Fission barriers and the inclusion of axial asummetry

Ragnarsson, Ingemar; Nilsson, Sven Gösta; Larsson, S E

Published in:
Physics Letters. Section B: Nuclear, Elementary Particle and High-Energy Physics

1972

F:560

Reprinted from

PHYSICS LETTERS

Volume 38B, No. 5, 6 March 1972

FISSION BARRIERS AND THE INCLUSION OF AXIAL ASYMMETRY

S. E. LARSSON, I. RAGNARSSON and S. G. NILSSON
Department of Mathematical Physics, Lund Institute of Technology, Lund, Sweden

pp. 269 – 273

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM

FISSION BARRIERS AND THE INCLUSION OF AXIAL ASYMMETRY

S. E. LARSSON, I. RAGNARSSON and S. G. NILSSON
Department of Mathematical Physics, Lund Institute of Technology, Lund, Sweden

Received 16 January 1972

The nuclear potential-energy surface has been calculated on the basis of the modified-oscillator model with the inclusion of the degrees of freedom of axial asymmetry. For the heavier actinides the inner barrier is found to be reduced by up to one MeV. This is in agreement with the results obtained by Pashkevich. For superheavy elements a similar reduction is obtained.

The potential energy surface of actinide and superheavy nuclei has recently been calculated [1-4] on the basis of the Strutinsky-Swiatecki shell correction method [1]. Several shape parameters have been considered, as P_2, P_4 and P_6 and in addition also reflection asymmetric degrees of freedom. Various potentials and parametrisations have been employed by the authors cited and others. Recently Pashkevich [5] has studied the influence on the fission barrier by the inclusion of axial asymmetry. Preliminary results of more recent calculations have been reported by Götz et al. [6] and are presently beeing completed by these authors for actinide and superheavy elements. One should also mention the inclusion of the Y_{44} and Y_{33} degrees of freedom considered by the Greiner [7] group.

Axially asymmetric collective shape coordinates of nuclei were introduced in the early papers by Bohr [8]. Spectroscopic data so far available appear to indicate that there is a strong preference in nature for relatively pure prolate spheroidal shapes in the established regions of nuclear deformations.

However, according to the calculations by Baranger and Kumar [9] as well as to the theoretical and empirical analysis e.g. by Johnson and Foucher [10] there is a transitional region near $A \approx 190$ as well as e.g. one near ^{132}Ba, where the equilibrium deformations appear to change from prolate to oblate and where intermediate distortions into the γ-plane may develop. We have therefore first applied our calculations to the $A \approx 190$ region as a test case.

The present calculations have been based on the modified oscillator potential with parameters as given in ref. [2]

$$V = V_{\mathrm{osc}} - 2\kappa \cdot \hbar \overset{\circ}{\omega}_0 \, l_t \cdot s - \kappa\mu \cdot \hbar \overset{\circ}{\omega}_0 (l_t^2 - \langle l_t^2 \rangle) ,$$

where the ellipsoidal distortion of V_{osc} is defined by

$$V_{\mathrm{osc}} = \hbar\,\omega_0 \cdot \rho^2 \left\{ 1 - \tfrac{2}{3}\,\epsilon' \sqrt{\frac{4\pi}{5}} \left[\cos\gamma \cdot Y_{20} - \frac{\sin\gamma}{\sqrt{2}} \times (Y_{22} + Y_{2-2}) \right] \right\} .$$

This corresponds to the choice of three independent oscillator frequencies

$$\omega_x = \omega_0 \{1 - \tfrac{2}{3}\epsilon' \cdot \cos(\gamma + \tfrac{2}{3}\pi)\}$$
$$\omega_y = \omega_0 \{1 - \tfrac{2}{3}\epsilon' \cdot \cos(\gamma - \tfrac{2}{3}\pi)\}$$
$$\omega_z = \omega_0 \{1 - \tfrac{2}{3}\epsilon' \cdot \cos\gamma\} .$$

This type of nonisotropic oscillator potential was first studied by Newton [11]. In the above formulation the potential is easily generalised to include for instance hexadecapole moments by addition of $\epsilon_4 \rho^2 P_4$ etc. in V_{osc}. In the present paper we limit ourselves to the case $\epsilon_4 = 0$ (see below).

