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# THE LOW-ENERGY SPECTRUM OF $^{29}\text{Si}$ AND THE EXISTENCE OF LARGE OBLATE DISTORTIONS ASSOCIATED WITH CERTAIN INTRINSIC ORBITALS

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**Abstract:** Equilibrium calculations involving different odd-particle excitations for  $^{29}\text{Si}$  and neighbouring nuclei are reported. Minima corresponding to very large oblate distortions are confirmed to be associated with a recently found rotational band in  $^{29}\text{Si}$  as suggested in ref. <sup>1</sup>).

## 1. Introduction

In a recent publication <sup>1</sup>) experimental evidence was given for a very strongly oblate rotational band based on the orbital  $[303 \frac{7}{2}]$ . The existence of strong oblate distortions for at least some intrinsic configurations of  $^{29}\text{Si}$  is reasonable in view of the fact that the ground state band of  $^{28}\text{Si}$  is associated with a strong intrinsic oblate quadrupole moment <sup>2,3</sup>). An analysis of the  $^{29}\text{Si}$  spectrum was made by Bromley *et al.* <sup>4</sup>) in 1957 suggesting oblate shapes for the lowest states of that nucleus. Renewed theoretical studies of the total potential-energy surface based on a generalized harmonic-oscillator potential <sup>5</sup>) and a subsequent normalization of the computed total energy (so as on the average to reproduce the liquid-drop behaviour) by the Strutinsky shell correction method <sup>6</sup>) is presently being undertaken for large regions of the periodic table. One aim of this study is to find the  $A$  and  $Z$  dependence of the nuclear potential parameters with important application to the problem of the superheavy elements <sup>7</sup>). The theoretical equilibrium configurations are found to explain remarkably well several dominant features of the  $^{29}\text{Si}$  spectrum.

## 2. Single-particle energies and the total nuclear energy

The following nuclear potential <sup>5</sup>), involving essentially quadrupole ( $\epsilon$ ) and hexadecapole ( $\epsilon_4$ ) degrees of freedom has been considered (particularly in the beginning of the  $A = 16$ –40 region the latter degree of freedom is of great importance)

$$V = \frac{1}{2}M\omega_0^2(\epsilon, \epsilon_4)(1 - \frac{2}{3}\epsilon P_2 + 2\epsilon_4 P_4) - \kappa\hbar\omega_0[2I_t \cdot s + \mu(I_t^2 - \langle I_t^2 \rangle_N)].$$

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The parameters  $\kappa$  and  $\mu$  are assumed to vary with  $A$  in a simple way over the periodic

table. For the  $A \approx 25$  region we have (as previously) assumed <sup>8)</sup>  $\kappa_n = \kappa_p = -0.08$ ,  $\mu_n = \mu_p = 0$ . We have furthermore assumed that in this particular region pairing is negligible ( $G_n = G_p = 0$ ). The Coulomb energy is calculated by a numerical method due to Nix, as in ref. <sup>7)</sup>. The total nuclear energy is subsequently evaluated as a function of  $\epsilon$  and  $\epsilon_4$  by employment of the Strutinsky shell correction method <sup>6)</sup>. In the

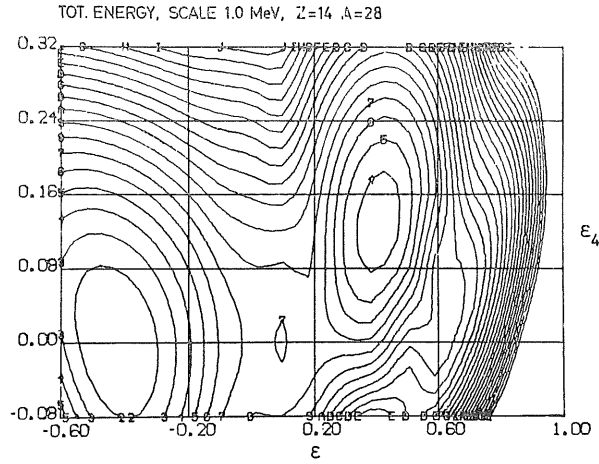


Fig. 1. The potential energy surface in the  $(\epsilon, \epsilon_4)$  plane for  $^{28}_{14}\text{Si}$ .

