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The Interrogative Model of Inquiry and Inquiry Learning

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Abstract Hakkarainen and Sintonen (2002) praise the descriptive adequacy of Hintikka’s *Interrogative Model of Inquiry* (IMI) to describe children’s practices in an inquiry-based learning context. They further propose to use the IMI as a starting point for developing new pedagogical methods and designing new didactic tools. We assess this proposal in the light of the formal results that in the IMI characterize interrogative learning strategies. We find that these results actually reveal a deep methodological issue for inquiry-based learning, namely that educators cannot guarantee that learners will successfully acquire a content, without limiting learner’s autonomy, and that a trade-off between success and autonomy is unavoidable. As a by-product of our argument, we obtain a logical characterization of serendipity.

1 Introduction

Epistemological models distinguish *contexts of discovery* from *contexts of justification*, and usually proceed from the assumption that inferences carried in the former cannot be rationalized. This is often expressed by saying that there can be no *logic of discovery*, only a *logic of justification*—where ‘logic’ is intended in a broad sense, including e.g. probability theory. However, some models of inquiry have explicitly tackled discovery of new facts, as part of problem-solving strategies. In particular, Hintikka’s *Interrogative Model of Inquiry* (IMI) presents a formal approach to discovery, and describes inquiry as a two-player game where one player, *Inquirer*, asks ‘small’ instrumental questions to the other player, *Nature*, in order to answer a ‘big’ research question. Within this framework, it may be shown that, for a wide variety of contexts, question-based discovery is grounded in deduction. Hintikka interprets the IMI as a vindication Sherlock Holmes’ method, in which deduction guides interrogation.

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For the above reasons, Hakkarainen and Sintonen (2002) argue that Hintikka's IMI offers an epistemological basis for *inquiry-based learning*. Proponents of inquiry-based learning consider that learners' progresses should not be assessed solely by evaluating whether they have acquired certain contents. Rather, evaluation should also take into account how well learners have developed analytical and experimental strategies to acquire new knowledge. The core assumption shared by proponents of inquiry-based learning is that children can actually develop autonomously highly sophisticated learning strategies, that mimic scientific method. If they are correct, designing learning environments that encourage children to develop such strategies would be more effective to prepare them to engage as adults in scientific inquiry, or at the very least in critical thinking, than e.g. relying on rote learning and normalized testing.

Hakkarainen and Sintonen present an empirical study that supports the descriptive adequacy of the IMI to inquiry contexts, and vindicates the core assumption of inquiry-based learning. Specifically, the claim that what the IMI characterizes as strategic reasoning actually occurs in an inquiry-based learning context involving elementary school children. Based on their observations, they suggest to mine the IMI for methodological principles, in order to develop new pedagogical practices and design didactic tools. However, and by their own admission, Hakkarainen and Sintonen rely on the conceptual apparatus of the IMI alone, and disregard the formal results which in the IMI characterize knowledge-seeking strategies. Whether this formal characterization actually supports Hakkarainen and Sintonen's methodological proposal thus remains an open question.

This paper addresses this question, and in so doing uncovers a deep methodological difficulty for inquiry-based learning. On the one hand, the study carried by Hakkarainen and Sintonen clearly shows that children engaged in cognitive processes that undoubtedly displayed the characteristics of discovery processes that the IMI describes formally as strategic and guided by deduction. On the other hand, in the light of the formal results of the IMI, these cognitive processes can also be shown to be such that they cannot be guaranteed to yield successful acquisition of a determined content. Moreover, the same formal results show that if inquiry-based learning is 'scaffolded' so as to guarantee successful learning of an intended content, the learner's autonomy is in fact virtually destroyed. The challenge for inquiry-based learning methodology, and the design of inquiry learning environments, is that a trade-off between success and autonomy is unavoidable.

We first introduce the formal model of the IMI (sec. 2), explain how it relates strategic questioning to deduction (sec. 3), and then turn to abduction in information-seeking strategies, and its relation to deduction (sec. 4). Each of these sections illustrates the model with one of Hintikka's favorite Sherlock Holmes example (the Case of Silver Blaze). Once the model is in place, we outline the study presented by Hakkarainen and Sintonen and discuss whether its conclusions are actually supported by the IMI (sec. 5). Finally, we conclude on

the Socratic method, and how it illustrates the difficulty to promote learners' autonomy through question-driven learning.

