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# SC-LDPC Codes over the Block-Fading Channel: Robustness to a Synchronisation Offset

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**Abstract**—Spatially-Coupled LDPC (SC-LDPC) codes have been recently shown to be very efficient for transmissions over nonergodic channels, in particular over block-fading channels [1]. In fact, it is possible to design a SC-LDPC code with any given code diversity [2]. In this work, we investigate the performance of SC-LDPC codes over block-fading channels, assuming a mismatch (or offset) between the first bit of a transmission packet and a first bit of a codeword. Such a mismatch is called the synchronisation offset, and it has a negative impact on the code diversity. We propose a data-allocation scheme for SC-LDPC codes that allows to obtain a robustness to the synchronisation offset. Combined together with the code design from [2], it allows to design efficient SC-LDPC codes, whose performance degrades only slowly under imperfect transmission conditions.

## I. INTRODUCTION

The mobile-radio channel can be modelled as a slow, flat fading together with additive noise. In many cases (e.g., short-range, high-throughput data communications), the channel coherence time (time where the channel fading is constant) is much longer than one symbol duration. Thus several symbols are affected by the same fading coefficient. An example of such a channel model is the *block-fading* channel introduced in [3]. In block-fading channel, coded information is transmitted over a finite number of fading blocks to provide *diversity*. The diversity order  $d$  of the code is an important parameter that gives the slope of the word error rate (WER) of the decoder. The WER defines the performance of the code for given channel conditions. Codes achieving diversity equal to the number of fading blocks in a codeword are said to be full diversity codes. In [4], a family of LDPC block codes, called *root-LDPC* codes, are proposed that provide full diversity<sup>1</sup>  $d = 2$  over a block-fading channel with two fading gains per codeword. [4] motivated a number of further results on the design of full-diversity "root" codes, see [5], [6]. However, for a rate  $1/2$  code all these examples still have  $d = 2$ , and the boundedness of the root structure, on which they are built on, makes it a complicated task to extend these designs to  $d > 2$ . Therefore, regardless a quite large number of previous works on coding over block-fading channels, the question of a systematic design of a code in order to meet *any* arbitrary diversity order  $d$  is not solved completely yet. Clearly, such

a constraint is difficult to meet with block codes, and one should probably move to convolutional structures, known to be suitable to get  $d > 2$ .

Recently, we have started the investigation whether *spatially coupled low-density parity-check (SC-LDPC) codes* can be used for transmissions over block-fading channels [1], [7]. Indeed, it happens [1] that SC-LDPC codes have very good performance over the block-fading channel once the coupling parameter  $m_{cc}$  (or the so called *memory*) of the ensemble is increased. The reason for such a good performance comes from the fact that SC-LDPC codes merge both block-code and convolutional-code designs. Moreover, this allows to choose among a large number of potentially good code structures (the so called *protographs*), on which SC-LDPC codes are based. Very recently [2], we have proposed an algorithm, recursively building a protograph structure of a SC-LDPC code ensemble. The construction allow to attain the target slope of the word error rate performance over the block-fading channel, i.e., allowing to get a desired diversity order  $d$ .

This work is the start of our study on SC-LDPC performance over block-fading channels *under imperfect transmission conditions*. We would like to confirm that, in addition to their good performance over block-fading channels, SC-LDPC codes are also robust to a synchronisation mismatch and imperfect channel state estimations. In this paper we focus ourselves on one particular mismatch model that is the *synchronisation offset*: the receiver gets transmission packets, each of them experiencing a different fading gain  $\alpha$ , however, the first bit of each transmission packet does not match with the first bit of a coded packet, i.e., the sequence of transmitted packets are shifted with respect to the sequence of coded packets. This implies that each coded packet (codeword), sent by transmitter, experiences two different fading gains (one corresponding to the first, and another - to the next transmission slot). Note that such a synchronisation offset model describes well the transmission mismatch which happens for instance: a) in HSDPA and LTE systems, when the frequency hopping is operated independently of the channel encoding operation (this is usually a result of the clock mismatch after a long sleep mode); b) during the handover, when two base stations are not perfectly synchronised among them; or c) in high-speed mobility scenarios, when the Doppler effect is significant.