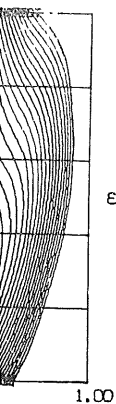
odd case the computer program places the odd particle in the orbital of specific  $\Omega$  and parity that is available nearest to the Fermi surface.

The  $^{28}_{14}\text{Si}$  potential energy surface for the  $0^+$  state is exhibited in fig. 1. Note the strongly oblate minimum corresponding to  $\epsilon = -0.43$ ,  $\epsilon_4 = 0.00$ , which under the assumption of homogeneous charge correspond to an intrinsic quadrupole moment  $Q_0 = -54 \text{ fm}^2$ . The apparent secondary prolate minimum situated roughly 2 MeV over the ground state oblate minimum may or may not be a proper minimum as the barrier in the gamma plane as encountered in the calculations by Cusson <sup>9)</sup> may not be sufficient to sustain the zero-point vibration.

### 3. Discussion of the $^{29}\text{Si}$ spectrum

Let us now consider the theoretical results for the  $^{29}\text{Si}$  case, see figs. 2 and 3. (Actually fig. 3 refers specifically to  $^{29}\text{P}$  whose potential energy surface differs from that of its mirror nucleus only with respect to the Coulomb term, whose difference in this case is negligible.) In fig. 2 the potential energy surface corresponding to  $\Omega = \frac{7}{2}$  is illustrated. It is found to exhibit only one well-developed minimum corresponding to  $\epsilon = -0.49$ ,  $\epsilon_4 = 0.00$ . In fig. 3 we give cuts through the two-dimensional energy surfaces corresponding to specified  $\Omega$  and parity in such a way that for each  $\epsilon$ -value we let  $\epsilon_4$  assume the value that minimizes the energy. In this way fig. 3 is obtained,

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 this particular region pairing is  
 calculated by a numerical method  
 subsequently evaluated as a func-  
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plane for <sup>28</sup>Si.

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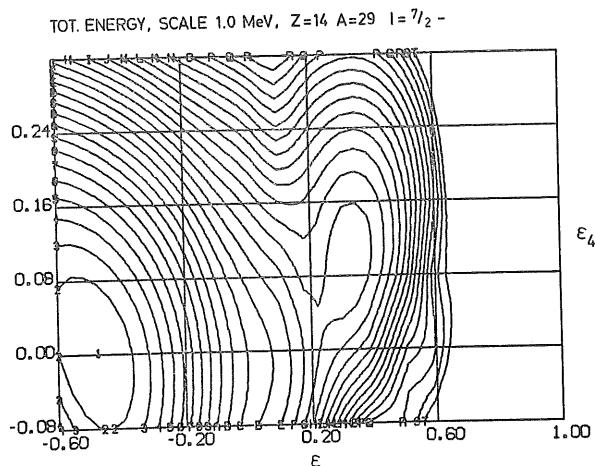


Fig. 2. Same as fig. 1 valid for <sup>29</sup>Si with the odd particle placed in an orbital with  $\Omega = \frac{7}{2}$  and negative parity.