In terms of pure ellipsoidal shapes corresponding to $\epsilon_4 = 0$ the surface and Coulomb energies can be expressed as simple integrals amenable to numerical computer evaluation. A convenient expression for E_{coul} was kindly developed for us by Nilsson [12]. We are presently trying to incorporate a program for evaluating the Coulomb energy also for the case that $\epsilon_4 \neq 0$, while for the surface energy ϵ_4 is easily included.

An example of the results of the test calculations in the $A \approx 190$ region is provided in fig. 1. In this preliminary version the results are given

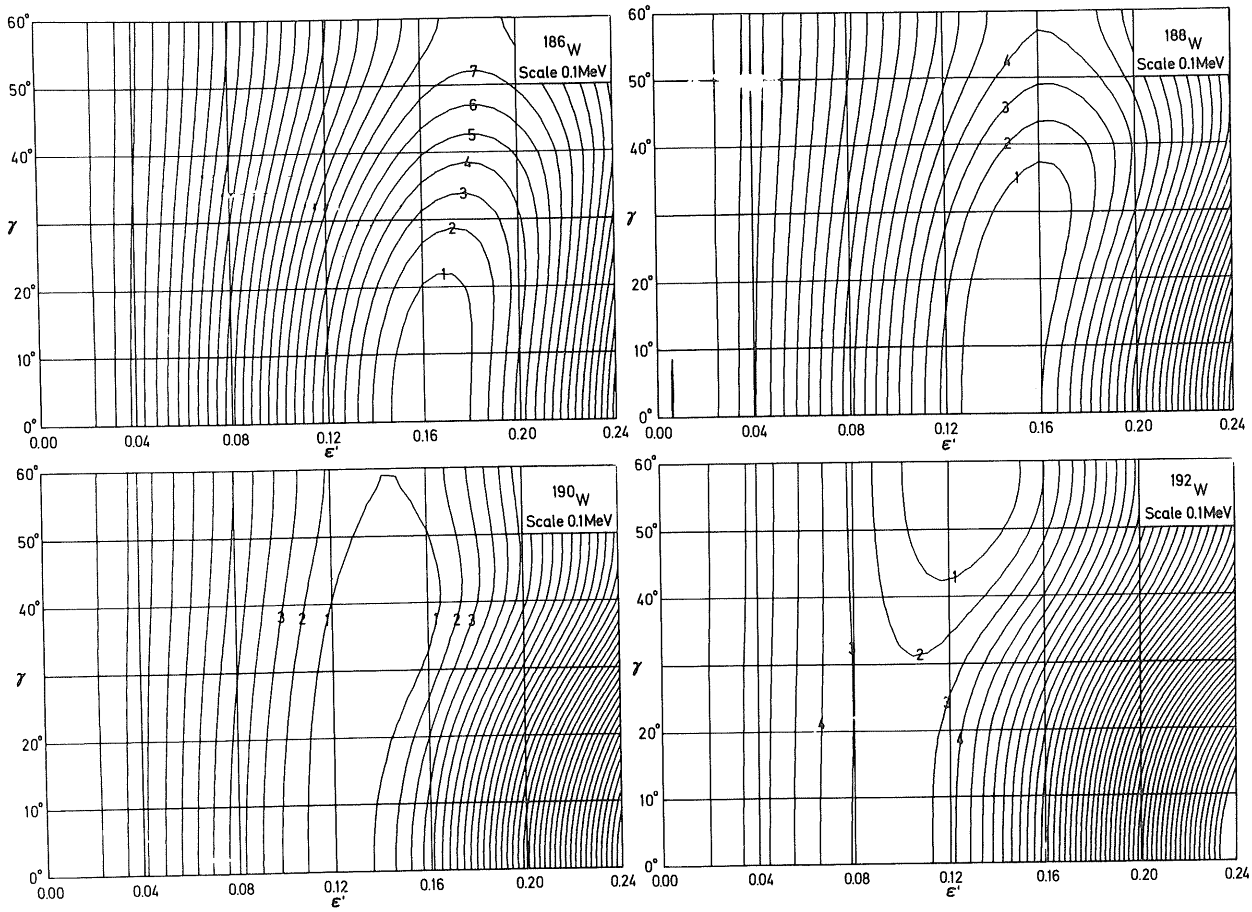

Fig. 1. Potential-energy surfaces for a series of W isotopes with A = 186 to 192 in terms of the distortion parameters ϵ and γ. Note that for data-technical convenience we have used a right-angle plot. Small deficiencies in the plotting program account for the fact that equipotential lines do not intersect the axes at right angles. Equipotential lines are numbered in units of 0.1 MeV. The minimum is placed at 0 MeV.

in right-angle plots with ϵ along the x-axis and $\gamma = 0 - 60^{\mathrm{o}}$ along the y-axis rather than in the more appropriate plots as suggested by the work of Bohr [8] and used by Kumar and Baranger [9].

Similar calculations have been carried out for Os and Pt in addition to W. For W nearly pure prolate, equilibrium deformations occur for $A \leqslant 186$ while for A = 188, 190 we obtain equilibrium values of $\gamma \approx 20 - 40^{\mathrm{o}}$. Furthermore A = 194, 196 have purely oblate minima. For Os and Pt the situation is very similar although intermediate γ values there occur for $A \approx 190$, 192 and stable oblates first for A = 194 according to this model. (With the inclusion of ϵ_4 these results appear to be slightly modified.) Our results with respect to the static potential energy surface are in good agreement with those of ref. [9].

We have extended the calculations to larger distortions (fig. 2) where a secondary minimum was predicted by Gustafsson et al. [13] for $A \approx 195$. The inclusion of the γ degree of freedom is found to leave the secondary minimum point unaffected although the barrier against electromagnetic decay back to the spherical ground state is considerably reduced.

A few of the nuclei in the actinide region studied are displayed in fig. 3. The first of the two-peak barriers is lowered by an amount of a few hundred keV for Pu, the reduction increasing to a little more than one MeV for Fm and beyond. Normal isotopes of Th and U are found to be stable or nearly stable to axially asymmetric distortions. These results are in good agreement with those obtained by Pashkevich [5].

