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2014

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Organizational coordination and costly communication with boundedly rational agents[*]

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April 9, 2014

Abstract

How does costly communication affect organizational coordination? This paper develops a model of costly communication based on the weakest-link game and boundedly rational agents. Solving for the stochastically stable states, we find that communication increases the possibilities for efficient coordination compared to a setting where agents cannot communicate. But as agents face a trade-off between lowering the strategic uncertainty for the group and the costs of communication, the least efficient state is still the unique stochastically stable one for many parameter values. Simulations show that this is not just a long run phenomena, the stochastically stable state is the most frequent outcome also in the short run. Making communication mandatory induces efficient coordination, whereas letting a team leader handle communication increases efficiency when the leader expects others to follow and has enough credibility. The results are broadly consistent with recent experimental evidence of communication in weakest-link games.

Keywords:	Organizational coordination, Commmunication, Stochastic stability,
	Bounded rationality, Simulation
JEL Classification:	D23, L22, L23, C73

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1 Introduction

When tasks are specialized and interdependent, coordination becomes important for organizations and the ability to facilitate coordination is often put forward as a major reason for their existence (e.g. Simon, 1991; Grant, 1996). Consequently, understanding why groups of agents may or may not be able to coordinate their actions, and how coordination mechanisms should be designed, is one of the keys to explaining and improving organizational efficiency.[1]

Communication may seem as an obvious way to coordinate groups of agents by simply transferring information about what agents intend to do. However, when agents act strategically, communication is not straightforwardly translated into efficient coordination as shown by experimental results with communication in coordination games.[2] Moreover, recent experiments indicate that whereas costless communication and/or mandatory communication may produce efficient coordination in weakest-link games, *costly* communication is much less successful – even if the costs are very small in relation to the potential gains of efficient coordination (Blume and Ortmann, 2007; Kriss et al., 2012). As arguably all organizational communication takes time and is in some sense costly, these results are important to understand.

We develop a model based on the weakest-link game that provides a theoretical explanation for this phenomenon. Message costs, even very small ones, imply that agents have to consider whether their message will change their colleagues' course of action. There is thus a trade-off between lowering the strategic uncertainty for the group and the costs of communicating, and also an incentive to free ride on other agents' communication, despite the common interest in coordination. Making communication mandatory removes the adverse incentives, and introducing a team leader improves efficiency as long as the team leader has enough credibility. Thus, our results indicate that organizations need to structure communication to achieve efficient coordination and suggest a reason for why inter-organizational cooperation and new collaborations – where such structures are often missing – frequently result in coordination failures (e.g. Hoopes and Postrel, 1999; Heath and Staudenmeyer, 2000).

To build intuition for how communication affects coordination, we first develop a simple model and solve for the stochastically stable states.[3] While stochastically stable states can be interpreted as the likely long-run state of a system or a process, related experimental studies use only a few periods (8-10 periods are common). To compare the short-run properties of the model and to relax

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[1]See Lawrence and Lorsch (1967), Sinha and Van de Ven (2005), Grandori and Soda (2006) and Sherman and Keller (2011) for evidence of the difficulties in choosing appropriate coordination mechanisms.

[2]E.g. Cooper et al. (1992); Weber et al. (2001); Chaudhuri et al. (2009); Wilson (2012), and Andersson and Holm (2013). See also Camerer (2003), Devetag and Ortmann (2007), and Camerer and Weber (2013) for reviews of experiments with coordination games, including many using different forms of communication.

[3]See Freidlin and Wentzell (1984) for a general exposition of the theory of randomly perturbed dynamical systems, and Foster and Young (1990), Kandori et al. (1993), and Young (1993, 1998) for the first developments of stochastic stability in game theory.

some of the simplifying assumptions, we run simulations. This exercise also allows us to use parameter values that have not been used in experiments so far.

We think the highly interdependent actions in the weakest-link game capture the essence of organizational coordination problems where the lowest quality of individual inputs disproportionally affects the quality of the output for the organization. This is for example the case in many types of services.[4] The experiments with communication in coordination games (and experimental results from many other games), as well as careful studies of coordination in organizations (e.g. Heath and Staudenmeyer, 2000), suggest that subjects may not be perfectly rational in terms of foresight and in their use of available information. To capture this, agents in our model are boundedly rational. As in Kandori et al. (1993), Robles (1997), and Riedl et al. (2012), our agents choose myopic best replies, have limited information processing capabilities, and may occasionally experiment or make mistakes. We believe these behavioral assumptions are a reasonable approximation of the behavior of real world organizational members, as well as experimental subjects, in settings characterized by high levels of strategic uncertainty. We stay close to how models without communication have modelled actions to clearly illustrate the effect of communication. When agents are not allowed to communicate, the results from our model – that all agents choosing the lowest ranked action is the unique stochastically stable state – is also similar to earlier results (Robles, 1997; Riedl et al., 2012).[5]

When agents can choose whether or not to communicate, we find that communication may solve the coordination problem, but only if the costs of communication are small enough and the incentives to coordinate on the efficient action are strong enough. The simulations furthermore show that the stochastically stable states are not only long run phenomena. With the experimental conditions used in Kriss et al. (2012), the least efficient state is stochastically stable, and the most frequent outcome in the short run. Our simulation results with voluntary costly communication are overall similar to the experimental results of Kriss et al. (2012), but with the difference that the exact level of message costs seems less important in our model.

An organization may also have the authority to structure communication by imposing rules or routines for how its members should communicate. We examine analytically how two simple routines – making communication mandatory and assigning one agent to be the team leader – change the outcome. In line with the experimental results in Blume and Ortmann (2007) and in Kriss et al. (2012), all agents coordinating on the payoff dominant action is the unique stochastically stable state with mandatory communication. A team leader may improve coordination, but the

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[4]Security, safety, data collection or other general quality assurances are further examples. Camerer (2003) mentions airplanes before departure, joint production of documents in law firms, accounting firms, and investments banks, and that production functions like the Cobb-Douglas with large exponents or Leontief functions also have similar properties.

[5]Starting with Van Huyck et al. (1990), a large experimental literature shows that play in the weakest-link game (or "minimum effort game") with groups of more than three subjects almost invariably converges towards the least efficient equilibrium when subjects are not helped by any coordination mechanism (Engelmann and Normann (2010) is the single exception we have found).

team leader must both expect other agents to choose the communicated action and have enough authority or credibility for efficient coordination to occur. In accordance with intuition, the result also indicates that this will become more difficult as the size of the group increases. While there are no experiments that exactly match our setup for this routine, credibility influences the impact of communication in the weakest-link experiments of Brandts et al. (2012), and Kriss and Eil (2012). In the latter study, expectations are also important for efficiency: team leaders sometimes fail to use costly communication that would help their groups. Regarding group size, free-form and costless communication by a team leader improves efficiency when addressing groups of two, but not groups of nine and ten subjects in Weber et al. (2001), and it improves but does not always yield full efficiency when the leader addresses groups of four in Cooper (2006), Brandts and Cooper (2007), and Brandts et al. (2012).

To the best of our knowledge, there are no similar theoretical models of communication in the previous literature on organizational coordination. The cheap-talk literature examines the effects of pre-play communication on outcomes in a variety of games (e.g. Crawford and Sobel, 1982; Farrell and Rabin, 1996). Closest to our paper, Ellingsen and Östling (2010) model cheap-talk by agents using level-k models of strategic thinking. They find that as long as truth-telling is lexicographically preferred to lying, *costless* communication facilitates coordination in common interest games with positive spillovers and strategic complementarities, such as the weakest-link game.[6] Models of *costly* communication mostly analyze sender-receiver games with perfectly rational agents and examine how outcomes vary with the degree of private information and/or conflicts of interest between sender and receiver (e.g. Austen-Smith, 1994; Dewatripont and Tirole, 2005; Gossner et al., 2006; Calvo-Armengol et al., 2009; Wilson, 2012). But as subjects have a common interest in achieving efficient coordination and the parameters of the experimental game are common knowledge, private information and conflicts of interests are unlikely explanations of the coordination difficulties in the situations we are interested in. Our game is also repeated in contrast to the one-shot settings typically used in the costly communication literature (Gossner et al. (2006) is an exception). Compared to Dewatripont and Tirole (2005), Calvo-Armengol et al. (2009) and Wilson (2012), we simplify and treat only the sending of information as costly.[7]

We proceed in the following way: section 2 outlines the model and presents the analytical results. Section 3 describes the simulation model and results, while section 4 contains concluding remarks.

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[6]See also e.g. Crawford (2003) and Wengström (2008) for results with level-k models in hide-and-seek and price competition games, respectively.

[7]There is also a related and recent literature in mechanism design that seeks to characterize feasible and optimal mechanisms given incentive and communication constraints, see e.g. Van Zandt (2007) and Mookherjee and Tsumagari (2012).

2 The model

This section contains the model and analytical results. We start in section 2.1 with a description of the weakest-link game and how agents choose messages and actions, while section 2.2 presents results with and without communication as well as when simple routines are used to structure the agents' communication.

2.1 A model of communication in weakest-link games

We consider a finite set of agents $N = \{1, 2, ..., n\}$, $n \geq 2$. Let $A_i = \{1, 2, ..., K\}$ be the set of actions for agent i. Actions are represented by integers where 1 is the lowest ranked action and K is the highest ranked. Let $M_i = A_i \cup \{\varnothing\}$ be the set of available messages, where the empty message represents the case of no communication. The set of all possible combinations of messages is denoted $M = \prod_{i \in N} M_i$ and the corresponding set of actions $A = \prod_{i \in N} A_i$. Agents' tasks in every period $t = 1, 2, ...$ of the infinitely repeated game are to choose a message $m_i^t \in M_i$ in the communication stage, and an action $a_i^t \in A_i$ in the action stage.

We structure the communication and action stages in a way similar to the experimental conditions of Blume and Ortmann (2007) and Kriss et al. (2012): each agent sends one message per period and this message is sent to all other agents. Furthermore, messages are sent simultaneously so agents do not learn the other agents' messages before sending their own. After receiving messages, agents simultaneously choose an action in the action stage. Let $m^t = \{m_1^t, m_2^t, ..., m_n^t\}$ be the set of sent messages and let m_{-i}^t denote agent i's received messages in period t. The set of chosen actions in period t are denoted $a^t = \{a_1^t, a_2^t, ..., a_n^t\}$. The cost of sending message m_i^t is $c(m_i^t)$ and we assume that $c(m_i^t) = c(m_j^t) \, \forall i, j \in N$. Moreover, $c(m_i^t) = c > 0$ in all periods and for all $m_i^t \neq \varnothing$, while $c(\varnothing) = 0$ (i.e. not communicating is costless). Receiving messages is not costly.