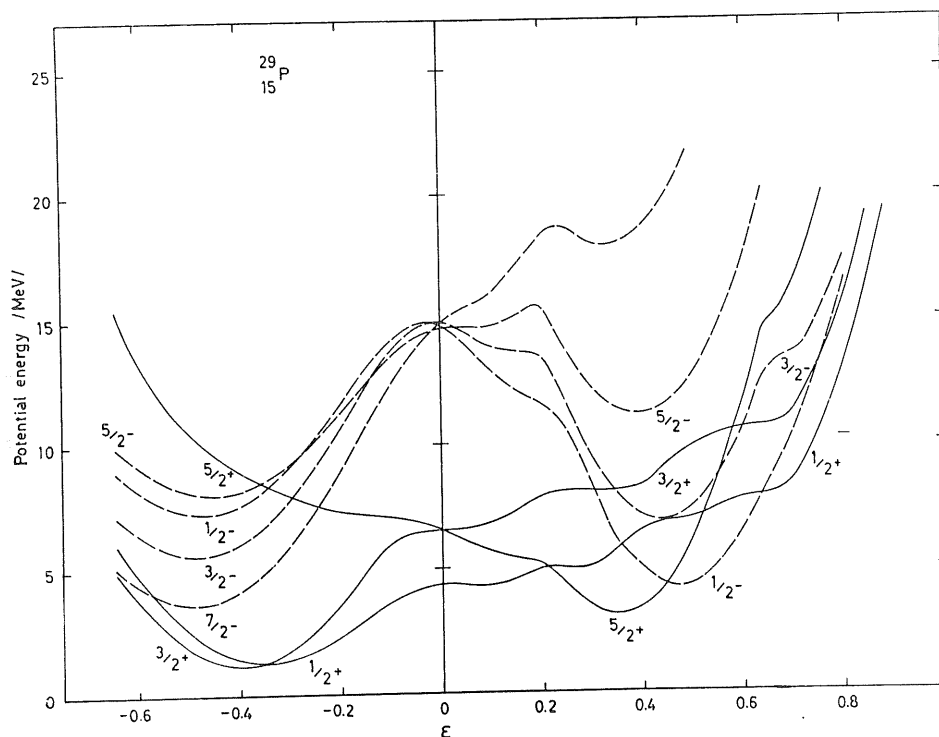


Fig. 3. The total energy for <sup>29</sup>P as a function of distortion for the odd particle associated with alternative orbitals of given  $\Omega$  and  $\pi$ . Except for the difference in Coulomb energy this diagram should be equal to that of <sup>29</sup>Si. The diagram represents cuts through surfaces of the type given in fig. 2. For each  $\epsilon$  the lowest energy with respect to  $\epsilon_4$  is plotted.

representing the lowest configurations of given  $\Omega$  and  $\pi$  for each distortion. From this figure one should expect an oblate  $\frac{1}{2}^+$  configuration ( $\varepsilon = -0.34$ ,  $\varepsilon_4 = 0.01$ ) and an oblate ( $\varepsilon = -0.39$ ,  $\varepsilon_4 = 0.02$ )  $\frac{3}{2}^+$  configuration at about the same energy and a  $\frac{5}{2}^+$  prolate one ( $\varepsilon = 0.35$ ,  $\varepsilon_4 = 0.11$ ) about 2 MeV higher. The strongly deformed oblate  $[303 \frac{7}{2}]$  configuration (corresponding to  $\varepsilon = -0.49$ ,  $\varepsilon_4 = 0.00$ ) occurs theo-

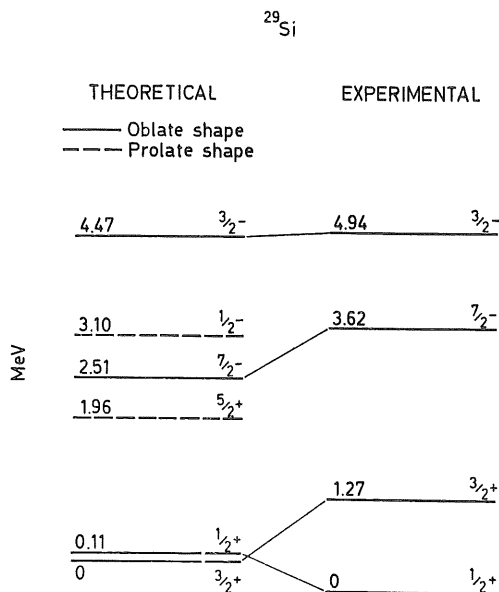


Fig. 4. Experimental and theoretical intrinsic spectra of  $^{29}\text{Si}$ .