<sup>1</sup>This is the optimal diversity order for codes of rate  $1/2$  from [4].

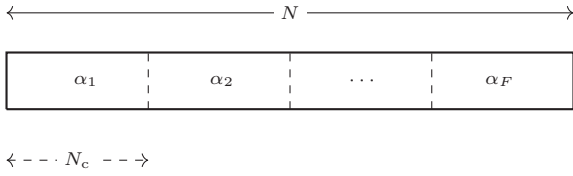


Fig. 1. Impact of the block-fading channel to a codeword of length  $N$ , sent through  $F$  coherence intervals of length  $N_c$  each.  $\alpha$ 's are fading coefficients of corresponding intervals.

The paper is organized as follows. The transmission model and SC-LDPC codes are defined in Section II. Section III-A describes the calculation of the diversity order for a fixed SC-LDPC code ensemble, as well as the method to design a SC-LDPC code with arbitrary diversity order. We also consider different allocation protocols of SC-LDPC coded bits within the transmission stream. Our approach to design a transmission scheme of a target diversity order is given in Section III-B; it is also explained how the non-zero synchronisation offset can be taken into account. Section IV provides numerical results. Finally, Section V concludes the paper.

The principal parameters, used in the paper, are coloured in blue the first time when they are mentioned in the text.

## II. SYSTEM MODEL

### A. Block-Fading Transmission

Let us start with the classical *block-fading transmission model*. Assume that a codeword of length  $N$  is transmitted over a block-fading channel, having a coherence time  $N_c = N/F$  (measured in bits). Hence, the codeword can be seen as divided in  $F$  transmission packets, each of them being affected during the transmission by a fading gain  $\alpha_j$ ,  $j = 1, \dots, F$ , see Fig. 1 for illustration. The received symbols,  $y_i$ ,  $i = 1, \dots, N$ , have the following form,

$$y_i = \alpha_j x_i + n_i. \quad (1)$$

Here  $j = 1 + \lfloor (i-1)/N_c \rfloor$  and  $\lfloor \cdot \rfloor$  represents the floor operator. The input symbols  $x_i$  are assumed to be chosen from BPSK alphabet and  $n_i$  are Gaussian random variables with zero mean and variance  $\sigma_n^2$ . The symbols are normalized to  $x_i = \pm 1$  and the fading coefficients  $\alpha_j$  are Rayleigh distributed with  $\mathbb{E}[\alpha_j^2] = 1$ .

In what follows, we will also consider the *block-fading transmission model with a synchronisation offset*, defined as follows. Assume that  $L$  codewords of length  $N$  are transmitted over a block-fading channel, having a coherence time  $N_c = N/F$ . The received symbols,  $y_{m,i}$  with  $i = 1, \dots, N$  and  $m = 1, \dots, L$ , have the following form,

$$y_{m,i} = \alpha_j x_{m,i} + n_{m,i} \quad (2)$$

where  $j = (m-1)F + 1 + \lfloor (i-1)/N_c - \delta \rfloor$ , with  $\delta$  being the synchronisation offset,  $0 \leq \delta \leq 1$ . Note that  $\delta = 0$  corresponds to the classical block-transmission model, while  $\delta = 1$  corresponds to the classical transmission model, given by (1), in which  $j = \lfloor (i-1)/N_c \rfloor$ . In these two extreme cases, any of  $L$  codewords can still be divided into  $F$  equal

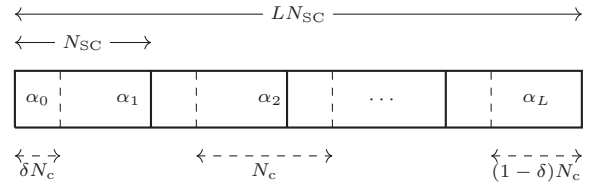


Fig. 2. Illustration of the synchronisation offset in the case of  $F = 1$  and  $N = N_{SC}$ : a sequence of  $L N_{SC}$  coded bits from  $L$  sequentially generated codewords is affected by  $L + 1$  fading gains during the transmission over the block-fading channel with  $\delta > 0$ . Straight vertical lines delimit the codewords, dashed vertical ones - coherence intervals of length  $N_c$ .

transmission packets. For  $0 < \delta < 1$ , however, a received codeword  $y_{m,1}, \dots, y_{m,N}$  is affected by  $F + 1$  fading gains: its first  $\delta N_c$  bits are affected by  $\alpha_{(m-1)F}$ ,  $F - 1$  equal parts of  $N_c$  bits each are affected by respective fading gains  $\alpha_{(m-1)F+1}, \dots, \alpha_{(m-1)F+F-1}$ , and its last  $(1-\delta)N_c$  bits are affected by  $\alpha_{mF}$ .