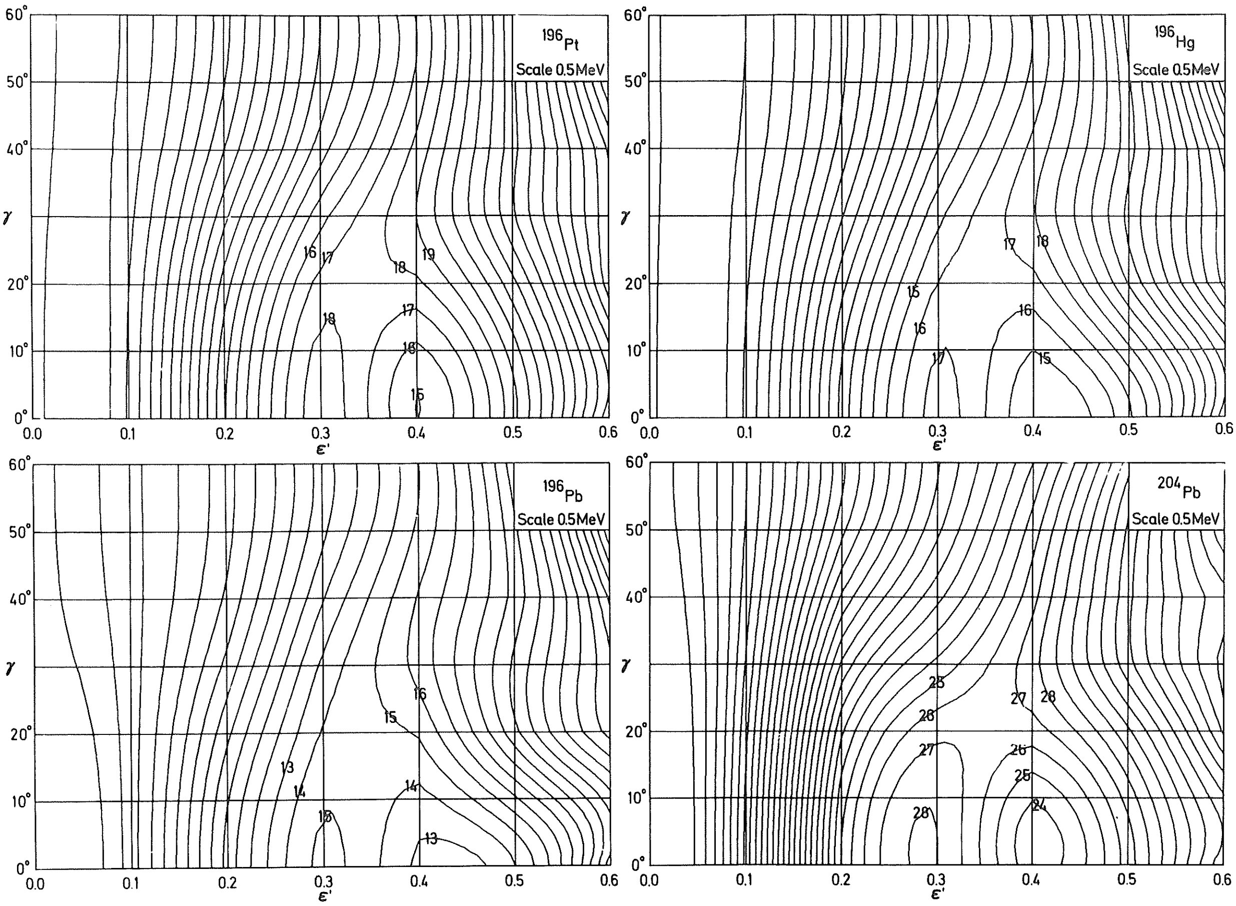

Fig. 2. Potential-energy surfaces in terms of ϵ and γ for some $A \approx 200$ elements. Note the secondary minima for $\epsilon \approx 0.4$ - 0.5 and the reduction of the barrier due to the γ degree of freedom. Equipotential lines are numbered in units of 0.5 MeV with spherical minimum at 0 MeV.

The difference is essentially a factor of about two and probably reflects the number of nuclear shells included; in our case N_{max} = 14. When the energy reduction due to γ, calculated for the case $\epsilon_4 = \epsilon_3 = \epsilon_5 = 0$, is applied to the barriers calculated by Möller [14], with ϵ_4, ϵ_3 and ϵ_5 included, fig. 4 is obtained, showing an improved agreement with the semi-empirical values as estimated by Björnholm and Lynn [15].

The fission barriers for nuclei near Z = 114, N = 184 are presently of great interest. The reduction of the inner barrier due to γ-asymmetry turns out relatively modest as shown in fig. 5, for the lighter of the exhibited nuclei less than one MeV.

The inclusion of the axial degree of freedom is found to reduce the inner one of the fission barriers in the actinide region by up to one MeV, improving the agreement with the empirically estimated barrier heights.

In the A = 190 - 210 region we have directed our attention to the secondary minimum. The barrier separating this from the ground state is found to be similarly reduced by of the order of one MeV. It appears that in fortunate cases a few isomeric states should still be observable around $A \approx 200$.