retically at about 2.5 MeV of excitation. The next excited oblate odd state is  $[301 \frac{3}{2}]$  with  $\varepsilon = -0.48$ ,  $\varepsilon_4 = -0.01$  predicted about 2 MeV higher in energy. The low-lying even-parity levels have earlier been identified by Bromley *et al.* <sup>4)</sup> as associated with weakly oblate ( $\varepsilon = -0.15$ ), Coriolis coupled orbitals of  $\Omega = \frac{1}{2}^+$  and  $\Omega = \frac{3}{2}^+$ . This interpretation is roughly in agreement with the theoretical results mentioned above apart from the larger theoretical distortions (the energy surface is, however, relatively flat and no pairing is included). The  $[202 \frac{5}{2}^+]$  orbital is probably involved at some place in the spectrum above a few MeV of excitation. As to other higher excited even-parity states we are aware that other degrees of freedom, not directly represented by minima in these potential energy surfaces, are expected to occur, e.g. in the form of particle-hole excitations.

(On the other hand when all these kinds of excitations are introduced, there is, of course, a clear redundancy of states, as for each distortion there exists a complete set of states once all particle-hole excitations are considered. Therefore, for each  $\Omega$  and parity, usually only the lowest-lying excitations can be reliably obtained by the method employed here.) The emphasis in this discussion is placed on the odd-parity states

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observed in ref. <sup>1)</sup>. In view of the discussion above, the assignment of the odd-parity states should be comparatively reliable, as the minima are very deep both for the  $[303 \frac{7}{2}]$  state and the  $[301 \frac{3}{2}]$  state, and as they indeed are the lowest odd-parity excitations of the respective  $\Omega$  and parity. Both these orbitals originate from the spherical  $f_{7/2}$  orbital. One may note that both theoretically and experimentally the orbital associated with the  $\frac{5}{2}^-$  orbital occurs higher in energy than the  $\frac{3}{2}^-$  orbital. This reordering of the  $\frac{7}{2}$ ,  $\frac{5}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$  sequence is a non-diagonal effect of the  $P_2$  deformation. The experimental and theoretical intrinsic spectra are given in fig. 4.

Two members,  $I = \frac{7}{2}$  and  $\frac{5}{2}$ , of the  $\frac{7}{2}^-$  band are observed <sup>1)</sup>, the band head at 3.62 MeV, or about 1 MeV higher than theoretically expected. The discrepancy is even smaller for the  $\frac{3}{2}^-$  excitation which experimentally occurs at 4.94 MeV. The E2 matrix element for the interband transition of the  $\frac{7}{2}^-$  band corresponds to an intrinsic quadrupole moment of  $|Q_0| = 66 \pm 9 \text{ fm}^2$ . The theoretical equilibrium distortions would correspond to a  $Q_0 = -62 \text{ fm}^2$ , in excellent agreement with experiments. There remains a problem of the empirical identification of the  $[330 \frac{1}{2}]$  rotational band, which is seen at an excitation of 3.09 MeV in <sup>25</sup>Al. According to the calculations the  $[303 \frac{7}{2}]$  orbital is never found to occur as a ground state in this region of nuclei.

It is tempting to conjecture that similar oblate configurations associated with  $[404 \frac{9}{2}]$  may be expected to occur for  $Z$  and  $N$  about 28, provided the single-particle preference for oblate distortions is strong enough to overcome the prolate tendency associated with the  $N = 28$  shell <sup>9)</sup>. No theoretical calculations corresponding to the ones presented here are presently available, however, in the relevant regions of nuclei. Results pertaining to the entire  $A = 16-40$  region as obtained by the method described will be published by Ragnarsson.

The comparison of experimental and theoretical level energies (see fig. 4), shows that the agreement is much less impressive than in heavier regions exhibiting stable equilibrium distortions. Some of the partial failure in this light region is probably due to the fact that the neglected gamma-degree of freedom here plays a role important enough to invert the predicted level order. In addition the single-particle orbital occupied by the odd particle may by itself radically effect the equilibrium shape, as is exemplified by the present case. This leads to some orthogonality problems for the equilibrium configurations calculated, as was pointed out above.

Hartree-Fock calculations in fairly large numbers are available for the <sup>28</sup>Si case, indicating oblate equilibrium shapes for that nucleus (for a review and further references see Ripka <sup>10)</sup>). Unfortunately, for the odd <sup>29</sup>Si case no such calculations are available for comparison.

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