2 The Game of Inquiry

2.1 Learning and information-seeking as questioning

Early formulations of the IMI date back to the 1980s but we will consider the model (for this exposition) as a generalization of algorithmic learning-theoretic models that appeared in the 1990s, esp. the 'first-order paradigm' of Martin and Osherson (1998). Schematically, the latter model characterizes a *problem* as a pair $\langle T, Q \rangle$, where T is a background theory expressed in some (first-order) language \mathcal{L} ;¹ and Q is a (principal) *question*—usually, but not necessarily, a binary question—that partitions possible states of Nature compatible with T , denoted hereafter $\mathbf{S}(T)$. Nature chooses a state $s \in \mathbf{S}(T)$ and a *data stream* (an infinite sequence of basic sentences of \mathcal{L}) that in the limit fully characterizes the features of s expressible in \mathcal{L} ; then Nature reveals one datum at a time. A learning strategy is a function taking as arguments finite segments of the data stream, and returning either an answer in Q or '?' (suspension of judgment).

The model of Hintikka et al. (2002) generalizes the above by dropping some idealizing assumptions. Nature, instead of a complete data stream, chooses a set A_s of *available answers* in s , that can be expressed by sentences in \mathcal{L} of arbitrary complexity (and may then be analyzed by 'analytical' moves). A_s determines which properties and entities are resp. observable and identifiable. The data stream is built by Inquirer, using instrumental questions to supplement the information T gives her about s , and may therefore remain incomplete.² An *interrogative learning strategy* takes as argument a finite sequence of data, and outputs a (possibly empty) subset of 'small' questions (aimed at generating the extension of the data sequence) along with the current conjectured answer to Q (or suspension of judgment). Finally, Q may be a *why*- or *how*-question about some $q \in \mathcal{L}$, in which case Inquirer *assumes* that q_i holds, and aims at finding conditions which, together with T , entail q . The answer to such a *why*- or *how*-question 'compacts' the whole line of

¹ A first-order language \mathcal{L} can express statements about individuals, their properties and relations; combinations of such statements (with Boolean operators *not*, *and*, *or*, and *if...then...*); and their existential and universal generalizations (with quantifiers *there exists...* and *for all...* respectively). A *basic* sentence of \mathcal{L} contains only individual names and relations symbols, i.e. no Boolean operator other than (possibly) an initial negation, and no quantifier. In what follows, we implicitly restrict the meaning of 'deduction' to 'first-order deduction'—i.e. relations between premises and conclusions couched in some first-order language.

² Introducing A_s weakens the assumptions that: (a) data streams are always complete in the limit; (b) all predicates (names) of \mathcal{L} denote observable qualities (identifiable objects); and: (c) a datum needs no analysis. The IMI also drops the idealization that: (d) Nature always chooses s in $\mathbf{S}(T)$, and: (e) all answers in A_s are true in s . Cases where (d-e) hold define the special case of *Pure Discovery* (cf. 3.1).

inquiry whereby q is shown to be entailed by T together with *ad explanandum* conditions (Hintikka and Halonen, 1997; Hintikka, 2007, ch. 7) (also, § 4.2).

The success of Inquirer’s strategy depends in part on the set of questions she is ready to ask at a given point (which evolves throughout inquiry), and in part on A_s . Hintikka calls *range of attention* the set of *yes-no* questions Inquirer considers at any given time (Hintikka, 1986), but the role of this set of questions is left implicit in the results presented in (Hintikka et al., 2002). We will make it explicit, since it is critical to understand how the IMI bears upon learning practices of empirical agents.

Together with T , the answers to instrumental questions induce an *information bi-partition* over $\mathbf{S}(T)$: the first cell comprises scenarios compatible with the answers, and the other, those which are not. At the outset, the first cell is identical with $\mathbf{S}(T)$: all possible states compatible with T are *indiscernible* from each other, and s is assumed to be one of them. The partition is refined when new answers are accepted. Answers gradually ‘hack off’ scenarios incompatible with them. The assumptions that T and A_s are truthful may be revised in the course of inquiry, thereby reopening possibilities. Instrumental questions may also trigger ‘sub-inquiries’ (e.g. *why*- and *how*-questions, or questions with statistical answers requiring parameters estimation) about some problem $\langle T', Q' \rangle$ —where T' extends T with some of the answers already obtained from A_s when investigating $\langle T, Q \rangle$ —possibly suspending investigations of $\langle T, Q \rangle$ proper.