After the action stage, payoffs in the weakest-link game with costly communication are given by a function $\pi : M \times A \to \mathbb{R}$, defined for each agent i in every period t as

$$\pi_i(a^t, m_i^t) = \alpha \underline{a}^t - \beta a_i^t - c(m_i^t) \tag{1}$$

where α and β are parameters of the game, $\alpha > \beta > 0$, and $\underline{a}^t \equiv \min_{j \in N}\{a_j^t\}$ is the lowest ranked (minimum) action played by some $j \in N$. Thus, an agent's payoff is increasing in the minimum action and decreasing in the action chosen by the agent, and the higher the ratio of β to α the weaker the incentives to choose a higher ranked action. The payoff function is common knowledge and the same for all agents in every period. If the game is restricted to the action stage, the assumption that $\alpha > \beta > 0$ implies that the combinations where all players choose the same action constitute the strict Nash equilibria of the game.

The next step is to describe how agents choose messages and actions. First, we assume that all

agents follow the same decision-making process, characterized by myopic best replies, limited information processing, and mistakes and experiments. As in the experiments of Blume and Ortmann (2007) and Kriss et al. (2012), we let agents observe all messages whereas they are only informed about the minimum action in each period.[8] Note that we do not assume that agents know they are identical, or that the details of the decision-making process is common knowledge. To form their expectations about play in the current period agents use information from the previous period; that is, in t agent i uses the history of play in the form of received messages m_{-i}^{t-1} and the minimum action $\underline{a}^{t-1}$ in period $t-1$.

Throughout, we use m and a to denote prospective messages and actions; that is, when agents think about what message to send and what action to take. Starting with how expectations are formed in the communication stage, we assume that the subjective probability of action a becoming the minimum action in any period t, before any message is actually sent, is influenced by i) the content of i's own prospective message m; ii) other agents' messages in the previous period m_{-i}^{t-1}; and iii) if $\underline{a}^{t-1} = a$, i.e. if a was the minimum action in the previous period or not.

Let $q_i^t(a|m) \equiv Pr\left(\underline{a}^t = a | m, m_{-i}^{t-1}, \underline{a}^{t-1}\right)$ be i's communication stage subjective probability of a becoming the minimum action in period t, should i send $m_i^t = m$ (as m_{-i}^{t-1} and $\underline{a}^{t-1}$ are the same regardless of which message is being contemplated, we leave them out of $q_i^t(a|m)$ to ease notation). That is, $q_i^t(a|m)$ represents agent i's subjective assessment, *before* any actual messages are sent, of how likely it is that action a becomes the minimum action in period t, should i send $m_i^t = a$. This assessment is done such that, for each $m \in M_i$, $\sum_{a=1}^K q_i^t(a|m) = 1$. We make four more specific assumptions about the $q_i^t(a|m)$, which are further discussed below:

Assumption 1: Agents form expectations based on a distribution they believe is stationary; i.e. they expect the empirical frequencies of other agents' messages in $t-1$ to be the same in period t.

Assumption 2: For $t > 1$ and all $i \in N$ and all $a, a' \in A_i$, if $m_j^{t-1} \neq a$ for all j and $\underline{a}^{t-1} \neq a$, then in t, $q_i^t(a|m = \varnothing) = q_i^t(a|m = a') = 0$. So, besides the initial period,[9] if there is no indication of a, either by communication or by earlier play, then agent i places probability 0 on action a being the minimum in period t, unless agent i contemplates to send $m = a$.

Assumption 3: The subjective probabilities are influenced by the frequencies of messages, not their labels. That is, if we for instance change the labels on messages and actions equal to a in $t-1$ to a' and call this new history $\hat{m}^{t-1}$ and $\hat{\underline{a}}^{t-1}$, then $m = a$ given m^{t-1} and $\underline{a}^{t-1}$ affect $q_i^t(a|m)$ exactly as $m = a'$ affect $q_i^t(a'|m)$ given $\hat{m}^{t-1}$ and $\hat{\underline{a}}^{t-1}$.

Assumption 4: For all $i, j \in N$ and all $a \in A_i$, $q_i^t(a|m)$ is non-decreasing in the number of $m_j^{t-1} = a$, if $m = a$, and if $m = \varnothing$ and $\underline{a}^{t-1} = a$.

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[8]That agents can only observe the minimum action and not individual actions of other agents is the most commonly used informational condition also in the experimental literature without communication, but see Devetag and Ortmann (2007) for some exceptions.

[9]Given the assumptions about the agents' decision-making process, the initial expectations are of no consequence for our first results, but we discuss this issue in section 3.2.

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The first two assumptions are relatively strong, but have counterparts in several other game-theoretical learning models. Agents are assumed to treat the empirical distribution of play as stationary in for example fictitious play (Fudenberg and Levine, 2009). Many models in which expectations are based on empirical frequencies of past play include an assumption similar to the second (e.g. Robles, 1997; Young, 1998; Riedl et al., 2012). Our results are however qualitatively similar if we allow for a small, but larger than zero, subjective probability of actions not indicated by messages or the previous minimum action (results available on request). As this complicates both expressions and proofs without adding much intuition, we prefer to use assumption 2 to keep the model simple. The third assumption states that there is nothing intrinsically special about certain actions in terms of how expectations change due to agent i's own communication. If some actions are focal points, so that messages indicating such actions are expected to influence other agents' choice of action more than others, this would be a violation of the assumption. The fourth assumption implies that messages indicating a certain action never decrease agents' expectations of this action becoming the minimum action, and may increase these expectations. It also implies that sending the empty message may increase, but not decrease, the subjective probability placed on the last period's minimum action.

Given these assumptions and the common knowledge of the payoff function, we formulate the subjective expected payoff of action a in time t when contemplating which message to send as (henceforth, h denotes actions higher or equally ranked, and l denotes lower ranked actions, compared to some a):

$$\mathbb{E}\left(\pi_i(a,m)\right) = \sum_{h=a}^{K} q_i^t(h|m)a\left(\alpha - \beta\right) + \sum_{l=1}^{a-1} q_i^t\left(l|m\right)\left(\alpha l - \beta a\right) - c(m). \tag{2}$$

Because the lowest ranked action played by any agent is always payoff-determining, the risk associated with playing a decreases when the subjective probabilities of a and all higher ranked actions increase. Therefore, the expected payoff of a becoming the minimum increases with *all* $q_i^t(h|m)$, such that $h \geq a$; i.e. the term $\sum_{h=a}^{K} q_i^t(h|m)$ in equation (2). Consequently, $\mathbb{E}\left(\pi_i(1,m)\right) = \alpha - \beta - c(m)$, regardless of the history of play. As all actions are higher ranked than 1, action 1 always determines payoffs if played by any agent. For this reason, it can never be a best reply message to send $m_i^t = 1$.

To determine a best reply message, we are interested in the total or aggregate expected payoff conditional on a certain message, denoted $\mathbb{E}(\pi_i(m))$. What we have in mind is a procedure where agents contemplate each possible message, compare the expected payoffs, and then choose the message that yields the highest expected payoff. However, the expected payoffs for single actions can be aggregated into $\mathbb{E}(\pi_i(m))$ in several different ways. For our first results, we assume the following:

$$\mathbb{E}\left(\pi_i(m)\right) = \sum_{a=1}^{K} \mathbb{E}(\pi_i(a,m)). \tag{3}$$

That is, the agent sums the expected payoffs for the individual actions. The best reply correspondence for messages is then:

$$BR_i^m = \{m \in M_i : \mathbb{E}\left(\pi_i(m)\right) \geq \mathbb{E}\left(\pi_i(m')\right) \ \forall\, m' \in M_i\}. \tag{4}$$

If there is more than one message that is a best reply, we assume that the agents choose between these messages by randomizing uniformly. Mistakes and experiments are also possible: agent i chooses a best reply message according to the above procedure with probability $1 - \varepsilon$, and with a (small) probability ε chooses a message in M_i by uniform randomization.[10]

In the action stage, we assume that agents best-reply to expectations given by the frequencies of received messages and the minimum action in the previous period. When an agent receives messages from some but not all other agents, agents assume that the non-communicating agents will play the minimum action in the previous period. Let $p_i^t(a)$ denote the probability assigned by agent i to a being the minimum action. The expected payoff of an action a in period t is then

$$\mathbb{E}(\pi_i(a)|m^t, \underline{a}^{t-1}) = \sum_{h=a}^{K} p_i^t(h)a(\alpha - \beta) + \sum_{l=1}^{a-1} p_i^t(l)(\alpha l - \beta a). \tag{5}$$

where $p_i^t(h) = \frac{1}{n}\sum_{j \in N} p_{ij}^t(h)$ and $p_i^t(l) = \frac{1}{n}\sum_{j \in N} p_{ij}^t(l)$, and

$$p_{ij}^t(h) = \begin{cases} 1 & \text{if } m_j^t = h \\ 1 & \text{if } m_j^t = \varnothing \text{ and } \underline{a}^{t-1} = h \\ 0 & \text{otherwise} \end{cases}$$

and

$$p_{ij}^t(l) = \begin{cases} 1 & \text{if } m_j^t = l \\ 1 & \text{if } m_j^t = \varnothing \text{ and } \underline{a}^{t-1} = l \\ 0 & \text{otherwise.} \end{cases}$$

That is, the subjective probabilities of each action being the minimum are equal to the frequencies to which the action has been indicated by messages, or indicated by the combination of the empty message and being the last period's minimum action. The procedure implies that $\sum_{a=1}^{K} p_i^t(a) = 1$. As in the communication stage, we assume that the expected payoff of any action a increases in the sum of subjective probabilities put on all higher ranked actions and the action itself, i.e. the

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[10]As discussed by for example Bergin and Lipman (1996), van Damme and Weibull (2002), and Blume (2003), the assumption of a uniform distribution of mistakes/experiments need not be innocuous. We make it here for simplicity, and test different distributions in section 3.

term $\sum_{h=a}^{K} p_i^t(h)$ in equation (5). Agents thus use the frequencies of messages to determine the subjective probabilities of actions, so again it is only the number of messages that counts, not their labels. Note that agents do not expect others to always choose the action indicated by their message. If so, then the best reply action would be $a \leq \min(m_j^t)$ and lower only if at least one other agent send the empty message and the minimum action in period $t-1$ was some $a' < \min(m_j^t)$.

With probability $1 - \varepsilon$ agents choose an action in the best reply correspondence for actions

$$BR_i^a = \{a \in A_i : \mathbb{E}(\pi_i(a)|m^t, \underline{a}^t) \geq \mathbb{E}(\pi_i(a')|m^t, \underline{a}^t) \, \forall a' \in A_i\} \tag{6}$$

and with probability ε agents use a uniform randomization over all actions in A_i. We make the following assumption about the probabilities of mistakes/experiments:

Assumption 5: $\varepsilon = \varepsilon$. That is, an agent is as likely to make a mistake or experiment in the communication stage as in the action stage. Furthermore, we assume that both ε and ε are identical for all agents and independent both across agents and over time.

