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OPTIMUM DATA RATE IN CELLULAR SYSTEMS

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Abstract – The spectrum efficiency in a cellular radio system is affected by the data rates used in the system. High data rates may result in low spectrum efficiency if the frequency reuse is low. The spectrum efficiency may be enhanced by using adaptive modulation with carefully chosen data rates. We present an analytical method, based on a lower bound on BER, for finding the best data rates. The method results in the best code rates to be used with QPSK, 8-PSK, and 16-QAM in an adaptive fashion. The general conclusion is that a large number of carefully chosen data rates enhances the spectrum efficiency.

I. INTRODUCTION

It is important to choose the channel coding rate carefully in a cellular radio system. This is true irrespective of the type of modulation scheme. The code rate must be chosen to allow a small cluster size, and it must also give a high throughput. These two demands are often contrary to each other, and it is not obvious for a designer how to find the best code rates.

The purpose of this paper is to analytically find the best code rates in an adaptive-modulation arrangement when the optimality criterion is the spectrum efficiency in Mbps/MHz/area-unit. The modulation arrangement may use one of three signalling modes consisting of alphabet sizes $\{4, 8, 16\}$ together with code rates $\{R_4, R_8, R_{16}\}$. By calculating the spectrum efficiency for all combinations of modes, we can find the combination which gives the best spectrum efficiency. Switching between modes generates some signalling between transmitter and receiver, but this signalling has been neglected in this paper. However, the analysis is general and can be applied to all FDMA/TDMA cellular systems.

Many parts of the transmission system are modelled as perfect, e.g., perfect synchronization and perfect filters. The radio channel is modelled as slow and flat Rayleigh fading, and the coding arrangement is optimal in additive white Gaussian noise (AWGN). The result is an average lower bound on the cluster size of the cellular system for a given bit error rate (BER). However, this assumes perfect channel side information and infinite decoding delay. It is also important to point out that the values of spectrum efficiency obtained are optimal only when the cellular system is modelled accurately.

The paper is organized as follows. It starts with an explanation of how the spectrum efficiency is calculated. After that follows a description of the cellular system model, the propagation model, and the coding arrangement. The results of the efficiency-calculation are presented in tabular form, and are followed by conclusions. Finally, the method and the results are discussed.

II. METHOD

Method for Finding Spectrum Efficiency

Spectrum efficiency in a cellular system is often defined as the total throughput in a cell-cluster divided by the cluster's total area and bandwidth [1]. This section departs with a study of the throughput in a cellular system, and arrives at a simple expression for spectrum efficiency.

Assume that the total bandwidth assigned to a cell is divided into n_c channels each having a constant bandwidth W Hertz. A user is offered throughput $d_k(t)$ at time t when using channel k, and the momentary total throughput in a cell $T_{\text{cell}}(t)$ is the sum of all user throughputs,

$$T_{\text{cell}}(t) = \sum_{k=1}^{n_c} d_k(t). \qquad (1)$$

Hence, $T_{\text{cell}}(t)$ changes as each user's throughput changes.

Each user's throughput depends on the signal to interference ratio (SIR) when adaptive modulation is used. $T_{\text{cell}}(t)$ is therefore a function of the user-SIR's in a cell. The SIR Γ in a cellular system is a random variable due to shadow-fading of the signal. Moreover, the expected SIR is a function of the distance r between base station and mobile station. The probability density function of Γ is thus conditioned on r:

$$f_\Gamma(\Gamma|r). \quad (2)$$

The adaptive modulation scheme allows three different alphabet sizes $M \in \{4, 8, 16\}$. Associated with each alphabet size is a code rate: R_4, R_8, and R_{16}, where all three code rates are in the interval $[0, 1]$. Thus, three sets (modes) of M and R are formed: mode A, mode B, and mode C. The set {A,B,C} is a permutation of $\{4, 8, 16\}$, and it is introduced to give a simple expression for the average throughput $T(r)$ below. Three data rates are associated with the three modes: d_A, d_B, and d_C. The data rate of mode i, $i \in \{A, B, C\}$, is

$$d_i = R_i \log_2(i). \quad (3)$$

A minimum SIR, which must be fulfilled to sustain the specified BER, is also associated with each mode. These minimum signal to interference ratios are denoted SIR_A, SIR_B, and SIR_C, where $\text{SIR}_\text{A} \leq \text{SIR}_\text{B} \leq \text{SIR}_\text{C}$. If several modes fulfill the demand on BER, the mode with the highest data rate is chosen. This way the system adapts the modulation to a changing SIR.