### B. Channel Codes with High Diversity Order: SC-LDPC

A terminated SC-LDPC codeword has a length  $N_{SC}L$ , where  $N_{SC}$  is the length of each individual LDPC codeword. We further consider only protograph-based codes, since these provide a structured ensemble and have advantages in terms of implementation.

A protograph  $P$  is simply a bipartite graph consisting of  $n_c$  check sets' (CS) and  $n_v$  variable sets' (VS) nodes. Each node in a protograph therefore represents a subset of variable or check nodes of the bipartite graph for any LDPC code from the (uncoupled) LDPC ensemble. Edges in  $P$  establish a structure of connections which are allowed in a bipartite graph of the LDPC code (i.e., a variable node from subset  $V_i$  can be connected to a check node from subset  $C_j$  iff there is an edge between corresponding VS and CS nodes in  $P$ ).  $P$  is usually given by its *base matrix*  $\mathbf{B}$  of dimensions  $n_c \times n_v$ . E.g., the protograph of a (3, 6)-regular LDPC ensemble is  $\mathbf{B} = [3, 3]$ . Thus,  $n_v = 2$ ,  $n_c = 1$ , and each check node from the set  $C_1$  is connected three times to variable nodes from the subset  $V_1$  and three times to the subset  $V_2$ .

Starting from the initial protograph  $P$ , one creates a *convolutional protograph*  $P_c$ , describing the structure of the SC-LDPC ensemble with  $L$  coupled LDPC blocks and having the memory constraint  $m_{cc}$ ,  $m_{cc} \geq 1$ . Let  $\mathbf{B}_0, \dots, \mathbf{B}_{m_{cc}}$  be  $n_c \times n_v$  matrices with elements from  $\mathbb{N}$  such that  $\sum_{i=0}^{m_{cc}} \mathbf{B}_i = \mathbf{B}$ . Then,  $P_c$  is described by means of the following base matrix  $\mathbf{B}_{[1,L]}$  with dimensions  $(L + m_{cc})n_c \times L n_v$ :

$$\mathbf{B}_{[1,L]} \triangleq \begin{bmatrix} \mathbf{B}_0 & & & & \\ \vdots & & & & \\ \mathbf{B}_{m_{cc}} & & \mathbf{B}_0 & & \\ & & \vdots & & \\ & & & \ddots & \\ & & & & \mathbf{B}_{m_{cc}} \end{bmatrix}_{(L+m_{cc})n_c \times L n_v}. \quad (3)$$

Here the component matrix  $\mathbf{B}_i$ ,  $0 \leq i \leq m_{cc}$  at time  $t$ ,  $1 \leq t \leq L$ , defines the connection from current LDPC block to the LDPC block at  $t + i$ .

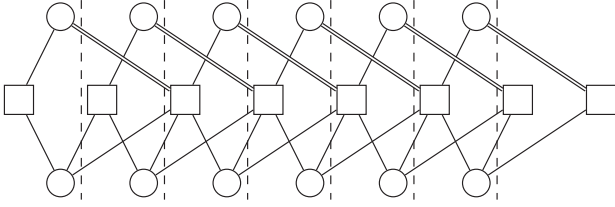


Fig. 3. Convolutional protograph of Example 1. Circles (boxes) represent variables (check) set nodes. Dashed lines separate VS nodes of  $L = 6$  LDPC blocks.

Equivalently,  $P_c$  can also be given by a bipartite graph, whose connections are drawn accordingly to  $\mathbf{B}_{[1,L]}$ .