This assumption is made for simplicity here, and we relax the equality between the probability of mistakes and experiments in the communication and action stages in the simulations. The decision-making process describes agents as myopic in that they choose best replies for just one period at a time, i.e. they are not forward-looking. Agents are limited in their information processing as we assume that they only use the previous period's information to form their expectations about communication and actions in the present period. The small probabilities of mistakes and experiments represents another aspect of bounded rationality. The model is a variant of the adaptive learning process first developed by Young (1993, 1998) and Kandori et al. (1993), and the decision-making process forms what Young (e.g. 1993) calls a regular, perturbed Markov process. Without communication, our model corresponds to one in Riedl et al. (2012), and is very similar to Robles (1997). The difference to Robles, who considers not only the weakest-link game but order-statistic games in general, is that agents in his model are informed of the whole empirical distribution of chosen actions in the previous period, not only the minimum action.[11]

For the results in the next section, we first find the *absorbing states* of the process – states that the process cannot leave without mistakes or experimentation – and second the *stochastically stable states*, which are roughly the absorbing states for which the minimum number of mistakes/experiments needed for the process to leave is the highest. Stochastically stable states can be interpreted as the states where the process is most likely to be found in the very long run; they need not be unique, but there is always at least one (Young, 1993). We describe this and related concepts

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[11]We depart from Young (1993, 1998) and others that have used a similar framework (e.g. Jackson and Watts, 2002; Goyal and Vega-Redondo, 2005) in that all agents choose an action in every period, instead of only one agent updating at t. Kandori et al. (1993) and Robles (1997) also let all players update their strategies in every period. The agents in Young (1993, 1998) furthermore use an individual random sample of the remembered history of play, which can be longer than one period, whereas Robles (1997) and Riedl et al. (2012) also let their agents use only the previous period. None of these models include communication between agents.

more in detail in section A.1 in the Appendix.

2.2 Analytical results

To derive some benchmark results, we first assume that agents cannot communicate. Best reply actions are then as defined by equations (5) and (6); i.e. since $m_i^t = \varnothing$ for all $i \in N$, $p_i^t(h) = 1$ if $\underline{a}^{t-1} = h$ and 0 otherwise, and $p_i^t(l) = 1$ if $\underline{a}^{t-1} = l$ and 0 otherwise. This yields a regular, perturbed Markov process P^ε on the state space A. As the game is restricted to the action stage, the assumption that $\alpha > \beta > 0$ implies that the combinations where all players choose the same action constitute the strict Nash equilibria of the game. We call the set of strict Nash equilibria $E = \{E_1, E_2, .., E_K\}$, where $1, ..., K$ corresponds to the ranking of actions.

Without communication, our results in the weakest-game are the same as in Riedl et al. (2012) and Robles (1997):[12]

Proposition 1: Let the agents' decision-making process be defined by P^ε and let the state space be A. Then the unique stochastically stable state in the weakest-link game without communication is E_1.

As this result is not new, we omit an individual proof (it is included as a case of proposition 2, see Appendix A for all proofs). The proposition indicates that the least efficient equilibrium, corresponding to all agents choosing action 1, is the most likely long-run outcome of the weakest-link game. While this is in line with much of the experimental evidence, note that the result holds regardless of the number of players and of the incentives to choose the payoff dominant action (the ratio of β to α). This seems intuitively less convincing and two-player experimental groups often manage to achieve efficient coordination (Van Huyck et al., 1990; Camerer, 2003).

The relevant state space for the perturbed Markov process $P^{\varepsilon,\varepsilon}$ defined by the decision-making procedure with communication is $S = M \times A$, and we denote a strategy profile $s \in S$ in period t as $s^t = (s_1^t, s_2^t, ..., s_n^t)$ where $s_i^t = (m_i^t, a_i^t)$. Let states where strategies are such that $s_i^t = (\varnothing, a_i^t) \, \forall \, i \in N$ be denoted E_a and the set of such states be E, i.e. we use, hopefully without any risk of confusion, the same notation for strategies corresponding to the strict Nash equilibria in the game without communication. Recall also that $q_i^t(a|m) = q_j^t(a|m)$ according to the stated assumptions so it suffices to check whether the results hold for one agent. This yields the following proposition:

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[12]See also Svejstrup Hansen and Kaarbøe (2002) for a discussion of results for order-statistic games in similar models, and Crawford (1995) for a different model of adaptive learning without communication that matches the short/medium run dynamics of the weakest-link and median action experiments run in Van Huyck et al. (1990) and Van Huyck et al. (1991). See also Honda (2012) for an illuminating theoretical explanation of equilibrium selection in the two-player version of the weakest-link game without communication and other coordination games.

Proposition 2: Let the agents' decision-making process be defined by $P^{\varepsilon,\varepsilon}$ and let the state space be S. Then, $E_a \in E$ are the only possible absorbing states. Suppose the process is in some E_a. If

(i) $q_i^t(h|m=h) < \frac{\beta}{\alpha} + \frac{c}{\alpha(h-a)}$ for every $E_a \in E$ and all pairs $a, h \in A_i$ such that $h > a$, and $\frac{n\beta}{\alpha} > 1$, then E_1 is the unique stochastically stable state;

(ii) $q_i^t(h|m=h) < \frac{\beta}{\alpha} + \frac{c}{\alpha(h-a)}$ for every $E_a \in E$ and all pairs $a, h \in A_i$ such that $h > a$, and $\frac{n\beta}{\alpha} \leq 1$, then all $E_a \in E$ are stochastically stable;

(iii) $q_i^t(h|m=h) \geq \frac{\beta}{\alpha} + \frac{c}{\alpha(h-a)}$ for at least one $E_a \in E$ and one pair a, h, then E_K is the unique stochastically stable state.

The proposition implies that only states where no agent communicates and all choose the same action can be absorbing states. Conditions (*ii*) and (*iii*) indicate that communication may help agents to avoid the least efficient (all agents play action 1) absorbing state. When agents' expectations that their message will sway the others to a higher ranked action, if "stuck" in some absorbing state, are high enough, the only remaining absorbing state is the most efficient (all agents play action K). A key to this result is that whenever the condition in (*iii*) holds for some $h > a$, it also holds for K as sending K is always at least weakly preferred to any other message, except possibly the empty message. In sum, although communication is not part of any stochastically stable state, the possibility of communication may help coordinate play on more efficient actions.

However, comparing these results to the experimental results in Kriss et al. (2012), we can note that only 4 out of 14 groups in their costly communication treatments manage to achieve a higher ranked minimum action than 1 in the eighth and final round of the experiment. Of these, only one group is coordinated on the highest ranked action. The threshold for $q_i^t(h|m=h)$ in the proposition also indicates that it would be difficult: using the experimental parameters in Kriss et al. (2012) implies that in E_1 (which yields the lowest possible threshold for the condition in (*iii*)), $q_i^t(K|m=K) = q_i^t(7|m=7) > 0.51$ would be needed in the low cost treatment and $q_i^t(7|m=7) > 0.54$ in the high cost treatment. That is, for the highest ranked action to be the stochastically stable state, an agent must expect that there is a larger than 50 percent probability that the group will switch from a minimum action of 1 to a minimum action equal to 7, should she send $m_i^t = 7$. Furthermore, most experimental subjects either send the empty message or $m_i^t = K = 7$ when they can choose whether to communicate or not, and the frequency of messages decline over the course of the experiment. Both the decline of communication over periods seen in the experiment, and the dominance of messages indicating the highest ranked action when the subjects communicate, are in line with proposition 2.

We have so far not made any assumptions on how $q_i^t(a|m)$ depends on the number of agents. It seems reasonable that more agents would make each agent less likely to expect that sending a message would affect the minimum action. If we add an assumption that $q_i^t(a|m)$ is decreasing in

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the number of agents (as we do in the simulations), the results and thresholds in proposition 2 still hold, but E_K would be less likely to be the stochastically stable state when the group is larger. As $n = 9$ in Kriss et al.'s experiments, using communication to break out of an inefficient state can be expected to be difficult. However, as the proposition should be interpreted as the likely long-run state and Kriss et al.'s experiment runs for eight periods, we should perhaps not make too much of this quantitative comparison. We return to the short run properties of our model in section 3.

Proposition 2 implies that just allowing agents to communicate may not be enough to induce coordination on efficient states. One of Kriss et al.'s conclusions is that "in some cases, communication may be effective only if its use by employees is mandatory" (p. 21). Our next result shows that making communication mandatory will help agents solve the coordination problem in our model as well. To create the routine *mandatory communication*, restrict the choice of messages to be $m_i^t \in A_i$ for all $i \in N$; that is, the empty message is not an option any more.[13] Choices of actions are made simultaneously as described by equations (5) and (6), and assumptions 1-5 still hold.

Proposition 3: If *mandatory communication* is in place and messages affect subjective probabilities, then the unique stochastically stable state in the weakest-link game with communication is $s = ((K,K)_1, ..., (K,K)_n)$.

The stated assumptions do not imply that agents must expect a message to have an effect on the subjective probabilities in the communication stage, but if it does, then proposition 3 implies that we are most likely to see agents coordinate on the highest ranked action. This is in line with the experimental results of Blume and Ortmann (2007) and Kriss et al. (2012), a pronounced majority of players both indicate by message and subsequently choose the highest ranked action. The intuition for the result is that once the empty message is no longer available, message costs are not important because of the assumption that they are equal, and the highest ranked action K is always one of the best reply messages. Thus, agents do not risk getting stuck on lower ranked actions and once the minimum action in period $t - 1$ is K, $m_i^t = K$ is the unique best reply message for all agents onwards. That is, the trade-off between lowering the strategic uncertainty for the group and costs of sending messages that exists when communication is voluntary disappear as soon as messages become mandatory.

Another way to coordinate agents is to impose restrictions on who gets to communicate. As tried experimentally in different ways by for instance Weber et al. (2001) and Brandts and Cooper (2007), an agent may therefore be assigned to the role of communicator (interpreted as a manager or a team leader). These two studies (and others mentioned in the introduction) use more free-form communication, so we do not exactly match the set-up in their experiments but model the routine *team leader* as follows: let the team leader be agent 1 and let the communication stage consist of

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[13]It does not matter for this result whether the empty message still can be sent by mistake or not.

agent 1 sending $m_1^t \in A_1$, while no other agent communicates. Agent 1 chooses a best reply message according to equations (2) and (4). Mistakes and experiments are still possible and equally probable in both stages of the game but only agent 1 can make them in the communication stage. As only agent 1 sends messages, assumption 1 about agents' messages is not in play any more, whereas we assume that agent 1's expectations in the communication stage follow assumptions 2-4.