An inquiry about $\langle T, Q \rangle$ terminates when Inquirer is able to tell whether the first cell of the partition (compatible with the answers and T) is identical with some $q_i \in Q$, i.e. suffices to identify s ‘enough’ to answer Q . This may sometimes be impossible (e.g. for inductive problems) but one can then strengthen T with additional assumptions (including e.g. extrapolations for unobserved values). It is also sometimes possible to devise methods that rather than waiting for an answer to Q , emit an initial conjecture and adopt a policy for changing it later in face of new data.³ The model handles retraction of answers by ‘bracketing’ and excluding them from further information processing, which may re-open Q by preventing identification of s . Bracketing can also be extended to handle revisions of T (Genot, 2009). Reasoning probabilistically from answers known to be uncertain is discussed in (Hintikka, 1987). Bracketing and probabilistic reasoning will be discussed in more details in § 3.1.

³ An example is the *halting problem*, in which one must determine whether the current run of a program p , that may execute either finitely many instructions, or loop an instruction indefinitely, is finite or infinite. An ‘impatient’ method that conjectures that p is currently at the beginning of an infinite run, and repeats this conjecture indefinitely unless p stops (in which case the method changes its assessment) *solves* the problem in the above sense on every possible run. Kelly (2004) discusses in details the relation between the halting problem and empirical inductive problems.

2.2 The Sherlock Holmes sense of “deduction”

An example from Sherlock Holmes inquiry in *The Case of Silver Blaze* ideally illustrates the type of reasoning the IMI captures. In this short story, Holmes assists Inspector Gregory in the investigation of the theft of Silver Blaze (a race horse) and the murder of his trainer. The principal question is: *who stole Silver Blaze and killed his trainer?* During the night of the theft, a stable-boy was drugged and Silver Blaze’s trainer was killed. Gregory holds a suspect, Fitzroy Simpson, and has already settled the following (instrumental) questions: *Does Simpson have motive? Did Simpson have an opportunity to commit the theft and the murder? Does Simpson own a weapon that could have been the murder weapon?* and *Can Simpson be placed at the crime scene?* All these questions have received a positive answer. Simpson is indebted from betting on horses. He visited the stables the evening before the theft, stopped the maid carrying the food, and was eventually driven out by the stable-boy and a watchdog. He owns a weighted walking stick that could have been the murder weapon. And finally, a scarf has been found found near the victim’s body, that the stable-boy and the maid recognized as Simpson’s.

Gregory’s questioning strategy is a staple of crime fiction: it’s a by-the-books strategy, that may be applied to any case of homicide, as it uses questions applicable to almost every potential suspect, once some descriptions (“the murder weapon”, “the crime scene”) have been specified for the current investigation. This strategy keeps questioning simple, as there are no strategic dependencies between questions. It gives a basis for quasi-probabilistic inferences, as a high ‘yes’ count increases suspicion (culprits usually have one), and a high ‘no’ decreases it (innocents usually have one). Although the former count may result from a coincidence, the probability remains low as long as answers are statistically independent. A conclusion put forward as a consequence of applying this strategy is acceptable provided that scenarios where answers would not be independent—i.e. when either the high ‘yes’ or ‘no’ counts have hidden common causes—are ruled out. In such scenarios, the method is known to be unreliable, as it may conclude to the guilt of an innocent who has been framed, or to the innocence of a culprit who has carefully premeditated and executed his plot. But as long as hidden common cause are not suspected, the hypothesis of Simpson’s guilt is strengthened by Gregory’s reasoning.

