2(k) \;\middle|\; Y_t \right\} \qquad (7.3)$$

where Y_t denotes the data available at time t i.e. {y(t), y(t-1),...}. By introducing the normalized variables

$$\eta = y/\sigma_e, \quad \beta = \hat{b}/\sigma, \quad \mu = -u\hat{b}/y \tag{7.4}$$

it can be shown that V_N depends on η and β only. The Bellman equation for the problem can be written as

$$V_T(\eta, \beta) = \min U_T(\eta, \beta, \mu) \tag{7.5}$$

where

$$V_0(\eta, \beta) = 0$$

and

$$U_T(\eta, \beta, \mu) = (\eta - \mu\eta)^2 + 1 + \left(\frac{\mu\eta}{\beta}\right)^2 + \int_{-\infty}^{\infty} V_{T-1}(y, b)\varphi(\varepsilon)d\varepsilon \tag{7.6}$$

where φ is the normal probability density and

$$y = \eta - \mu\eta + \varepsilon\sqrt{1 + \left(\frac{\mu\eta}{\beta}\right)^2}$$

$$b = \frac{\mu\eta\varepsilon}{\beta} + \beta\sqrt{1 + \left(\frac{\mu\eta}{\beta}\right)^2} - \frac{\mu\beta}{\eta}\varepsilon$$

see Åström (1978). When the minimization is performed the control law is obtained as

$$\mu_T(\eta, \beta) = \arg\min_u U_T(\eta, \beta, \mu) \tag{7.7}$$

The minimization can be done analytically for T = 1. We get

$$\mu_1(\eta, \beta) = \arg\min_u \left[(\eta - \mu\eta)^2 + 1 + \left(\frac{\mu\eta}{\beta}\right)^2\right] = \frac{\beta^2}{1+\beta^2}$$

Transforming back to the original variables we get

$$u(t) = -\frac{1}{\hat{b}(t)} \cdot \frac{\hat{b}^2(t)}{b^2(t)+\sigma^2(t)}\, y(t) \tag{7.8}$$

This control law is called one-step control or myopic control because the loss function V_1 only looks one step ahead.

For T > 1 the optimization can no longer be made analytically. Instead we have to resort to numerical calculations. For large values of T the solution can be approximated by

$$\mu(\eta, \beta) = \frac{0.56\beta + \beta^2}{2.2 + 0.08\beta + \beta^2} + \frac{1.9\beta}{\eta(1.7 + \beta^4)} \qquad \eta > 0, \ \beta > 0.$$

The control law is an odd function in η and β, see Åström and Helmersson (1983).

Some approximations to the optimal control law will also be discussed. The certainty equivalence control

$$u(t) = - y(t)/\hat{b} \tag{7.9}$$

is obtained simply by taking the control law (2.24) for known parameters and substituting the parameters by their estimates. The self-tuning regulator can be interpreted as a certainty equivalence control. Using normalized variables the control law becomes

$$\mu = 1 \tag{7.9'}$$

The myopic control law (7.8) is another approximation. This is also called cautious control, because in comparison with the certainty equivalence control it hedges and uses lower gain when the estimates are uncertain. Notice that all control laws are the same for large β i.e if the estimate is accurate. The optimal control law is close to the cautious control for large control errors. For estimates with poor precision and moderate control errors the dual control gives larger control actions than the other control laws.

A simulation of the dual control law for an integrator with variable gain is shown in Figure 8. Notice that the gain varies by an order of magnitude in size and that it changes sign at T = 2000. In spite of this the regulator have little difficulty in controlling the process. Also notice that the regulator does probing well before the gain changes time and that it jumps between caution and probing when the gain passes through zero.

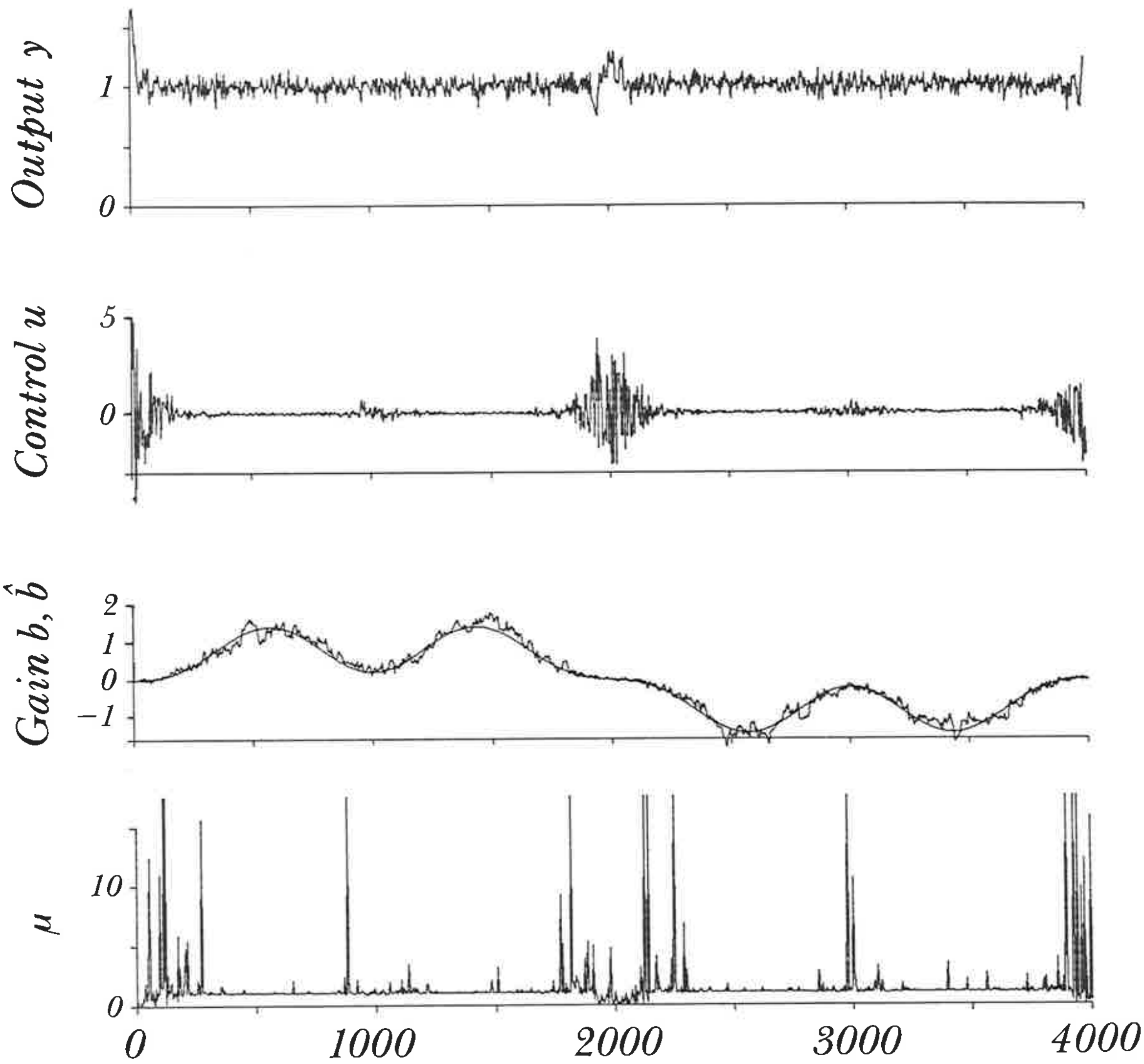

Figure 8. Simulation of dual control law applied to integrator with variable gain.

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