A team leader can have different levels of authority or credibility. We incorporate this notion by making the following assumption. Let $w_{i1}^t(a|m_1^t = a) \in [1, n]$ and $w_{i1}^t(a|m_1^t \neq a) = 0$ be the weights assigned by agent i to action a due to the team leader's message, if the team leader sends or does not send message a, respectively. Again, $w_{i1}^t(a|m_1^t = a)$ is not influenced by the labels of messages, so that $m_1^t = a$ has the same influence on $w_{i1}^t(a|m_1^t = a)$ as $m_1^t = a'$ has on $w_{i1}^t(a'|m_1^t = a')$ for all $a, a' \in A_i$ and all t. Furthermore, let $p_i^t(a) = \frac{1}{n}\left(w_{i1}^t(a|m_1^t) + \sum_{j>1} p_{ij}^t(a)\right)$ and let $\sum_{a=1}^{K} p_i^t(a) = 1$ for all $i \in N$.

These assumptions imply that if $w_{i1}^t(a|m_1^t = a) = n$, then $p_{ij}^t(a') = 0$ for all $j > 1$ and all $a' \neq a$. That is, when $w_{i1}^t(a|m_1^t = a) = n$, the team leader has absolute authority and the previous period's minimum action does not influence the expectations of the other agents. If $w_{i1}^t(a|m_1^t = a) = 1$ agent i does not assign a higher probability to the team leader's message than to the actions of other agents, which can be interpreted as the team leader having no more authority or credibility than any other agent. In the action stage, all agents choose actions simultaneously: agent 1 chooses $a_1^t = m_1^t$ and agents $i \in \{2, ..., n\}$ choose actions according to the modified equations (5) and (6), where w_{i1}^t take the place of p_{i1}^t.

Proposition 4: Let the routine *team leader* be in place. If (*i*) $q_1^t(K|m = K) \geq \beta/\alpha$ and (*ii*) $w_{i1}^t(K|m_1^t = K) \geq n\beta/\alpha$, then $s = ((K, K)_1, (\varnothing, K)_2, ..., (\varnothing, K)_n)$ is the unique stochastically stable state. If $q_1^t(K|m = K) < \beta/\alpha$ and/or $w_{i1}^t(K|m_1^t = K) < n\beta/\alpha$, then $s = ((1, 1)_1, (\varnothing, 1)_2, ..., (\varnothing, 1)_n)$ is the unique stochastically stable state.

For the routine to induce coordination on the highest ranked action, the team leader must both expect a message to result in the indicated action and have enough credibility/authority. As mentioned in the introduction, credibility and expectations matter in the experiments of Brandts et al. (2012), and Kriss and Eil (2012). Here, we also have the intuitive result that with the same level of credibility, it is more difficult for a team leader to lead a larger group to an efficient outcome. This is consistent with the difference between results in smaller and larger groups found in Weber et al. (2001); Cooper (2006); Brandts and Cooper (2007), and Brandts et al. (2012).

3 Simulation

To examine the short-run properties of the model and to relax some of the assumptions, we run simulations. However, there is a trade-off in this latter regard, as the simulations require that we determine how agents' own messages, and the previous period's minimum action and messages, affect expectations in the communication stage in exact terms. In the next section 3.1, we present the version of the model used in the simulation. Section 3.2 describes the parameter configurations and the results.

3.1 Model of communication for simulation

The model of communication described in section 2.1 assumes certain properties about the conditional expectations of agents, i.e. we assign probabilities to actions given messages and minimum actions in the form of $q_i^t(a|m)$, but the model is otherwise silent about how agents reason to reach these expectations. Here, we describe a process where agents reason about how other agents would react to their messages, which we then use in the simulations. Intuitively, each agent i first forms an expectation about what other agents' would believe under different messages sent by i. Second, agent i then computes the expected payoff for each of the other agents, thereby learning which message by i would trigger what payoff-maximizing action by any other agent. This results in a best reply correspondence for each agent. In this formulation, assumptions 1-4 still hold, but the effect of the agent's own message and previous period's messages and minimum action is embodied in equations rather than the general terms of assumption 4. We relax assumption 5 and assumptions about the distribution of mistakes/experiments when we run the simulations.

Let $q_{ij}^t(a|m) \equiv Pr_{ij}\left(a = \underline{a}^t | m, m^{t-1}, \underline{a}^{t-1}\right)$, so i, as before, uses i's own prospective message in period t, and the empirical distribution of messages and the minimum action in $t-1$ but this time to form expectations of j's subjective probabilities in t. More specifically, let

$$q_{ij}^t(a|m) = \frac{1}{n}\left(\mathbf{1}\left(m = a\right) + \sum_{j \in N\setminus\{i\}} \mathbf{1}\left(m_j^{t-1} = a\right) + |\varnothing| \times \mathbf{1}(\underline{a}^{t-1} = a)\right) \tag{7}$$

be i's expectation over j's subjective probability of action a becoming the minimum in period t in the case i should send message m. $\mathbf{1}(\cdot)$ are indicator functions equal to 1 whenever the conditions in parentheses hold. Unless agent i contemplates to make a change from communication to non-communication or the other way around, the term $|\varnothing|$ is just the number of empty messages sent in the previous period. If i contemplates a change from sending a substantive to the empty message (or from the empty to a substantive message), $|\varnothing|$ decreases (increases) by one. That is, if, for some

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$a \in A_i$

$$m = a \text{ and } m_i^{t-1} = \varnothing \Rightarrow |\varnothing| = \sum_{j=1}^{n} \mathbf{1}\left(m_j^{t-1} = \varnothing\right) - 1 \tag{8}$$

$$m = \varnothing \text{ and } m_i^{t-1} = a \Rightarrow |\varnothing| = \sum_{j=1}^{n} \mathbf{1}\left(m_j^{t-1} = \varnothing\right) + 1 \tag{9}$$

where $\mathbf{1}(\cdot)$ is an indicator function equal to 1 when an agent sent the empty message in $t-1$. This formulation constrains $\sum_{a=1}^{K} q_{ij}^t(a|m) = 1$ for each $m \in M_i$ and all t, except for the initial period. We describe the initial expectations we use in section 3.2. Then, agent i can calculate each agent j's expected payoff for $a > 1$ as

$$\mathbb{E}\left(\pi_{ij}(a,m)\right) = \sum_{h=a}^{K} q_{ij}^t(h|m)a(\alpha - \beta) + \sum_{l=1}^{a-1} q_{ij}^t(l|m)(\alpha l - \beta a). \tag{10}$$

As before, each agent's payoff of $a = 1$ is always safe, and equal to $\alpha - \beta$. Agent i does not have to take into account any message costs for agent j, as these represent sunk costs in the action stage for j and are not considered when choosing a best reply action.

Now, using the the expected payoff $\mathbb{E}(\pi_{ij}(a,m))$, agent i can evaluate the expected minimum action by checking each agent j's best reply to each of i's messages, and then choose the message that induces the highest ranked minimum action of the other agents. Formally, let

$$\Pi_j(m) = \{a \in A_i : \mathbb{E}\left(\pi_{ij}(a,m)\right) \geq \mathbb{E}\left(\pi_{ij}(a',m)\right) \forall a' \in A_i\} \tag{11}$$

be the set of actions such that they are an expected best reply to message m for agent j (from the point of view of agent i). If $\mathbb{E}\left(\pi_{ij}(a,m)\right) = \mathbb{E}\left(\pi_{ij}(a',m)\right)$ for some actions a and a', agents randomize uniformly among them (so $\Pi_j(m)$ becomes a singleton). Let $\Pi_{-i}(m) = \Pi_1(m) \cup ... \cup \Pi_{i-1}(m) \cup \Pi_{i+1}(m) \cup ... \cup \Pi_n(m)$ be the union of all agents' $j \neq i$ expected best reply sets. There is thus one $\Pi_{-i}(m)$ for each $m \in M_i$ and $K+1$ in total for every agent i. Agent i then compares the payoffs of the lowest ranked action in each $\Pi_{-i}(m)$ – the minimum, denoted a_{min} – and then chooses the message corresponding to the set with the minimum yielding the highest payoff. We denote this collected set of minimum actions by Π_i^{min}. Best reply messages are found in

$$BR_i^m = \{m \in M_i : \pi_i\left(a_{min}\right) \geq \pi_i\left(a'_{min}\right) \forall a_{min}, a'_{min} \in \Pi_i^{min}\} \tag{12}$$

where

$$\pi_i\left(a_{min}\right) = a_{min}\left(\alpha - \beta\right) - c\left(m\right). \tag{13}$$

If there is more than one message in this best reply correspondence, we again assume that agents

randomize uniformly between them. The implication of the above procedure is that the only probabilistic judgement in the communication stage is made when assessing the impact of a certain message on other agents' choice of best replies. In the action stage, the decision-making is exactly as described by equations (5) and (6).

3.2   Simulation results

We start by comparing our results to Kriss et al. (2012) in the next section, and then examine the model at a more general level by expanding the range of the parameters of the game in section 3.2.2. Each configuration of the parameters run for eight periods, as in the Kriss et al. (2012) experiments. When there are non-zero probabilities of mistake and experiments, we run each configuration 100 times. For most of the results below, we report averages of these 100 repetitions. The simulations without mistakes and experiments are not repeated.[14]

We use three different levels of mistake/experiment probabilities in the communication and action stage: 0, 10, and 20 percent.[15] In contrast to assumption 5, mistake and experiment probabilities do not have to be same in both stages. We also include simulations with alternative distributions to the uniform. The *DoubleDist* distribution captures the idea that experiments and mistakes may be more likely to be close to the originally intended message/action. The probability that a mistake/experiment occurs is thus the same as with the uniform distribution, but doubling the distance from the best reply message/action reduces the probability of being mistakenly chosen by half. Assume for example that there are 5 messages (including the empty message) and that the best reply message is 2. Under uniform probability, each message has a 20 percent chance of being chosen when a mistake/experiment occurs. Under the "double-distance-half-likely" type, the empty message would have the probability of 11.1 percent, 1 of 22.2 percent, 2 of 44.4 percent, 3 a chance of 22.2 percent, and 4 a 11.1 percent chance. The distribution works identically for actions (but there is no action corresponding to the empty message of course).

Another possibility is that if agents experiment in the communication stage, they may be more prone to try higher ranked messages. Such experiments seem less likely in the action stage, as the possible payoff loss of trying higher ranked actions is much greater. We therefore try two other distributions in the communication stage: the *HighestMsg* distribution stipulates that *all* experiments result in the highest ranked message being sent, and the *Exponential* distribution (with rate parameter $\lambda = 1$ in all cases) make messages progressively more likely, the higher their rank. Given the

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[14]Note that there is a chance component also when mistakes/experiment probabilities are zero, as agents resolve the choice between best reply messages/actions with equal expected payoff by randomizing uniformly. As these ties are rare and to keep the number of simulations at a manageable size, we choose not to repeat these runs.