In the superheavy region, finally, for $Z \leqslant 114$, $N \leqslant 184$, the barrier reduction is generally less than one MeV but somewhat larger above those limits of Z and N.

As no inertial parameters associated with the axial degrees of freedom are presently available beyond the $A \approx 190$ region, no proper estimates of the corresponding reductions of fission half-lives can be made. However, it appears reason-

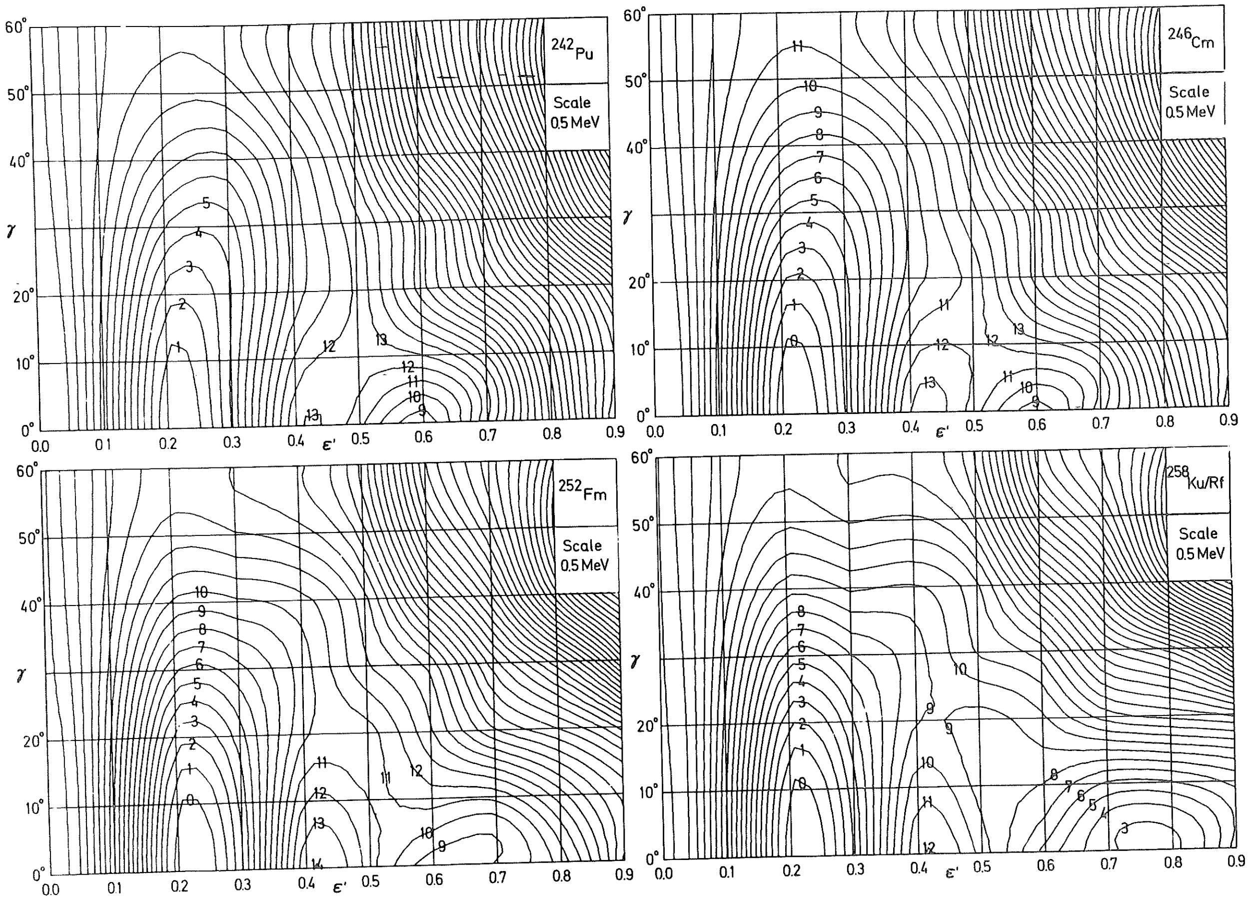

Fig. 3. Same as fig. 2 for ^{242}Pu, ^{246}Cm, ^{252}Fm and ^{258}Ku/Rf. The lowest energy in the figures corresponds to 0 MeV and the first equipotential line is labeled "1" for ^{242}Pu. In the three other cases the lowest equipotential line is instead labeled by "0".

able to expect a reduction of up to a few powers of ten of the superheavy element half-lives.

References

[1] V. M. Strutinsky, Nucl. Phys. A95 (1967) 420.

[2] S. G. Nilsson, C. F. Tsang, A. Sobiscewski, Z. Szymanski, S. Wycech, C. Gustafsson, I. L. Lamm and B. Nilsson, Nucl. Phys. A131 (1969) 1.