Holmes describes the case as one where “[t]he difficulty is to detach the framework of fact—of absolute undeniable fact—from the embellishments of theorists and reporters” (Conan Doyle, 1986, p.522). Holmes’ own expectations are instrumental in his decision to investigate,⁴ but he does not favor any hypothesis, even for the sole purpose of testing it first. In particular, although Holmes conceded that Simpson’s guilt is the ‘natural’ hypothesis, he does not consider it as the first to be investigated. Instead, he proceeds trying to identify the thief, narrowing down the range of suspects without explicitly

⁴ Holmes confesses that “[he] could not believe it possible that the most remarkable horse in England could long remain concealed [and] expected to hear that he had been found, and that his abductor was the murderer” (Conan Doyle, 1986, p.522).

listing them, attempting instead to find discriminating properties, using *yes-no* questions. One of them is whether the dog kept in the stables has barked at the thief. Holmes does not ask the question explicitly, but obtains an answer from Gregory in the following dialogue:

“Is there any point to which you would wish to draw my attention?”
 “To the curious incident of the dog in the night-time.” “The dog did nothing in the night-time.” “That was the curious incident,” remarked Sherlock Holmes.” (Conan Doyle, 1986, p.540)

The definite description “*the* curious incident” can refer to either of two circumstances: the watchdog’s barking at the thief, or failing to do so. However, only one circumstance is compatible with Simpson’s guilt, since the watchdog kept in the stables is the very dog that the stable-boy set after Simpson in the preceding evening. Eventually, Holmes sums up the conclusions he drew learning that the dog had barked against Simpson in the evening, and remained silent during the night:

I had grasped the significance of the silence of the dog, for one true inference invariably suggests others. The Simpson incident had shown me that a dog was kept in the stables, and yet, though someone had been in and had fetched out a horse, he had not barked enough to arouse the two lads in the loft. Obviously the midnight visitor was someone whom the dog knew well. (Conan Doyle, 1986, p.540)

Holmes’ instrumental question may seem irrelevant to those who do not anticipate his reasoning, and Holmes’ reputation plays a role in their judgment: the horse’s owner does not consider the incident significant, but Gregory and Watson do, knowing that Holmes seldom attends to insignificant facts. Holmes trusts his assumptions and reports about the facts, and conservatively so: the ‘yes’ count vs. Simpson could make one doubt that the dog is a good watchdog, but Holmes never contemplates ‘bracketing’ this assumption. Finally, Holmes’ conclusion reduces the set of potential suspects by ruling out Simpson without tracking probabilities, because there is no range of suspects over which distribute them.

3 Deduction in inquiry

3.1 Pure Discovery

The inquiry game described in §2.1 is with asymmetric information: Inquirer does not know whether her assumptions are correct (formally, if $s \in \mathbf{S}(T)$), nor which answers are available (in A_s), and whether those available are reliable. Nevertheless, as illustrated by Sherlock Holmes’ method in *The Case of Silver Blaze*, one can take evidence at face value as long as possible, then follow a line of deductions, possibly taking educated guesses, rather than considering from the outset several cases in parallel. Indeed, Holmes usually reconsiders his

grounds for accepting evidence, or relying on background assumptions, only in the face of contradictory evidence. His method amounts to address (at least initially) any problem-solving situation as what Hintikka calls a problem of *Pure Discovery* (PD):

“[Pure discovery] means a type of inquiry in which all answers are known to be true, or at least can be treated as being true. If so, all we need to do is to find out what the truth is; we do not have to worry about justifying what we find.” (Hintikka, 2007, p. 98).

A distinctive feature of the IMI is to take Pure Discovery (hereafter PD) as the ‘default’ mode of inquiry: PD reasoning is maintained as long as it is possible to do so, uncertain answers are disregarded whenever possible, and attempts are made to reach conclusions that involve only whatever premises and answers one can ‘treat as being true’. However, a given context can turn out not to be a PD-context in a variety of ways, and Inquirer’s strategy must be modified accordingly. Contradiction may arise between expectations based on deductions from T , and answers to some questions. Or multiple sources may give incompatible answers to the same questions. Or some action undertaken on the basis of conclusions arrived at earlier stages of inquiry, may fail to produce the expected result. Even if none of the above occurs, it may prove impossible to deduce a unique answer to the principal question Q from T and answers to instrumental questions. One then may have to settle for partial answers, weighted by the amount of justification available for them.

The IMI handles contexts in which conflicts occur mainly through *defeasible* reasoning. When premises in T turn out to be unsafe, i.e. when grounds for justification cannot be ignored anymore, or when answers are deemed uncertain, they can be ‘bracketed’ (Hintikka et al., 2002; Genot, 2009) so that no further inquiry step depends on them: no consequence is inferred from them, and no further question is asked using them as presuppositions. ‘Bracketing’ allows for the circumscription of a ‘safe’ PD subcontext, and for maintaining PD behavior for as long as possible, and is reversible: premises and answers can be reinstated in the light of further evidence. Moreover, if one’s current evidence proves insufficient, and no complete answer can be obtained, the IMI accommodates probabilistic reasoning, as a ‘logic of justification’ (Hintikka, 1987, 1992). Subsequently, the IMI addresses the issues arising in PD-contexts, before considering any other type of context.