[15]The experimental literature examining the relationship between stated beliefs and strategy choices often find frequencies of choices that are not best reply replies to stated beliefs at least as high as these probabilities of mistakes/experiments, see e.g. Nyarko and Schotter (2002), Costa-Gomes and Weizsäcker (2008), and Manski and Neri (2013).

payoff structure of the game, we expect the highest ranked message to be experimented with more often and it is the second most common message after the empty message in Kriss et al. (2012) (by a wide margin). However, the *HighestMsg* distribution leaves little room for mistakes, which is why we include the *Exponential* distribution that does allow for lower ranked messages to be chosen by mistake.

In the initial round, we use a uniform randomization to create a vector of non-empty messages that agents use to form expectations about which messages they think other agents will send in period 1. Agents then send best reply messages conditional on these expectations as described in section 3.1. In the action stage, agents best reply to sent messages as before, but as there is no minimum action in the previous round, agents use only the messages to form their expectations and the empty message puts equal weight on all actions. So if all messages are empty in the initial round, which may happen, agents randomize uniformly over all available actions.

### 3.2.1 Comparison to experimental results

The two treatments in Kriss et al. (2012) use a ratio of $\beta/\alpha = 10/20$, 9 agents, and 7 actions and let message costs be equal to 1 or 5. In Figure 1 we show the full empirical distributions of minimum actions in round eight of the simulations when message costs are either 1 or 5, and there are mistake/experiment probabilities greater than zero in both the communication and action stages. We show results for all four communication stage distributions of mistakes/experiments, while both the *Uniform* and the *DoubleDist* distributions are included in the action stage. Although it is evident that the distributions are wide-ranging (all seven actions are represented as the minimum action in both), it is also clear that the action corresponding to the stochastically stable state in our model with uniform mistakes/experiments (action 1 for both configurations) is the most frequent among the minimum actions already in round 8 with both levels of message costs and for all distributions. The overrepresentation of action 1 among the minimum actions is somewhat less pronounced if we only allow for a 10 percent probability of mistakes and experiments, but the results for the different distributions are otherwise similar (results not shown). Note also that the distributions are similar regardless of the exact level of the message costs.

In Kriss et al.'s treatment with message costs = 5, all six groups have a minimum action of 1 in the eighth round. With message costs = 1, the distribution is the following: four groups play action 1, two groups play 3, and one group each play action 5 and action 7.[16] Thus, action 1 is the most frequent minimum action also in the experiments.

Table 1 contain the mean and range of our simulated minimum actions in round eight, where the minimum action is averaged over 100 repetitions for each configuration. As can be seen in Figure 1, the *Exponential* and *HighestMsg* distribution yield higher ranked minimum actions on

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[16]Information about the average action in round eight is not included in Kriss et al. (2012), so we cannot compare our results to the distribution of the average action in their experiments.

Figure 1: Distributions of minimum actions in round 8
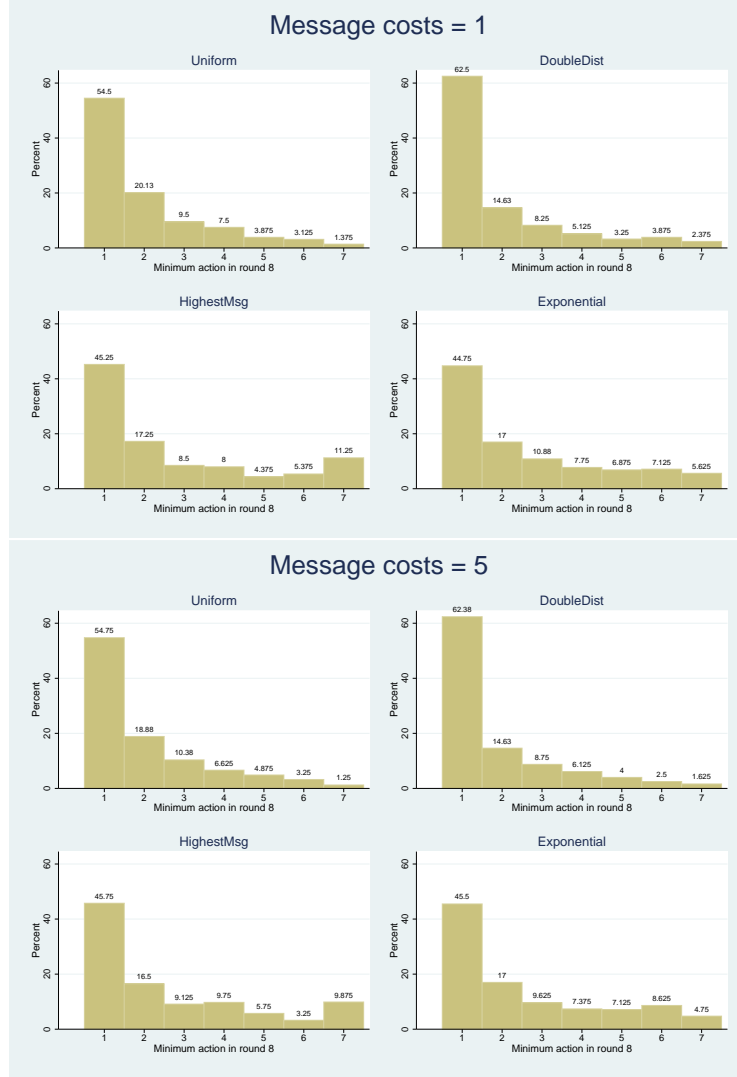
average, but the difference to the other two distributions is not very large. With message costs = 1, the average minimum action in round eight in Kriss et al. is 2.75. This average is included in the range of 5 out of 8 combinations of distributions and is closest to the *DoubleDist-Exponential* combination, which is 2.92. As our means and ranges are very similar for message costs = 5, these are further away from the experimental mean where all six groups end up with a minimum action of 1 in round eight.

These results reflect a feature confirmed also in the results with a wider range of parameters (presented in the next section): our estimates are not very sensitive to increases in the cost of messages. Kriss et al.'s subjects do on the other hand seem to react to the different costs, although one reason may be that we are comparing our averages for the minimum actions over groups to

Table 1: Mean and range, *Average minimum action* in round 8

	Panel A: Message costs = 1			
	(1)	(2)	(3)	(4)
DISTRIBUTION	*Uniform*	*DoubleDist*	*HighestMsg*	*Exponential*
Uniform	1.80	1.71	2.21	2.22
	[1.60,1.98]	[1.53,1.89]	[1.77,2.87]	[1.52,2.86]
DoubleDist	2.25	2.15	3.19	2.92
	[1.65,2.81]	[1.77,2.50]	[1.95,4.45]	[1.91,3.78]
	Panel B: Message costs = 5			
	(1)	(2)	(3)	(4)
DISTRIBUTION	*Uniform*	*DoubleDist*	*HighestMsg*	*Exponential*
Uniform	1.71	1.73	2.01	2.08
	[1.50,1.86]	[1.45,1.91]	[1.28,2.82]	[1.64,2.73]
DoubleDist	2.35	2.05	3.24	3.09
	[1.95,2.88]	[1.70,2.34]	[2.06,4.39]	[2.1,4.09]

The table display the mean and, in square brackets, the upper and lower
bound of the range of average minimum actions in round 8 for each
combination of communication and action stage distributions.

a very small sample. As mentioned, there are 8 groups in the low message cost treatment and 6
in the high message cost treatment in their experiments. A hypothetical seventh group in the high
message cost treatment coordinating on a minimum action of 6 or 7 in round 8 would raise the
mean so that it would be included in 5 out of 8 ranges for example. A behavioral reason may be
that the salience of costs differ, message costs = 1 may be treated as negligibly small for example,
while higher message costs may loom larger and enter into the calculations of subjects' expected
payoffs. A related possibility is that the lower message costs make agents experiment with sending
messages of higher actions more often, which our simulations do not allow for. The difference in
the pattern of communication is not large between the two treatments, but both messages in general
and the highest ranked message are more frequent in the low cost treatment.

3.2.2 Regression results

To be able to separate the effects from different variables and to report the general results in a
succinct way, we run OLS regressions with the *average action*, and the *average minimum action*
in round eight as dependent variables (both variables are averaged over 100 repetitions in con-
figurations with positive mistake/experiment probabilities). As independent variables, we include
indicator variables for each increment of the variables used to determine the configurations, using
the category with the lowest value as the reference category throughout.

The following variables determine the configurations. *Number of agents* and *Number of ac-
tions*: both the number of agents and actions are varied between 2-10 in increments of two for the

regressions. These are denoted as for example *agents$_2$* if the configuration uses two agents. *Message costs*: the cost of sending messages is increased in increments of two, starting from 1 and up to 9. *Action mistake probabilities* and *Communication mistake probabilities*: we use three different levels of mistake/experiment probabilities in both the communication and action stage: 0, 10, and 20 percent.

Uniform, *DoubleDist*, *HighestMsg* and *Exponential*: We include two distributions of mistakes and experiments in the action stage, and four in the communication stage as described in the previous section. All are included as indicator variables, and the uniform distribution is the reference category in all regressions (*DoubleDist(actions)* is the indicator variable for the *DoubleDist* distribution in the action stage).

β/α-*ratio*: We keep α constant at 20, while β varies between 8-12 in increments of one, so *ratio$_r$* $\in \{0.40, 0.45, 0.50, 0.55, 0.60\}$, $r = 1, 2, 3, 4, 5$. The mid-point 0.50 is the most commonly used ratio in the experimental literature. As β increases, the incentives to choose a higher ranked action becomes weaker. Thus, we expect a negative relationship between the β/α-*ratio* and our outcome variables.

Using the full ranges described above yields 45,000 configurations (and a total number of simulations well over 4 million). The results of the OLS regressions are shown in table 2.[17] Columns (1)-(2) use the average action in round 8 as the dependent variable and columns (3)-(4) use the average minimum action in round 8. Columns (1) and (3) contain all configurations regardless of whether mistakes and experiments are possible, while columns (2) and (4) contain specifications where there are non-zero probabilities of mistakes and experiments in both stages.[18] In columns (2) and (4), we use the categories where mistake and experiment probabilities are 10 percent as reference categories instead of the 0 percent category.

The results for the average and minimum actions are similar for most variables over the two types of specifications, so we discuss them together. We expect that increasing the *number of agents* should make it more difficult to use communication to break out of inefficient states, and to increase the number of occasions where some agent makes a mistake or experiments. The first effect should imply lower average actions and average minimum actions, while direction of the second effect depends on the distribution used. The estimates get progressively more negative as we increase the number of agents. The largest change is the jump from 2 (the reference category) to 4 agents.

Message costs are not a large influence on either the average or minimum action in round 8. Except for the highest cost category in the specifications with all configurations included, the

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[17]Indicator variables for actions are suppressed in table, but included in all specifications. The results for the number of actions are perhaps less interesting as when more actions are available, the average and minimum actions increase mechanically.

[18]Using an intermediate specification where mistakes and experiments are possible in at least one stage does not change the results much.