*Example 1:* Let  $\mathbf{B} = [3, 3]$  and also let

$$\mathbf{B}_0 = [1, 1], \mathbf{B}_1 = [0, 1], \mathbf{B}_2 = [2, 1], \quad (4)$$

i.e.,  $m_{cc} = 2$ . Then the base matrix  $\mathbf{B}_{[1,L]}$  of  $P_c$  can be directly obtained from (3), while the bipartite graph of  $P_c$  with  $L = 6$  coupled blocks is presented in Fig. 3.

The interest of considering SC-LDPC codes over block-fading channels is that one can design a protograph  $P$  (and thus,  $P_c$ ) to obtain a target diversity order. This is explained in the following section.

### III. DIVERSITY ORDER $d$ AND METHODS TO INCREASE $d$ IN THE CASE OF SC-LDPC CODES

#### A. Definition of Diversity and Its Calculation

Our main performance measure of a code family  $\mathcal{F}$  over the block-fading channel is the diversity order  $d$ :

$$d \triangleq \sup_{C \in \mathcal{F}} \lim_{\gamma \rightarrow \infty} \frac{-\log P_e(\gamma, C)}{\log \gamma}. \quad (5)$$

Here  $C$  is a code belonging to  $\mathcal{F}$ ,  $\gamma$  denotes the channel signal-to-noise ratio,  $P_e(\gamma, C)$  is the error probability of code  $C$  after decoding. As we are focused on a graph-based codes, the iterative decoding algorithm is assumed to be used. Then, it can be shown that  $d$  can be computed as the smallest number of received packets with deep fades (i.e.  $\alpha = 0$ ) that cannot be recovered under iterative decoding of a code from given LDPC family  $\mathcal{F}$ .

Assume a SC-LDPC code used for a block-fading channel. Let  $\delta = 0$ . Assume the following holds:

the allocation of  $LN_{SC}$  coded bits to transmission packets is done in such a way that all bits, corresponding to the same VS node in  $P_c$ , are allocated to the same transmission packet and thus experience the same fading gain during the transmission.

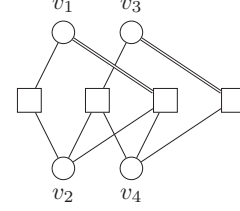
Under the condition above, the diversity order  $d$  is solely dependent on the structure of  $P_c$ . In fact it is related to the *minimum stopping distance*  $s_{min}$  of the equivalent convolutional code, given by  $P_c$ :

*Definition 1:* A stopping set in a protograph  $P_c$  is a subset  $\mathcal{S}$  of VS nodes in  $P_c$  such that:

- if a VS node at block time instant  $t$  belongs to  $\mathcal{S}$ , then any other VS node at  $t$  also belongs to  $\mathcal{S}$ ;
- VS nodes from  $\mathcal{S}$  are connected to a set of CS nodes  $\mathcal{C}$ , and each node from  $\mathcal{C}$  is connected to at least twice.

We denote by  $s$  the size of the stopping set  $\mathcal{S}$ ,  $s = |\mathcal{S}|$ . Then the minimum stopping distance  $s_{min}$  is defined to be the size of the smallest stopping set in  $P_c$ .

*Example 2:* Let us consider  $P_c$  in Fig. 3. Then it can be checked that the smallest stopping set has the following configuration:



Now note that, if the received bits, related to VS nodes of  $\mathcal{S}$ , are in deep fade, then they cannot be corrected by iterative decoding. Therefore, one can easily verify that

$$d \leq s_{min}.$$

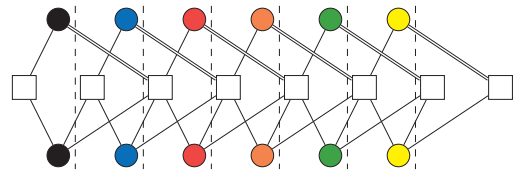
What concerns the exact value of  $d$ , it is dependant on the value of  $F$  and on the actual allocation pattern of SC-LDPC coded bits to transmission packets of the block-fading channel. To make it clear, let us consider an example:

*Example 3:* Consider the setup of Example 1. From the configuration of its minimum stopping set in Example 2, one obtains  $d \leq s_{min} = 4$ .