Mismatch between Inquirer’s range of attention and the contextually available answers (represented by A_s) is the prime issue of interrogative inquiry. Mismatch occurs when either Inquirer asks a question which has no answer in A_s , or fails to ask a relevant question whose answer is in A_s —as with Gregory, failing to ‘ask’ about the dog. A related issue is the strategic problem of choosing the next best ‘small’ question given one’s current information (T and past answers). Mismatch is an issue in PD and non-PD contexts alike, as it encompasses the use of ‘control’ questions which can be answered given one’s current information. How Inquirer addresses these problems depends on how she manages her range of attention.

3.2 Building blocks of interrogative strategies

The IMI counts *inferential* and *interrogative* moves on a par with each other. Hintikka calls *presupposition* of a question the statement that opens a question, i.e. makes possible to ask it meaningfully. Specifically, disjunctions make *whether*-questions possible, by determining a range of alternatives, and existential statements make possible to ask *what*-, *where*-, *which*- and *who*-questions. Subsequently, the fundamental ‘rule’ of the game of inquiry is that a question can be asked as soon as its presupposition has been inferred (making it available for an interrogative move). With our extended notion of range of attention, the rule can be rephrased as: *a question enters the Inquirer’s range of attention when its presupposition is obtained by an inferential move*. This ‘rule’ is critical in how the IMI captures the dynamics of discovery of new facts through ‘small’ questions, as a goal-directed process.

Questioning strategies supervene on one’s current information (T and the answers accepted so far), which is mined out for open questions. Inquirer is *not* assumed to be aware of all the consequences of her current information. For example, Inquirer’s information partition may exclude that “neither A nor B ” holds in the state of Nature s , which in turn entails that “either A or B ” holds in s . But the question whether A or B holds will not enter Inquirer’s range of attention before she has established that T entails that “ A or B ” holds. Once Inquirer performs the inference, she may choose to raise the question “Which of A or B holds?”—or a sequence of *yes-no* questions about A and B —and use it to refine her information partition. If no answer is obtained, she may need to *reason by cases*, or mine T (and past answers) to find equivalences between A and B on the one hand, and some A' and B' on the other, so as to reformulate her questions. The same holds *mutatis mutandis* for statements like “There is an x s.t. $\phi(x)$ ”—where $\phi(\cdot)$ stands for some description which qualifies x . Such statements open *wh*-questions about the x (object, person, location, etc.) satisfying the description. If one fails to obtain an answer, one can introduce some ‘arbitrary’ name α standing for the (so far unknown) object satisfying the description, avoiding any other assumption about α other than $\phi(\alpha)$, until (possibly) a referent for α is identified. Again, it may be possible to mine T to obtain a description $\psi(\cdot)$ such that T (possibly together with past answers) entails that “If x is s.t. $\psi(x)$, then it is s.t. $\phi(x)$ ” and ask the question about $\psi(\cdot)$ instead.

In the case of Silver Blaze, Gregory’s strategy is based on a deduction from his background knowledge, providing him with a testable reformulation of the question *Is Simpson guilty?* The derivation could be obtained from a general truth formulated as follows: *if x is a murderer, then x has a motive, had the opportunity, owns a murder weapon, and was present at the crime scene*. Gregory’s strategy can be viewed as an application of *modus ponens*, that assumes the natural hypothesis (Simpson is the culprit) and derives observable consequences. It thus uses ‘small’ *yes-no* questions that test properties occurring in the consequent of the general truth. The strategy only warrants a *partial* ans-

wer, because the general truth is not an equivalence, as someone may satisfy the description in the consequent without being a murderer.