Table 2: *Average action* and *Average minimum action* in round 8

VARIABLES	(1) Avg action	(2) Avg action	(3) Min action	(4) Min action
$agents_4$	-0.719***	-0.694***	-0.800***	-0.761***
	(0.0208)	(0.0267)	(0.0221)	(0.0289)
$agents_6$	-1.122***	-1.327***	-1.239***	-1.452***
	(0.0205)	(0.0254)	(0.0219)	(0.0281)
$agents_8$	-1.541***	-1.985***	-1.716***	-2.284***
	(0.0212)	(0.0255)	(0.0224)	(0.0278)
$agents_{10}$	-1.804***	-2.321***	-1.996***	-2.651***
	(0.0219)	(0.0266)	(0.0231)	(0.0289)
$msgcosts_3$	-0.00772	0.00228	-0.00801	0.00196
	(0.0194)	(0.0221)	(0.0205)	(0.0245)
$msgcosts_5$	-0.00982	-0.00168	-0.00998	-0.00127
	(0.0193)	(0.0221)	(0.0205)	(0.0244)
$msgcosts_7$	-0.0228	-0.00914	-0.0234	-0.0101
	(0.0194)	(0.0221)	(0.0205)	(0.0245)
$msgcosts_9$	-0.0587***	-0.0340	-0.0611***	-0.0370
	(0.0194)	(0.0222)	(0.0205)	(0.0245)
$actionmistake_1$	-1.262***		-1.335***	
	(0.0156)		(0.0161)	
$actionmistake_2$	-1.588***	-0.281***	-1.754***	-0.425***
	(0.0158)	(0.0144)	(0.0167)	(0.0158)
$commistake_1$	0.565***		0.562***	
	(0.0156)		(0.0164)	
$commistake_2$	0.918***	0.455***	0.939***	0.429***
	(0.0159)	(0.0144)	(0.0167)	(0.0157)
$DoubleDist(actions)$	0.310***	0.500***	0.380***	0.557***
	(0.0123)	(0.0140)	(0.0130)	(0.0155)
$DoubleDist$	-0.239***	-0.343***	-0.253***	-0.375***
	(0.0176)	(0.0213)	(0.0185)	(0.0231)
$HighestMsg$	0.606***	0.756***	0.639***	0.826***
	(0.0168)	(0.0188)	(0.0177)	(0.0206)
$Exponential$	0.386***	0.723***	0.480***	1.032***
	(0.0182)	(0.0194)	(0.0192)	(0.0215)
$ratio_2$	0.0324	-0.0313	0.0309	-0.0350
	(0.0199)	(0.0236)	(0.0212)	(0.0259)
$ratio_3$	-0.300***	-0.294***	-0.363***	-0.407***
	(0.0193)	(0.0225)	(0.0203)	(0.0244)
$ratio_4$	-1.193***	-1.179***	-1.283***	-1.323***
	(0.0198)	(0.0223)	(0.0210)	(0.0248)
$ratio_5$	-1.224***	-1.209***	-1.316***	-1.357***
	(0.0197)	(0.0225)	(0.0209)	(0.0249)
Constant	2.854***	2.391***	3.002***	2.594***
	(0.0310)	(0.0375)	(0.0328)	(0.0413)
Mean, dep var	3.44	3.13	3.40	3.13
Observations	45,000	20,000	45,000	20,000
R^2	0.701	0.775	0.686	0.756

Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1
Indicator variables for the number of actions are included in all
specifications but are left out of the table.

estimates are very close to zero (and the magnitudes are also rather small for message costs = 9). This can actually be seen also in the conditions of proposition 2: while the stochastically stable state depends to some degree on message costs, the ratio has greater influence. Beyond the introduction of costly communication, the exact levels of message costs is therefore not a huge influence on the outcome in either version of our model.

The coefficients representing the frequency of mistakes/experiments in the action stage are negative and large, while mistakes and experiments have a positive but smaller influence in the communication stage. This is in line with the proof of proposition 2, where it turns out that mistakes/experiments in the action stage for most parameter values determine the stochastically stable state – often resulting in the most inefficient state being stochastically stable. However, in some cases when incentives are strong and groups are small, communication mistakes also become important, in which case all absorbing states become stochastically stable. In the action stage, almost all of the effect comes from going from no mistakes/experiments (the reference category) to having at least some, as the coefficients on the variables are of similar magnitude for both $actionmistake_1$ and $actionmistake_2$. In the communication stage, a 20 percent mistake/experiment probability increases the average action by almost 1, and adds almost 0.5 compared to the 10 percent probability. This difference between the stages is due to the *HighestMsg* and *Exponential* distributions, which increase the play of higher ranked actions.

The *DoubleDist* distribution affects the results differently depending on whether we are in the communication or action stage. Compared to the uniform distribution, *DoubleDist(actions)* makes it less likely that play drops all the way to a lower ranked action by just one mistake in the action stage. This makes sense since any mistake tends to be closer to the intended action than under the uniform distribution where a far lower action played by mistake is quickly followed by far lower play of other agents. On the other hand, in the communication stage, the *DoubleDist* distribution makes it less likely that a mistaken/experimental message indicates a much higher ranked action than the current state. The *HighestMsg* and the *Exponential* distribution increase both the *average action* and the *average minimum action* in round 8, as expected.

Lastly, the coefficients for *ratios* are mostly negative and becoming progressively more negative the higher the ratio becomes. This is in line with the model's predictions as a higher ratio represents weaker incentives to play a higher ranked action. $Ratio_2$ is however positive relative to $ratio_1$ as the reference category in column (1) and (3), which may seem unintuitive (although $ratio_2$ is never significant). One explanation is that the stochastically stable state is the same for a large share of the configurations, regardless of the ratio. Thus, we expect to see convergence over time. An interpretation in terms of behavior that fits with how agents in our simulations react in the initial round could be that with a higher ratio, agents are relatively certain that others will indicate the highest ranked action with a message. Therefore, they abstain from doing so themselves (i.e. take

the chance to free ride on other agents' messages).[19] However, if many or all agents think in this way, few will actually send a message. When faced with an unexpected situation in the action stage agents may therefore find it difficult to assess what will happen and choose an action more at random, which frequently yield low ranked minimum actions.

4   Concluding remarks

This paper develops a model to examine how communication affects organizational coordination when actions are highly interdependent among boundedly rational agents. The results imply that efficient coordination may be difficult to achieve when communication is costly, as the stochastically stable state is often the least efficient coordinated state. Even if message costs are small compared to the potential gains of efficient coordination, the costs introduce a trade-off for agents between lowering the strategic uncertainty for the group and the costs of communication. This effect of communication costs may explain the contrasting results in experiments with costly and costless communication.

We also use a version of the model in simulations to examine its short run properties. The stochastically stable states often have considerable explanatory power also in the short run, as these states are overrepresented in the empirical distribution of minimum actions after eight rounds (especially when groups are large). The difficulties experienced by experimental subjects to coordinate on efficient states when communication is costly is clearly present also in our simulations.

These results suggests that organizations may improve the efficiency of groups by lowering communication costs, but also that lower communication costs are often not enough to achieve efficient coordination. That organizations can structure communication by imposing formal rules and routines may therefore be more important and, under certain conditions, necessary for efficient coordination. Situations where such structures are often missing, like new collaborations between teams or organizations, frequently result in coordination failures. We examine two simple routines, *mandatory communication*, and the assignment of a *team leader*. Mandatory communication implies that sending and choosing the payoff dominant action is the unique stochastically stable state. A team leader may also induce efficient coordination but only when he or she has enough authority or credibility, and expects to be able to persuade the group to choose the communicated action.

While our model is broadly consistent with recent experimental results, it is of course in some aspects a drastic simplification of human decision-making. But we think that the modelling of costly communication is at least one step towards richer game-theoretical models of organizational coordination; models that allow for more general ways of communication and are informative about

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[19]In Kriss et al. (2012), the modal message in the first round of the costly communication treatments is the empty message; 45.8 and 53.7 percent of the subjects send this message in low and high cost treatment respectively, so it is not uncommon that agents choose not to communicate in the first round.

how communication and routines can be mixed to achieve efficient coordination. Interesting future developments in this direction would be to let agents communicate sequentially and send more than one message, to generalize the number of periods agents remember, and to model communication in other coordination games, for example the median action game.

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A Proofs of propositions

We start in the next section by defining the concept of stochastic stability and how stochastically stable states can be computed, as well as some properties of unperturbed and perturbed Markov processes that we use in the proofs. A fuller description of these concepts can be found in for example Young (1998) (especially chapter 3, which we follow closely below). The proofs of the propositions follow in sections A.2 – A.4.

A.1 Stochastic stability

A discrete-time Markov process on a finite state space X specifies the probability that the process changes from state x to state y from one period to the next for each state $x, y \in X$ (Young, 1998). In our model, the largest state space we use is $S = M \times A$, which is clearly finite. The transition probability of moving to state $s = (m^{t+1}, a^{t+1})$ in period $t+1$ conditional on being in $s' = (m^t, a^t)$ in t is determined by the frequencies of messages and the minimum action in t, as well as the probabilities of mistakes and experiments. As long as the mistake/experiment probabilities are non-zero but small, they imply that the process can be regarded as a perturbed Markov process, in the sense that the transition probabilities are slightly distorted versions of some original process, called P^0. Young (1993, 1998) calls such processes *regular perturbed Markov processes*, denotes them P^ε, and define them to have certain characteristics, which we describe below.

Definition: P^ε is aperiodic and irreducible for all $\varepsilon \in (0, \varepsilon^*]$, where $\varepsilon^* > 0$.

Aperiodic means that the process can return to a state x at irregular times. A process is irreducible if there is a positive probability of moving from any state to any other state in a finite number of periods. Because mistakes and experiments are possible in every period in our setting, any state can be reached with positive probability from any other state.

As P^ε is irreducible for every $\varepsilon > 0$, it has a unique stationary distribution μ^ε (Young, 1993). Again following Young (1993, 1998), a state x is *stochastically stable* if

$$\lim_{\varepsilon \to 0} \mu^\varepsilon(x) > 0, \tag{14}$$

i.e. any state that the limiting distribution puts positive probability on is a stochastically stable state. The limit $\lim_{\varepsilon \to 0} \mu^\varepsilon(x) = \mu^0(x)$ exists for every x, and the limiting distribution μ^0 is a stationary distribution of P^0. It follows in particular that every regular perturbed Markov process has at least one stochastically stable state. To describe a way to find this state or states, we need to define some other concepts as well.

Definition: A *recurrent class* of P^0 is a collection of states such that no state outside the class is accessible from any state inside it, i.e. the probability of leaving a recurrent class is zero. A state is called *absorbing* if it constitutes a singleton recurrent class.

Let $E = \{E_1, E_2, ..., E_K\}$ denote the set of recurrent classes of the unperturbed process. An irreducible process, like the perturbed one, has only one recurrent class, which consists of the whole state space. There is in general several different ways of reaching an absorbing state or a recurrent class.