[3] M. Bolsterli, E. O. Fiset, J. R. Nix and J. L. Norton, Phys. Rev. C, to be published.

[4] M. Brack, J. Damgaard, H. C. Pauli, A. Stenholm Jensen, V. M. Strutinsky and C. Y. Wong, Rev. Mod. Phys. (1972), to be published.

[5] V. V. Pashkevich, Nucl. Phys. A133 (1969) 400.

[6] U. Götz, H. C. Pauli and K. Alder, Nucl. Phys. A175 (1971) 481 and private communication.

[7] R. Fraser, J. Grumann and W. Greiner, Phys. Letters 35B (1971) 483.

[8] A. Bohr, Mat. Fys. Medd. Dan. Vid. Selsk. 26, No. 14 (1952).

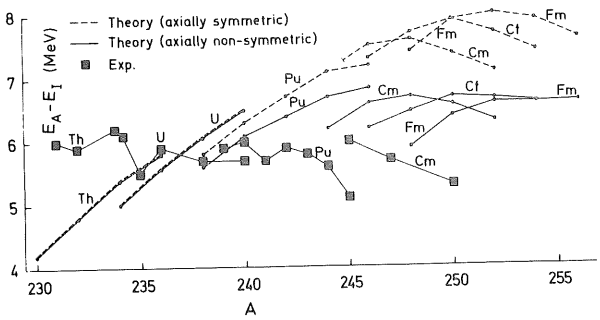

Fig. 4. Reduction of calculated inner fission barriers due to the inclusion of γ. In the original calculations ϵ_4, ϵ_3 and ϵ_5 are included in the finding of the "minimal projection" barriers [2,14]. From these potential energy values the effects due to γ (calculated for $\epsilon_4 = \epsilon_3 = \epsilon_5 = 0$) are subtracted. For comparison the empirical inner barrier heights as given by Björnholm and Lynn [15] are exhibited.