Holmes formulates a different question (*Who stole Silver Blaze and killed his trainer?*) and the way he arrives at the instrumental question that specifies it, and the conclusion(s) he draws from the answer, all proceed from deductive inferences. But Holmes' 'small' question has the form *Is it the case that A or not?*, where *A* is: "the dog barked at the thief", and the possibility to ask it depends on the language he uses alone (irrespective of the current information state). Generally, for some language \mathcal{L} , any grammatically correct statement *A* or description $\phi(a)$ built with the vocabulary of \mathcal{L} (where *a* is a proper name or an indexical like 'this' or 'that') can in principle be built into a *yes-no* question without the need of further inference from one's current information.⁵ How the question whether the dog barked entered Holmes's range of attention, but not Gregory's, is trivially deductive: the deduction that *A or not A* from *any* background theory *T* would be valid, and thus its being deductive does not suffice to explain how it was arrived at.

Two other aspects of the case of Silver Blaze will be significant for our comparison with empirical data. The first is that Holmes' initial principal question is based on a *false presupposition*—namely, that there is a single individual who stole Silver Blaze and murdered his trainer. Holmes' instrumental questions are selected in order to help answer that question. The information Holmes obtains after asking about the dog eventually leads him to revise this presupposition. Narrowing down the range of suspects that satisfy the condition of "not being barked at by the watchdog", Holmes comes to suspect, and later establish, that the answers to the questions *Who stole Silver Blaze?* and *Who killed Silver Blaze's trainer* cannot be the same. Interestingly, because the deduction of the presupposition of the *instrumental* question about the dog is trivial, it is *independent* of Holmes initially incorrect assumption that the thief and the murderer are the same individual, which served as presupposition for his *principal* question.

The second significant aspect is that Holmes' mention of "the curious incident with the dog in the night-time" can be viewed as an implicit suggestion to Gregory—to consider a control question (*Did the dog bark at the thief?*) and revise his conclusions in the light of its answers. At the time Holmes asks about the dog, Gregory has accepted the positive answer to the question *Is Simpson guilty or not?* But pending some assumptions about watchdogs shared by Gregory and Holmes, assuming that Simpson is guilty leads to expect that the dog *has* barked in the night (because the dog barked at Simpson earlier in the evening). This expectation is incompatible to what is known to have happened, and thus constitutes a *reductio* of the assumption that Simpson is guilty. Gregory needs then to revise either the background assumptions that the watchdog is well-trained, and barks at unauthorized visitors even after a first encounter; or the assumption that coincidences that would make Simpson

⁵ If *A* or $\phi(a)$ include vague terms (or imprecise categories), disambiguation is needed to obtain an answer, but *sequence* of *yes-no* questions (further specifying a 'prototype' in the current context) will suffice.

appear guilty can be neglected. In the latter case, new questions may enter Gregory’s range of attention; such as: *How did Simpson’s scarf ended at the crime scene?*—a question that, incidentally, Holmes later raises and answers, considering as relevant the fact that it is a scarf, and not that it is Simpson’s.

4 Deduction Abducted

4.1 Strategic reasoning in interrogation

The previous section has presented informally the relation between selection of questions, and deduction. Let us now consider how the IMI formally characterizes this relation. Hintikka describes the strategic problem of interrogative inquiry (neglecting the distinction between statements and propositions they express) in the following terms:

Strategic knowledge will in interrogative inquiry ultimately come down to a method answering questions of the following form: Given the list of the propositions one has reached in a line of inquiry, which question should one ask next? In view of the need of presuppositions, this amounts to asking: Which proposition should one use as the presupposition of the next question? (Hintikka, 2007, p.98).

Hintikka et al. (2002) present three formal results that can be combined to answer this question: the *Deduction Theorem*, the *Yes-No Theorem*, and the *Strategy Theorem*. The *Deduction Theorem* simply states that if a statement expressing an answer $q_i \in Q$ can be established *interrogatively* to hold in s assuming T , then that statement can be established to hold *deductively* (without using questions) from T and a finite subset A'_s of A_s . Equivalently: answers act as additional premises, and interrogative reasoning reduces to *deduction from T strengthened by a finite set of answers*.⁶ The *Deduction Theorem* is in fact rather trivial. It is an immediate consequence of the definition of what problems and solutions are, in learning-theoretic models. Nevertheless, it implicitly refines this definition, as solving problems requires to raise the ‘small’ questions, whose answers will be instrumental to solve a problem. The role of instrumental question generating the data stream is usually left implicit in learning-theoretic models. Notice that the Deduction Theorem shows how one can come to accept some answer q_i that *does not in fact hold* in s , i.e. if s is *not* in $\mathbf{S}(T)$, when e.g. $T \cup A'_s$ is consistent.