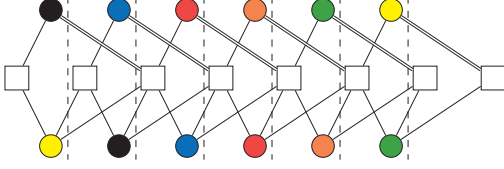
- Assume two cases,  $N_c = N_{SC}$  ( $F = 1$ ) and  $N_c = N_{SC}/2$  ( $F = 2$ ), and let us show that the respective values of  $d$  differ for the two cases.

For  $F = 2$ , there are two transmission packets per one LDPC block (each described by exactly two VS nodes in  $P_c$ ). So, one might allocate one VS node to one transmission packet (e.g., in the graph shown in Example 2, the bits related to  $v_1$  are transmitted in the packet 1, those related to  $v_2$  in the packet 2 etc.). In this case, the iterative SC-LDPC decoder will get stuck if there are at least four transmission packets in deep fade, i.e.,  $d = 4$ .

In the case of  $F = 1$ , we might, for instance, choose the allocation such that each LDPC codeword corresponds exactly to one transmission packet, as it is shown in the figure below, where VS nodes belonging to the same transmission packet are coloured the same. In this case, any two successive deep fades are enough to make the decoder stuck as the set of variable nodes, related to bits with deep-fade estimation, will form a stopping set. So,  $d = 2$ .



- Consider another allocation pattern for  $F = 2$ , and let us show that for different allocation patterns the respective values of  $d$  might be different. As the first example, let us take the allocation pattern drawn in the figure above and implying  $d = 2$ . Let us choose a different allocation pattern, for instance the one shown in the figure below: It can be verified that one should have at least



three transmission packets in deep fade in order to get a decoding failure, so  $d = 3$ .

The Example 3 shows that  $d$  is dependent on both  $F$  and the chosen allocation pattern  $\mathcal{A}$ . This motivates us to define the following quantity

*Definition 2:* Assume a SC-LDPC code, described by a protograph  $P_c$  with  $Ln_v$  VS nodes  $v_1, \dots, v_{Ln_v}$ . Let, for some  $F \geq 1$ ,  $\mathcal{A} = \{A_1, \dots, A_F\}$  be an ensemble of sets such that:

- $A_i \subset \{1, \dots, Ln_v\}$ ,  $1 \leq i \leq F$ ;
- $|A_i| = |A_j| = \frac{Ln_v}{F}$ , for all  $i, j$ ;
- $|\cup_i A_i| = Ln_v$ .

Then, the smallest stopping set  $\mathcal{S}(\mathcal{A})$  is defined as the smallest subset of  $\mathcal{A}$ , covering a stopping set in  $P_c$ . The size of such a smallest stopping set is called the *minimum stopping distance for given allocation  $\mathcal{A}$*  and is denoted by  $s_{min}(\mathcal{A})$ .

It can be shown, that for given allocation  $\mathcal{A}$ ,  $d = s_{min}(\mathcal{A})$ .

*B. Our Approach to Obtain a Target Diversity Order, for Both  $\delta = 0$  and  $\delta > 0$*

Using all the said above, we propose the following procedure to design a SC-LDPC transmission protocol, attaining a target diversity order  $d_{target}$ :

- 1) Design a protograph  $P_c$  having  $s_{min} \geq d_{target}$ . This point might be done by applying for instance our design procedure, proposed in [2].
- 2) Find an allocation pattern  $\mathcal{A}$ , having  $s_{min}(\mathcal{A}) = d_{target}$ . At our knowledge, there does not exist yet a systematic procedure of searching  $\mathcal{A}$ , and some exhaustive or random search might be implemented.

The last point might sound somewhat ambiguous, however one should remind that in most of cases  $P_c$  contains a reasonably small number of nodes, and efficient search for the best allocation is indeed feasible to do.