The average throughput $T(r)$ experienced by a mobile user on distance r from the base station is

$$\begin{aligned} T(r) \quad = \quad & d_A \int_{\text{SIR}_\text{A}}^{\text{SIR}_\text{B}} f_\Gamma(\Gamma|r) d\Gamma + \\ & d_B \int_{\text{SIR}_\text{B}}^{\text{SIR}_\text{C}} f_\Gamma(\Gamma|r) d\Gamma + \\ & d_C \int_{\text{SIR}_\text{C}}^{\infty} f_\Gamma(\Gamma|r) d\Gamma. \end{aligned} \quad (4)$$

In order to eliminate the conditioning on r, we specify a uniform customer density. The probability density function for r is

$$f_r(r) = \frac{2r}{R_0^2}. \quad (5)$$

Here, R_0 is the range of the base station. The base station's antenna is assumed to be elevated, and the mobile user can not come closer to it than $R_{\min}$, thus $R_{\min} \leq r \leq R_0$. The average throughput of a randomly picked user is denoted T_{user}, and is equal to

$$T_{\text{user}} = \int_{R_{\min}}^{R_0} T(r) f_r(r) dr = \frac{2}{R_0^2} \int_{R_{\min}}^{R_0} T(r) r dr. \quad (6)$$

The average throughput of a cell with n_c users (channels) is $T_{\text{cell}} = n_c T_{\text{user}}$. The relation between T_{cell} and $T_{\text{cell}}(t)$ in (1) is $T_{\text{cell}} = \text{E}[T_{\text{cell}}(t)]$, i.e., T_{cell} is the expected value of $T_{\text{cell}}(t)$ over time t. A whole cell cluster with size β has average throughput

$$T_{\text{cluster}} = \beta T_{\text{cell}} = \beta n_c T_{\text{user}}. \quad (7)$$

All cells are assumed to have constant cell area ζ, which is an accurate model only for a cellular system with homogeneous terrain and constant user density. The spectrum efficiency in Mbps/MHz/area-unit is now defined as

$$\eta \triangleq \frac{T_{\text{cluster}}}{\zeta \beta W \beta n_c} = \frac{T_{\text{user}}}{\zeta W \beta}. \quad (8)$$

The spectrum efficiency in (8) can be simplified by excluding the cell-area ζ and the bandwidth W because they are independent of the cluster size and each user's data rate. The final expression for η is thus

$$\eta \propto \frac{T_{\text{user}}}{\beta}, \quad (9)$$

which is used in the rest of the paper. Expression (9) does not give absolute values of the spectrum efficiency. However, it is only the change in η between different modes which is of interest here.

The parameter remaining to be calculated in (9) is the cluster size β. A one-to-one relationship (described below) exists between the distribution of Γ and the cluster size. The position in the cell where the SIR is lowest (on the average) is on the cell's edge. Therefore, Γ on the cell's edge must not go below SIR_A ($\text{SIR}_\text{A} \leq \text{SIR}_\text{B} \leq \text{SIR}_\text{C}$) more often than specified by the outage probability P_{out}. Mathematically,

$$P(\Gamma \leq \text{SIR}_\text{A}) = P\left(z \leq \log_e\left(\text{SNR}_\text{A}\right)\right) \leq P_{\text{out}} \quad (10)$$

must be fulfilled on the cell's edge, and $z = \log_e(\Gamma)$.

When both wanted signal and all interferers suffer fading with lognormal distribution, the SIR Γ has lognormal distribution, and therefore $z = \log_e(\Gamma)$ has normal distribution. The mean and variance of z

are denoted m_z and σ_z^2 respectively. Expression (10) can then be rewritten

$$1 - Q\left(\frac{\log_e(\text{SNR}_A) - m_z}{\sigma_z}\right) \leq P_{\text{out}}, \qquad (11)$$

where $Q(\cdot)$ is the normalized upper tail probability function for the normal distribution. The cluster size β is a function of both m_z and of σ_z, and β can thus be calculated when m_z and σ_z are known.

To complete the method we need a relation between the BER and $\{\text{SIR}_A, \text{SIR}_B, \text{SIR}_C\}$. Such a relation is presented in section Channel Model and Coding Arrangement. Then, given the BER, the code rates $\{R_4, R_8, R_{16}\}$, and P_{out}, we can find β and T_{user}. In the investigation below, spectrum efficiency is calculated for the modes that can be created from $M = \{4, 8, 16\}$ and $R = \{0.02, 0.04, \ldots, 1\}$.

Cellular System Model

The cellular system is modelled as a perfectly symmetric grid of equally sized hexagonal cells. As mentioned above, a mobile user never comes closer to the base station than $R_{\min}$. The range of the base stations are R_0, and therefore $R_{\min} \leq r \leq R_0$ where we have chosen $R_{\min} = 0.1R_0$.

Only cochannel interference is accounted for, and the interfering cochannel base stations can be geometrically arranged in so-called group-tiers. Without describing the geometry in detail, we will present an expression for the positions of the interfering base stations as a function of β. The expression gives the positions of the cochannel base stations illuminating a user when all base station antennas have 120° lobe widths. The expression, together with supplementary information about the parameters in it, renders it possible to repeat the results in this paper.