Definition: A *aa'-path* is a sequence of states $\zeta = (E_a = z_1, z_2, ..., z_q = E_{a'})$ that start in E_a and end in $E_{a'}$.

We also need a concept for how "difficult" it is for the process to move from a certain state to another:

Definition: The *resistance* of a one-period transition between two states z_i, z_j in a perturbed process, denoted $r(z_i, z_j)$, is the minimum number of mistakes or experiments required to make the transition, i.e. $r(z_i, z_j)$ is a positive integer, or zero if no mistakes or experiments are needed. The resistance of a *aa'*-path is the sum of the resistances on the path, i.e. $r(\zeta) = r(z_1, z_2) + r(z_2, z_3) + ... + r(z_{q-1}, z_q)$.

As it is impossible to leave a recurrent class or an absorbing state without mistakes/experiments, the resistance of a transition from a recurrent class E_a to another $E_{a'}$ is always positive. There can in general be many *aa'*-paths, but to find the stochastically stable we are going to be interested in the ones with the least resistance.

Definition: $r_{aa'} = \min r(\zeta)$ is the *minimum total resistance* needed to transition from E_a to $E_{a'}$ for all possible *aa'*-paths ζ.

Note that $r_{aa'}$ need not be equal to $r_{a'a}$. Young (1998, p. 55-56) describes how the stochastically stable states can be computed in a simple way: first, construct a complete directed graph with K nodes, one for each recurrent class. The directed edge $a \to a'$ from E_a to $E_{a'}$ is called *aa'* and the weight on the edge is equal to $r_{aa'}$. A rooted tree T is a set of $K-1$ directed edges such that from every node different from E_a, there is a unique directed path in the tree to E_a. The total resistance of T is the sum of the minimum resistances $r_{aa'}$ on the $K-1$ edges that compose it.

Definition: The *stochastic potential* $\gamma(E_a)$ of the recurrent class E_a is defined as the minimum resistance over all trees rooted at a. That is, denote the set of all trees rooted at E_a with $T(a)$, then the stochastic potential is

$$\gamma(E_a) = \min_{T \in T(a)} \sum_{k, k' \in T} r_{kk'}. \tag{15}$$

Stochastically stable states are the absorbing states that have the minimum stochastic potential, i.e. $\min_{E_a \in E} \gamma(E_a)$ (Young, 1993, Theorem 2).

Example: Consider the complete graph in Figure A.1, where the three recurrent classes E_1, E_2, and E_3 are represented by the three nodes and the resistances between these classes are shown by the adjoining numbers to the edges.

This example has nine rooted trees, three for each node. For example, the three trees rooted at E_1 have the following directed edges: $(23, 31)$; $(21, 31)$; $(32; 21)$. The stochastic potentials – the

Figure A.1: Pairwise resistances between recurrent classes
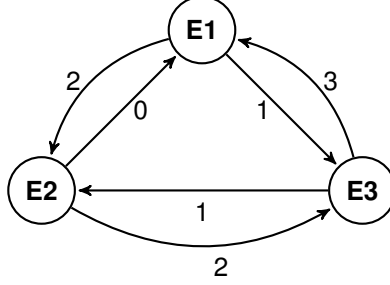
summed resistances on the tree with the minimum resistance for each E_a – are :

$$\gamma(E_1) = r_{32} + r_{21} = 1 + 0 = 1$$
$$\gamma(E_2) = r_{13} + r_{32} = 1 + 1 = 2$$
$$\gamma(E_3) = r_{21} + r_{13} = 0 + 1 = 1$$

Consequently, $\gamma(E_1)$ and $\gamma(E_3)$ have the same minimum stochastic potential and are therefore the stochastically stable states.

   All proofs use the idea in Riedl et al. (2012) that if a state can be reached with a number of uncoordinated mistakes, it can also be reached with the same number of "coordinated" mistakes. That is, if we for example assume that all mistakes are made in the action stage, one of the least resistance aa'-paths between any E_a and $E_{a'}$ is always one where all mistakes/experiments are of action a' (the path need not be unique). This is so since all combinations of the same number of mistakes/experiments have the same probability, as the distribution of mistakes/experiments is uniform. Also, for all E_a except E_1 and E_K, moves to both higher and lower ranked absorbing states are possible. Thus, if it always requires less mistakes/experiments to move to a lower ranked state, then $\gamma(E_1) < \gamma(E_a) \, \forall E_a \in E$. If it always requires more mistakes/experiments to move to a lower ranked state, then E_K has the minimum stochastic potential, i.e $\gamma(E_K) < \gamma(E_a) \, \forall E_a \in E$.

A.2   Proof of proposition 2

We start by stating two lemmas. The first shows that the expected payoff of $m = K$ is always weakly greater than all other messages except possibly the empty message and that sending a lower ranked message than the previous period's minimum is never a best reply. The second shows that the only candidates for absorbing states, and thus for stochastically stable states, are $E_a \in E$. As all agents use the same decision-making process and the same information, we need only to check the condi-

tions for one agent i.

Lemma 1: For all t, (*i*) $\mathbb{E}\left(\pi_i(a, m = K)\right) \geq \mathbb{E}\left(\pi_i(a, m = a')\right)$ for all $a, a' \in A_i$; and (*ii*) for all $l < \underline{a}^{t-1}, m = l \notin BR_i^m$.

Proof: For (*i*): First, $\sum_{h=a}^{K} q_i^t(h|m = K) \geq \sum_{h=a}^{K} q_i^t(h|m = a')$ for all $a, a' \in A_i$ as the number of messages indicating actions ranked higher than or equal to a is at least as many in the first term and messages affect probabilities only by their frequencies according to assumption 3. In turn, $\sum_{l=1}^{a-1} q_i^t(l|m = K) \leq \sum_{l=1}^{a-1} q_i^t(l|m = a')$ since $\sum_{a=1}^{K} q_i^t(a|m) = 1$. From equation (2), we can see that $m = K$ thus always implies at least as much weight on $a(\alpha - \beta)$, and as $a(\alpha - \beta) > \alpha a' - \beta a$ for all $a, a' \in A_i$ such that $a > a'$, $\mathbb{E}\left(\pi_i(a, m = K)\right) \geq \mathbb{E}\left(\pi_i(a, m = a')\right) \forall a, a' \in A_i$, which proves part (*i*).

To prove (*ii*), assume $\underline{a}^{t-1} = a$ and $m = l \in BR_i^m$. Then, $\mathbb{E}\left(\pi_i(m = l)\right) \geq \mathbb{E}\left(\pi_i(m = K)\right)$ by equation (4). Part (*i*) implies that this can hold at best with equality. If messages are able to affect subjective probabilities, then by assumption 3: $\mathbb{E}\left(\pi(l', m = K)\right) = \mathbb{E}\left(\pi(l', m = l')\right)$ for all $l' \leq l$. Furthermore, $\mathbb{E}\left(\pi(h, m = K)\right) \geq \mathbb{E}\left(\pi(h, m = l')\right)$ for all $h \geq a > l$ as the frequency of higher ranked messages is equal for l and lower ranked actions, while higher for all $h > l$ so that $\sum_{h>l}^{K} q_i^t(h|m = K) \geq \sum_{h>l}^{K} q_i^t(h|m = l)$. This implies that $\sum_{h>l}^{K} q_i^t(h|m = K) = \sum_{h>l}^{K} q_i^t(h|m = l)$ only if $m = K$ is not expected to increase $q_i^t(K|m = K)$ and therefore does not increase $\sum_{h>l}^{K} q_i^t(h|m)$ (which $m = l$ is never expected to do by assumption 4). But if messages do not increase subjective probabilities, $\sum_{h>l}^{K} q_i^t(h|m = \varnothing) \geq \sum_{h>l}^{K} q_i^t(h|m = l)$ for all $h > l$ as $q_i^t(a|m)$ is non-decreasing in $m = \varnothing$ by assumption 4. Therefore, as $c(\varnothing) = 0$ and $c(l) > 0$, $\mathbb{E}\left(\pi(m = \varnothing)\right) > \mathbb{E}\left(\pi(m = l)\right)$, contradicting $m = l \in BR^m$.∎

Lemma 2: Only $E_a \in E$ can be absorbing states of the unperturbed process.

Assume $\underline{a}^{t-1} = a$. Lemma 1 implies that no agent sends $m_i^t = l < a$ (but possibly the empty message). Then, as $\sum_{l=1}^{a-1} p_i^t(l) = 0$ according to equation (5), and $a(\alpha - \beta) > l(\alpha - \beta)$ for all $l < a$, playing a lower ranked action than a cannot be a best reply in t. This implies that we cannot go back to lower ranked actions being minimum actions in the unperturbed process. As agents are identical, either all send $m_i^t = h \geq a$ so that all agents' best reply actions are equal to h, or all send the empty message and a is the minimum action also in t and onwards.

Assume $s_i^t = (h, h) \forall i \in N$. Then in $t + 1$, by assumption 1, all agents expect the same distribution of other agents' messages. As $q_i^{t+1}(a'|m = \varnothing) = 0 \forall a' \neq h$ due to assumption 2, this and assumption 4 implies $q_i^{t+1}(h|m = h) = q_i^{t+1}(h|m = \varnothing) = 1$, and as $c(h) > c(\varnothing)$, agents send $m_i^t = \varnothing$. By equation (5), indicating action h or no communication has the same effect on $p_i^t(h)$ in this case. Then, as $\underline{a}^t = h$, h is the best reply action and strategy profile $s_i = (\varnothing, h)$ in $t + 1$ and onwards. Assume instead $s_i^t = (\varnothing, a) \forall i \in N$. Then, as $m_i^t = h$ was not a best reply for any agent,

$s_i = (\varnothing, a)$ in $t+1$ and onwards. As these scenarios hold for all $a \in A_i$ and all agents, the only possible absorbing states are $E_a \in E$.∎

This leaves $E_a \in E$ as candidates for stochastically stable states. Assume we are in E_a in $t-1$. By lemma 1, sending a higher ranked message is the only possibility besides the empty message. By assumption 1 and 2, $q_i^t(a|m=\varnothing) = 1$ and $q_i^t(a'|m=\varnothing) = 0 \,\forall\, a' \neq a$. By the same assumptions, for $h > a$: $\sum_{h>a}^{K} q_i^t(h|m=h) = q_i^t(h|m=h)$ and $\sum_{l=1}^{h-1} q_i^t(l|m=h) = q_i^t(a|m=h) = 1 - q_i^t(h|m=h)$. Using this and equation (2), we can see that whenever

$$a(\alpha - \beta) > q_i^t(h|m=h)h(\alpha - \beta) + \left(1 - q_i^t(h|m=h)\right)(\alpha a - \beta h) - c$$
$$\Rightarrow q_i^t(h|m=h) < \frac{\beta}{\alpha} + \frac{c}{\alpha(h-a)} \tag{16}$$

holds for all $a, h \in A_i$ there is no better reply to $s_i = (\varnothing, a)$ than itself.