All the aforementioned above works well to design a SC-LDPC code for a block-fading channel with  $\delta = 0$ . What happens in the case of  $\delta > 0$ ? In fact, the design procedure should stay the same, the only thing which changes are definitions of  $\mathcal{S}(\mathcal{A})$  and  $s_{min}(\mathcal{A})$ :

*Definition 3:* Let the ensemble  $\mathcal{A} = \{A_1, \dots, A_F\}$  be as given in Definition 2. For some  $\delta$ ,  $0 < \delta < 1$ , define the ensemble  $\mathcal{A}' = \{A'_1, \dots, A'_F\}$  such that

$$A'_i = \cup_{a \in A_i} \{a - \Delta, \dots, a + \Delta\},$$

where  $\Delta = \lceil \frac{\delta N_{SC}}{n_v} \rceil$ . Then, the smallest stopping set  $\mathcal{S}(\mathcal{A})$  is defined as the smallest subset of  $\mathcal{A}'$ , covering a stopping set in  $P_c$ . The size of such a smallest stopping set is called the *minimum stopping distance for given allocation and fixed  $\delta$*   $\mathcal{A}$  and is denoted by  $s_{min\delta}(\mathcal{A})$ .

It can be also shown, that, for fixed  $\delta$ ,  $d = s_{min\delta}(\mathcal{A})$ .

#### IV. SIMULATION SETUP AND RESULTS

In this section we provide finite length simulation results for the proposed SC-LDPC codes over block-fading channels, for both cases  $\delta = 0$  and  $\delta > 0$ . As a reference, we compare our code constructions with well-known root-LDPC codes, briefly introduced below.

##### A. Comparison Reference: Root-LDPC Block Codes [4]

Root-LDPC block codes, first proposed in [4], provide full diversity over a block-fading channel, i.e.,  $d = F = 1/R$ . Note that, for  $F = 2$ , a codeword of length  $N$  is spanned over two fading channels with fading gains  $\alpha_1$  and  $\alpha_2$  (see Fig. 1). The fading gains  $\alpha_1$  and  $\alpha_2$  are constant throughout the first and second half of the codeword, respectively. for an  $F = 2$ , the full diversity ( $d = 2$ ) is provided to the systematic information bits by using a special check node structure called *rootcheck* and by connecting only one information bit to every rootcheck.

##### B. Comparison of SC-LDPC and Root-LDPC Codes ( $\delta = 0$ )

Let us present the performance of the proposed SC-LDPC codes and compare them with root-LDPC codes. We choose all the codes (both SC-LDPC and root-LDPC) to be (3, 6)-regular (the code rate is  $R = 1/2$ ). Also, all the simulated codes were generated using a PEG<sup>2</sup> algorithm, so that their finite-length performance is not perturbed by the presence of too short cycles in the code structure.

To compare the proposed SC-LDPC codes with root-LDPC codes, let us assume  $N_{SC} = 2N_c$  (i.e.  $F = 2$ ). Also, let us assume that a perfect channel information at the receiver side is available. We pick the (3, 6) SC-LDPC code with the base matrix  $B$  from our Example 1 of Section II.

For a fair comparison, we are going to compare codes having the same *structural decoding latency*:

- A root-LDPC codeword of length  $N$  is decoded upon reception, so the latency of the decoder is  $N$ ;
- The decoding for SC-LDPC code is carried out using a sliding window decoder of size  $W$ , where iterative decoding algorithm is applied to the nodes within the window. So, at time  $t$  the decoder operates on  $W$  received words,  $\mathbf{y}_t, \mathbf{y}_{t+1}, \dots, \mathbf{y}_{t+W-1}$ , and only symbols in  $\mathbf{y}_t$  are decoded and sent to the decoder output. (Note that the intermediate decoding of the part  $\mathbf{y}_{t+1}, \dots, \mathbf{y}_{t+W-1}$  is useful as it helps the decoder to converge for next decoding times  $t + 1, \dots, t + W - 1$ ).

The latency of the sliding window decoder is  $WN_{SC}$ . So, for fair comparison with root-LDPC codes, we choose the codelength of the SC-LDPC code so that  $N = WN_{SC}$ .

<sup>2</sup>i.e. Progressive-Edge-Growth algorithm, see [8]

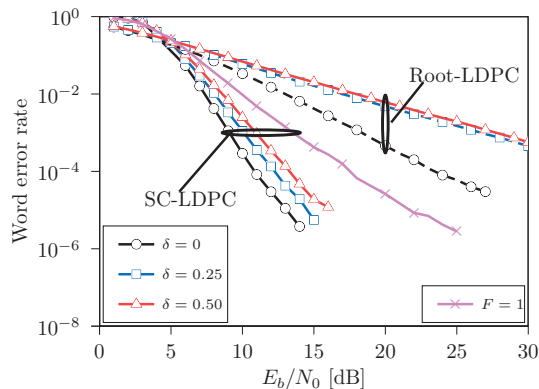


Fig. 4. WER for root-LDPC and SC-LDPC codes with and without offset  $\delta$ .