The position $z(n,p)$ of cochannel base station number n in group-tier p relative to the origin is described by using complex notation

$$\begin{aligned} z(n,p) &= \sqrt{\frac{2\pi}{\sqrt{3}}} R_0 \sqrt{\beta} \sqrt{p^2 - p|q| + |q|^2} \times \\ & e^{j\left[\frac{\pi}{3}\lfloor\frac{n}{p}\rfloor + \arcsin\left(\frac{\sqrt{3}}{2}\frac{q}{\sqrt{p^2 - p|q| + |q|^2}}\right) - \alpha\right]}, \\ & \left\{ \begin{array}{l} n = 3p, \ldots, 5p - \text{mod}(p,2) \\ q = \text{mod}(n,p) - \lfloor\frac{p}{2}\rfloor \end{array} \right. . \end{aligned} \qquad (12)$$

Here, the angle α depends on the actual allocation of carrier frequencies in the cells, and it changes with the cluster size. However, α is assumed constant in this simple model, and is chosen as $\alpha = \pi/6$. This will affect the outage probability less than 30%. Three group-tiers (resulting in 13 cochannel interferers) has been taken into account.

Propagation Model

The area mean power at distance r from a transmitter is described by

$$S(r) = S(R_0)\left(\frac{r}{R_0}\right)^{-\gamma}, \qquad (13)$$

where the propagation exponent γ is between 3.5 and 4.0 [2]. $S(r)$ is the average over the shadow fading, which, as mentioned before, is assumed to have lognormal distribution. Inspired by [3], Wilkinson's method [4, 5] is used for adding the independent lognormal random variables that model the cochannel interference. The random variables are unidentically distributed as a result of different distances to the interferers in a cellular system.

The SIR's standard deviation σ_z is often called the dB-spread when measured in dB. To imitate a GSM system as closely as possible, typical GSM-parameters are adopted in order to find a suitable value for σ_z. With outage probability $P_{\text{out}} = 0.05$, $\beta = 9$, and necessary $\text{SIR}_{\min} = 9$ dB in a GSM system [6], the cellular system model above gives that the dB-spread is 5.1 dB for each interferer when $\gamma = 3.5$.

The mobile user is assumed to move slowly enough for the slow fading to be virtually constant over many symbols. The sum of interference over a symbol is modelled as AWGN, which is plausible when the number of interferers is large.

Channel Model and Coding Arrangement

An average lower bound on the BER for given M and R in slow, flat Rayleigh-fading channel was derived in [7], and is reviewed here for the reader's convenience. A consequence of the slow and flat fading property is that the sampled output y of the matched filter in the receiver is

$$y = ce^{j\phi}x + u, \qquad (14)$$

when symbol x is transmitted. Here, $c \in \text{Rayleigh}(\sigma_c^2)$ is the channel's influence on the amplitude, $\phi \in \text{Rect}(0, 2\pi)$ is a random phase shift introduced by the channel, and the random variable u is AWGN. It is assumed that perfect channel side information eliminates $e^{j\phi}$. The power attenuation due to distance is fully taken care of in (13); the second moment of c can be set to 1. Thus, the variance of c is $\sigma_c^2 = 1/2$. The additive noise u and the representation of the transmitted symbol x are complex, i.e., $x, u \in \mathbb{C}$, while $c \in \mathbb{R}$.

Although perfect channel side information is available, the Rayleigh fading channel degrades the performance compared to an AWGN channel with constant SIR. An expression for the channel capacity $C_{\text{AWGN}}(c)$ of a two-dimensional signal constellation with equiprobable channel symbols in AWGN is presented in [8]. The average channel capacity C_{Ray} of the Rayleigh fading channel is found by averaging $C_{\text{AWGN}}(c)$ over all possible values of c, $c \in [0, \infty)$, [9]

$$C_{\text{Ray}} = \int_0^\infty f_C(c) C_{\text{AWGN}}(c)\, dc. \qquad (15)$$

Here, $f_C(c)$ is the density function of the Rayleigh distribution.

The average capacity is not a bound but an estimate of the capacity of a Rayleigh fading channel. However, the number of transmitted bits per channel access approaches C_{Ray} if the code words' length are allowed to approach infinity. Very long code words require very long decoding delays, and the decoding delay is thus unconstrained. So far, it has been assumed that the Rayleigh fading channel is memoryless. A virtually memoryless channel can be obtained by introducing large interleavers after the encoder.