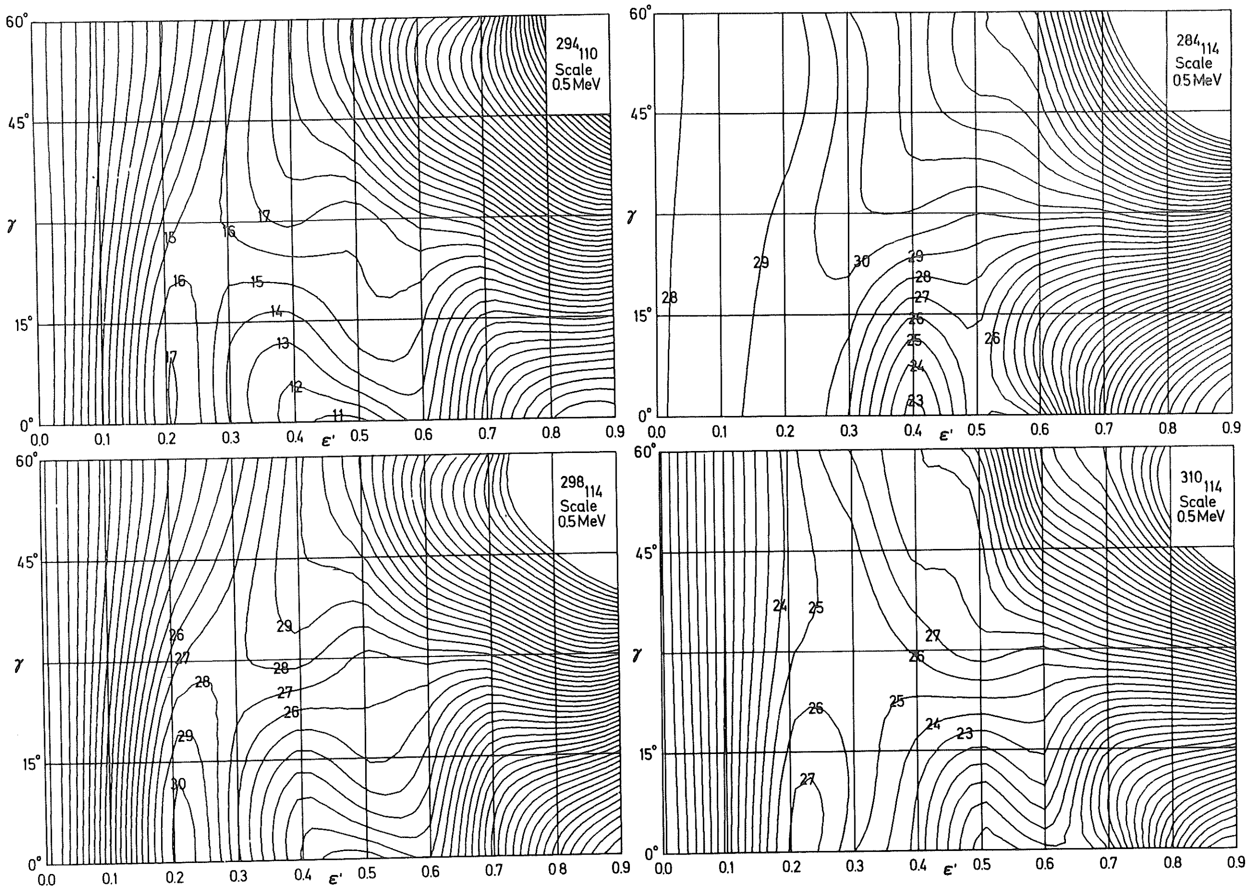

Fig. 5. Potential-energy surfaces for a few sampled superheavy elements in terms of ϵ and γ. Note the modest reduction of the inner barrier due to γ. Note also the inherent deficiencies in the plot at $\epsilon = 0$. The minimum for the spherical case corresponds to the entire γ-axis. The equipotential lines are labeled by computer in units of 0.5 MeV such that the lowest line corresponds to "1". Note that the nucleus $^{284}114$ is deformed in its ground state, while the other superheavy nuclei have spherical ground states.

[9] K. Kumar and M. Baranger, Nucl. Phys. A110 (1968) 529.
[10] A. Johnson and R. Foucher, to be published; A. Johnson, thesis, Faculté de Science de Paris (Paris 1971).
[11] T. D. Newton, Can. J. Phys. 38 (1960) 700.
[12] S. B. Nilsson, private communication.
[13] C. Gustafsson, I. L. Lamm, B. Nilsson and S. G. Nilsson, Ark. Fys. 36 (1967) 613.
[14] P. Möller, Nucl. Phys., to be published.
[15] S. Björnholm and L. Lynn, Rev. Mod. Phys., to be published.

* * * * *