To separate between the candidates for stochastically stable states whenever equation (16) hold, we check the number of mistakes/experiments needed to move from one absorbing state to another. In the action stage, the following must hold for a higher and lower ranked action than a to be a best reply and thus for a move to a new absorbing state:

$$p_i^t(h)h(\alpha - \beta) + \left(1 - p_i^t(h)\right)(\alpha a - \beta h) \geq a(\alpha - \beta) \Rightarrow p_i^t(h) \geq \frac{\beta}{\alpha} \tag{17}$$

$$l(\alpha - \beta) \geq \left(1 - p_i^t(l)\right)a(\alpha - \beta) + p_i^t(l)(\alpha l - \beta a) \Rightarrow p_i^t(l) \geq 1 - \frac{\beta}{\alpha} \tag{18}$$

This holds as we only need to consider coordinated mistakes/experiments, which imply that the frequencies of all other actions not h or a (l and a) are zero. Therefore, a is a certain payoff in first equation above, and l in the second as they are lower ranked.

Assume first that all mistakes/experiments are made in the communication stage, then given the definitions of $p_i^t(h)$ and $p_i^t(l)$ we can rewrite these conditions as

$$p_i^t(h) = \frac{1}{n}\sum_{j \in N} p_{ij}^t(h) \Rightarrow \sum_{j \in N} p_{ij}^t(h) \geq \frac{n\beta}{\alpha} \tag{19}$$

$$p_i^t(l) = \frac{1}{n}\sum_{j \in N} p_{ij}^t(l) \Rightarrow \sum_{j \in N} p_{ij}^t(l) \geq n\left(1 - \frac{\beta}{\alpha}\right) \tag{20}$$

where $p_{ij}^t(h)$ and $p_{ij}^t(l)$ is 1 if j makes a mistake/experiment and otherwise 0, as $m_i^t = \varnothing$ is the best reply message in E_a by the proof of Lemma 2.

Assume instead that all mistakes/experiments are made in the action stage (this is thus the situation the agents are in when communication is not possible, as in proposition 1). By equation (5), if $\underline{a}^t = h$, then $p_i^{t+1}(h) = 1$ and otherwise 0. That is, $a_i^t \geq h \,\forall\, i \in N$ must hold for h to be

the best reply in $t+1$. As this can only happen by mistake/experiment in t, it takes at least n mistakes/experiments to move to a higher ranked absorbing state. Similarly, $p_i^t(l) = 1$ in $t+1$ if $\underline{a}^t = l$. Moving to a lower ranked absorbing state requires only one mistake/experiment for any $l < a$. Thus, if $\frac{n\beta}{\alpha} > 1$, $\gamma(E_1) < \gamma(E_a) \, \forall E_a \in E$ such that $a \neq 1$ and E_1 is the unique stochastically stable state. If $\frac{n\beta}{\alpha} \leq 1$, then all $E_a \in E$ are stochastically stable as it requires just one mistake/experiment to move to any absorbing state (combinations of mistakes/experiments in the two stages always require more mistakes/experiments than one). This proves part (*i*) and (*ii*) of proposition 2.

For part (*iii*), assume again we are in E_a in $t-1$, and that at first $q_i^t(h|m = h) \geq \frac{\beta}{\alpha} + \frac{c}{\alpha(h-a)}$ for all pairs h, a such that $h > a$. It is enough that this holds with equality, as then agents randomize between sending h and the empty message. At some $t' \geq t$, enough agents send h so that this is also the best reply action, and by the proof of lemma 2, we cannot go back to lower ranked actions in the unperturbed process.

By lemma 1, if this condition hold for h, it must also hold for K. If i is contemplating either $m = h$ or $m = K$, then under assumption 2 the only other action with positive probability is a. In turn, by assumption 3 $q_i^t(K|m = K) = q_i^t(h|m = h)$, $q_i^t(a|m = h) = q_i^t(a|m = K) = 1 - q_i^t(h|m = h) = 1 - q_i^t(K|m = K)$. Then we can write the difference between the expected conditional payoffs of messages K and h as

$$q_i^t(K|m = K)K(\alpha - \beta) + \sum_{l=1}^{K-1} q_i^t(l|m = K)(\alpha l - \beta K) - q_i^t(h|m = h)h(\alpha - \beta)$$

$$-\sum_{l=1}^{h-1} q_i^t(l|m = h)(\alpha l - \beta h) = (K - h)\left(q_i^t(K|m = K)\alpha - \beta\right). \tag{21}$$

Expression (21) is always positive as $q_i^t(K|m = K)\alpha > \beta$ if $q_i^t(K|m = K) \geq \frac{\beta}{\alpha} + \frac{c}{\alpha(K-a)}$. So if equation (16) does not hold, all agents send $m_i^t = K$ and K is consequently the best reply action. In turn, the best reply to $s_i^t = (K, K)$, for all i, is $s_i^{t+1} = (\varnothing, K)$ (see Lemma 2), which is the only best reply to itself and the only absorbing state and stochastically stable state.

For any $h > a$, $\frac{\beta}{\alpha} + \frac{c}{\alpha(h-a)}$ is smallest when $a = 1$. Thus, whenever $q_i^t(h|m = h) \geq \frac{\beta}{\alpha} + \frac{c}{\alpha(h-a)}$ holds for some a, h, it also holds for h and $a = 1$. Assume that it holds for $h = K$ and $a = 1$, but does not hold for any other pair $a, a' \in A_i$. This implies that $E_2, ..., E_K$ are absorbing states. By the proof of part (*i*), unless $n\beta/\alpha \leq 1$, the move to E_1 requires the fewest mistakes/experiments among $E_a \in E$. Whenever in E_1, $s_i = (K, K)$ is the unique best reply, i.e. the resistance between the states is zero. A similar argument can be made when $q_i^t(h|m = h) \geq \frac{\beta}{\alpha} + \frac{c}{\alpha(h-a)}$ hold also for $a > 1$, which implies that E_K has the minimum stochastic potential whenever $q_i^t(K|m = h) \geq \frac{\beta}{\alpha} + \frac{c}{\alpha(h-a)}$ holds for some pair a, h and $n\beta/\alpha \leq 1$. This concludes the proof.∎

A.3 Proof of proposition 3

Assume $\underline{a}^{t-1} = a$. By Lemma 1, sending $m_i^t = K$ is at least weakly preferred to all other messages in any period. If BR_i^m contains more than one message, agents randomize uniformly between these. As $m = K \in BR_i^m$ holds for all t and there are finitely many messages, at some $t' \geq t$ enough agents will send $m_i^{t'} = K$ so that $\underline{a}^{t-1} = K$. If messages affect subjective probabilities, $q_i^{t'+1}(K|m = K) > q_i^{t'+1}(K|m = a')$, and $\sum_{h=a'}^K q_i^{t'+1}(h|m = K) \geq \sum_{h=a'}^K q_i^{t'+1}(h|m = a')$ for any $a' \neq K$. Therefore, $\mathbb{E}(\pi_i(m = K)) > \mathbb{E}(\pi_i(m = a'))$, and K is then the only best reply message. K is consequently the only best reply action in $t' + 1$. As $s = ((K,K)_1, ..., (K,K)_n)$ for the same reasons is the best reply to itself, it is the unique absorbing and stochastically stable state.∎

A.4 Proof of proposition 4

Assume $\underline{a}^{t-1} = a$ and that $m_1^{t-1} = K > a$. Then agent 1's message did not make K into a best reply for all agents in $t - 1$. As in equation (19) in the proof of proposition 2, this implies that $p_i^t(K) < \frac{\beta}{\alpha}$. According to our definitions, $p_i^t(K|m_1^t = h) = \frac{w_{i1}^t(K|m_1^t = K)}{n}$ when $\underline{a}^{t-1} = a$. Together with the assumption that $m_1^{t-1} = K$ did not change the minimum action into K, this implies that $w_{i1}^t(K|m_1^t = K) < n\beta/\alpha$. If this holds for K, it holds for all $h > a$ and messages do not change the minimum action. Then only $m_1^{t'} = a$ remain a best reply message for agent 1 and mistakes/experiments in the action stage are the only source of change from absorbing states.[20] Therefore, $s = ((1,1)_1, (\varnothing, 1)_2, ..., (\varnothing, 1)_n)$ is the unique stochastically stable state for the same reason as in the proof of proposition 2: it takes only one mistake/experiment to move to lowest ranked action from any other state and more to move higher ranked.

Assume instead that $w_{i1}^t(h|m_1^t = h) \geq n\beta/\alpha$ holds for all $h \in A_i$ (it either holds for all actions or none by assumption 3). Agent 1 sends a higher ranked message if it is expected to change the choices of the other agents. As Lemma 1 and equation (21) from the proof of proposition 2 holds for agent 1, the best reply message for agent 1 is $m_1^t = K$ in such a case. That is, agent 1 sends $m_1^t = K$ when

$$q_1^t(K|m = K)K(\alpha - \beta) + (1 - q_1^t(K|m = K))(\alpha a - \beta K) \geq a(\alpha - \beta)$$
$$\Rightarrow q_1^t(K|m = K) \geq \frac{\beta}{\alpha} \qquad (22)$$

Note that as agent 1 must send a message, we can disregard the message costs since these are always incurred. Thus, if $w_{i1}^t(K|m_1^t = K) \geq n\beta/\alpha$ and $q_1^t(K|m = K) \geq \frac{\beta}{\alpha}$,[21] then the only absorbing

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[20]Mistakes and experiments in the communication stage cannot be important here because if $w_{i1}^t(h|m_1^t = h) < n\beta/\alpha$ for all $h > a$, then the minimum action does not change because of a mistaken/experimental message from agent 1.

[21]It is enough that these condition holds with equality for the same reason as in part (*iii*) of the proof of proposition 2.

state and the unique stochastically stable state is

$$s = ((K,K)_1, (\varnothing,K)_2, ..., (\varnothing,K)_n). \blacksquare$$

Organizational coordination and costly communication with boundedly rational agents

JENS DIETRICHSON | TORSTEN JOCHEM

How does costly communication affect organizational coordination? This paper develops a model of costly communication based on the weakest-link game and boundedly rational agents. Solving for the stochastically stable states, we find that communication increases the possibilities for efficient coordination compared to a setting where agents cannot communicate. But as agents face a trade-off between lowering the strategic uncertainty for the group and the costs of communication, the least efficient state is still the unique stochastically stable one for many parameter values. Simulations show that this is not just a long run phenomena, the stochastically stable state is the most frequent outcome also in the short run. Making communication mandatory induces efficient coordination, whereas letting a team leader handle communication increases efficiency when the leader expects others to follow and has enough credibility. The results are broadly consistent with recent experimental evidence of communication in weakest-link games.

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