The discrete-input/discrete-output system from encoder input to decoder output is regarded as a binary symmetric channel (BSC) [10]. The capacity of a BSC, C_{BSC}, is a function of the BER, while C_{Ray} in (15) is a function of the SIR. The capacity of the Rayleigh fading channel lies within $0 \leq C_{\text{Ray}} \leq R \log_2(M)$, and due to the data processing theorem [8],

$$C_{\text{BSC}} R \log_2(M) \leq C_{\text{Ray}}. \qquad (16)$$

Equality in (16) gives a relationship between a specific BER through C_{BSC} and the appropriate average SIR-level through C_{Ray}. The minimum SIR sustaining a given BER with data rate $R \log_2(M)$ can thus be found.

Set of M	η	R_4	R_8	R_{16}	β
4	0.4	0.22			1.1
8	0.4		0.14		1.1
16	0.4			0.12	1.2
4,8	1.3	0.18	0.66		1
4,16	1.5	0.20		0.60	1
8,16	1.5		0.12	0.60	1
4,8,16	1.8	0.18	0.50	0.72	1

Table 1: Optimum code rates R_4, R_8, and R_{16} for all combinations of alphabet sizes M. The full set $\{4, 8, 16\}$ gives best spectrum efficiency η. The cluster size is constrained to $\beta \geq 1$, and the powerful coding arrangement gives cluster sizes close to 1.

Set of M	η	R_4	R_8	R_{16}	β
4	0.22	0.78			7
8	0.26		0.64		7.3
16	0.29			0.54	7.3
4,8	0.36	0.78	0.92		7
4,16	0.45	0.78		0.88	7
8,16	0.44		0.64	0.90	7.3
4,8,16	0.46	0.78	0.82	0.92	7

Table 2: Optimum code rates R_4, R_8, and R_{16} when the allowed cluster sizes are between 7 and 9. The set $\{4, 8, 16\}$ gives the best spectrum efficiency.

III. RESULTS

The analysis is divided into two cases, one where all cluster sizes $\beta \geq 1$ are allowed, and one where cluster sizes are confined to $7 \leq \beta \leq 9$. The first case deals with the maximum spectrum efficiency, while the second deals with the highest spectrum efficiency in existing systems. Calculated values on spectrum efficiency, optimum code rates, and cluster sizes are given in Table 1 for $\beta \geq 1$, and in Table 2 for $7 \leq \beta \leq 9$. It is obvious in Table 1 that the extreme coding arrangement is very robust against interference, and cluster sizes close to one results. Cluster sizes between 7 and 9 gives lower spectrum efficiency although the code rates are much higher than in the first case.

IV. CONCLUSIONS

The conclusion from Table 1 and Table 2 is that the spectrum efficiency is enhanced when the number of modes increases. This is natural since a large number of modes allows the communication system to adapt with greater precision to the changing channel. If only a limited number of alphabet sizes can be supported, then modes which cover the SIR-range as well as possible should be chosen. For example are modes based on $M = \{4, 16\}$ next best to modes based on $M = \{4, 8, 16\}$.

Another conclusion is that a low demand on throughput on the cell's edge gives the best spectrum efficiency. The largest spectrum efficiency in Table 1 is obtained when the data rate on the cell's edge is $R_4 \log_2(4) = 0.36$ bits per channel access, which accounts to 72 kbps with a bandwidth of 200 kHz. Such a low data rate on the edge facilitates a high throughput close to the base station, and the total spectrum efficiency benefits.

Although the throughput is part of the expression for spectrum efficiency in (9), it is not sure that a high value on spectrum efficiency gives high throughput everywhere in the cell. Additional demands on throughput must be made to guarantee a certain throughput everywhere in a cell.

V. DISCUSSION

A relevant question is if the values on spectrum efficiency presented here are optimum. The answer is yes if the real cellular system is accurately modelled by the model in section Cellular System Model. The model in this paper does not consider antenna diversity or dynamic channel allocation, which will give even higher values of spectrum efficiency. Larger alphabet sizes and more than three modes will also enhance the spectrum efficiency. Optimum data rates for this type of enhanced systems needs to be analyzed separately.

It is optimistic to believe that the results on optimum data rates presented here can be applied to a practical system. Perfect channel side information, infinite decoding delay, and perfect synchronization are hard to attain. However, the impact of a non-perfect system can be accounted for by adding an implementation loss in dB to the minimum SIR.

In any case, this paper shows that optimum data rates exist in a cellular system as a consequence of the frequency reuse mechanism. It also clarifies that a large number of modes and a low cell-edge throughput is necessary to obtain a high spectrum efficiency. As mentioned before, the signalling required for mode-switching has not been accounted for. This is justifiable in a system with a very slow channel and/or a very high bit-rate. Otherwise, only a limited number of modes can be used before the throughput decreases as a result of extensive signalling. To investigate this upper limit, the signalling protocol must be defined and an error bound for constrained block lengths in a Rayleigh fading channel found. This is a difficult problem and is not treated here.

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