Curves in black in Fig. 4 present the WER performance of root- and SC-LDPC codes, when  $N = 1000$  and  $W = 5$  (remind that  $F = 2$ ). The root-LDPC code of rate  $1/2$  provides the full diversity to the information bits in the codeword. However, due to the convolutional structure of the SC-LDPC codes, and to the best found allocation pattern  $\mathcal{A}$ , the designed codes achieve a much better performance compared to the root-LDPC codes. In this case the "effective"<sup>3</sup> diversity of the SC-LDPC can be computed numerically from the slope of the WER curve and is equal to 4. Figure 4 also shows a curve with  $F = 1$  using an allocation as in Example 3. As expected the diversity order of the code is reduced in this case.

### C. Case of Synchronisation Offset $\delta > 0$

Assume the simulation scenario described in the previous section. Now, a non-zero offset  $\delta$  is assumed.

Figure 4 also present simulation results for  $\delta = 0.25$  (blue curves with squares) and  $\delta = 0.50$  (red curves with triangles), both for root- and SC-LDPC codes. The performance for both root-LDPC code and SC-LDPC code deteriorates when a constant offset  $\delta$  is introduced. However, while in the root-LDPC case the diversity immediately degrades to 1, it degrades much slower in the SC-LDPC case. This happens thanks to the convolutional structure of SC-LDPC codes, allowing to obtain a gradual performance degradation. This gradual transition for SC-LDPC codes with  $\delta$  (for fixed parameter  $W$ ) is similar to decreasing  $W$  (for fixed  $\delta = 0$ ). This fact is demonstrated in Fig. 5, where one compares a SC-LDPC code from Section IV-B under  $\delta = 0.25$  and decoded with the window of size  $W = 7$  with the same code. It is seen that the WER curves for the two cases are almost identical. Hence the loss due to the synchronisation offset is compensated using a larger  $W$ , i.e., increased decoding latency. However, for a full-diversity code, in general the structure of the code needs to be modified together with the synchronisation offset and this also requires a CSI at the transmitter. In contrast to this, CSI at receiver is sufficient for SC-LDPC codes.

<sup>3</sup>By effective diversity we understand the average diversity, obtained by averaging over all possible channel error patterns. The effective diversity is in fact a lower bound to the diversity, given by (5).

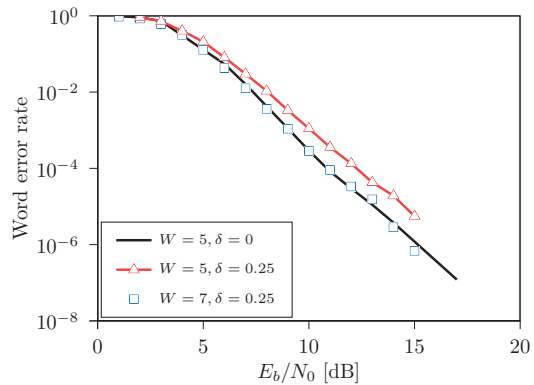


Fig. 5. WER for SC-LDPC codes with offset compensation.

## V. CONCLUSION

The results, presented in this paper, show that SC-LDPC codes are efficient for transmission over block-fading channels. Moreover, SC-LDPC codes happen to be robust to a synchronisation offset, in contrary to existing full-diversity block codes. The paper also considers various ways of allocating coded bits to transmission packets – all the possible approaches imply different values of the code diversity. However, all of them can be studied by means of a general framework of stopping sets. A future work on the subject will focus on two aspects: 1) search procedure of best allocation patterns in classical (block-fading) and more practical-oriented setups (mismatched transmissions, imperfect CSI, etc.); 2) estimation of the transmission cost of SC-LDPC code-allocation schemes in terms of latency, decoder complexity etc.

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