Perhaps more surprising, the *Yes-No Theorem* is no less straightforward. It states that a statement expressing an answer q_i can be established interrogatively from T and A_s iff that statement can be established interrogatively from T and A_s *using yes-no questions only*. The Yes-No Theorem is best understood

⁶ Because of the possibility of mismatch, the converse of the Deduction Theorem only holds on the condition that elements of A_s needed to obtain (interrogatively) q_i from T are answers to questions in Inquirer’s range of attention.

as stating that every interrogative argument can be *reconstructed* with *yes-no* questions alone.⁷ But the *Yes-No Theorem* also implies that it is possible in principle to solve interrogatively a problem $\langle T, Q \rangle$ without any *non-trivial* deduction from T . Trivial deductions from T are also necessary to formulate *control* questions: if one ‘goes along’ with expectations based on T , and takes its consequences at face value, one will not test for potential contradictions (assuming that T itself is consistent). Hence, contradictions between T and facts can only be revealed using *yes-no* questions trivially deduced from T .

Finally, the *Strategy Theorem* rests on an observation about *deductive* proofs. Obtaining the shortest proof for a conclusion c from a set of premises P (when c actually follows from P) requires to: (a) examine the least number of cases; and: (b) introduce the smallest number of (arbitrary) names. Proof rules that open cases and introduce names in deductive reasoning, are *the same as* inferential rules that open questions in interrogative reasoning. Taking $P = T$ and c to be a statement expressing some q_i in Q , answers in A_s eliminate cases, and dispense from introducing arbitrary individuals. Given the Deduction Theorem, this means that, when q_i can be interrogatively established given T and A_s , the shortest *interrogative* derivation is identical with the shortest *deductive* derivation of a statement expressing q_i from T and a finite subset A'_s of A_s . More informally: the best selection of questions, which depends on the best strategy for obtaining presuppositions in T , mirrors the best strategy to select premises from $T \cup A_s$. Therefore, anticipations about deductions from a strengthened theory T can guide the selection of questions whose answers could actually strengthen T . In a slogan: *deductive skills carry over to interrogative skills*.

As long as some conclusion c and some set of premises P are formulated in a first-order language \mathcal{L} , there is always a finite proof that c follows deductively from P ; when it does. However, first-order consequence is not fully decidable: there may not be a finite proof that c does *not* follow from P , when it does not. Subsequently, the Strategy Theorem entails that there cannot be any general mechanical (algorithmic) method for solving interrogative problems by: (1) trying first to deduce some a statement that expresses some $q_i \in Q$ from T ; (2) use questions to strengthen T with A_s if step (1) is not successful; and: (3) if step (2) is also unsuccessful, reiterate (1) and, if necessary, (2) with some potential answer $q_j \neq q_i$ in Q . However, it *does* entail that having some idea about *which cases compatible with T would have to be ruled out* to deduce a statement that expresses some $q_i \in Q$ from a strengthened version of T , gives a good idea of *which question one should ask* to establish interrogatively q_i from T (if answers were obtained).

⁷ This understanding eschews the issue of possible mismatch between A_s and Inquirer’s range of attention. In the left-to-right direction, every *whether*-question about A or B , or *wh*-question about $\phi(\cdot)$, that receives (say) answer A or $\phi(a)$ suffices for the *yes-no* questions about A or $\phi(a)$ to enter Inquirer’s range of attention for the purpose of reconstructing an argument. The antecedent of the right-to-left direction holds when the *yes-no* questions are already in the range of attention (the consequent is satisfied trivially).

4.2 Abduction and *yes-no* questions

Hintikka has suggested that the Strategy Theorem offers important insights about *abduction* (Hintikka, 1988, 2007, ch. 2), esp. in contrast with inference to the best explanation (IBE). The latter occurs when Inquirer accepts (defeasibly) one of the answers, without having established it interrogatively. Such a reasoning can be rationalized, e.g. by assuming a probability distribution over the refined partition; and an acceptance rule that fires if probabilities are raised (conditional on past answers) over a fixed threshold. Gregory’s strategy is an illustration of IBE so construed, where the acceptance rule ‘fires’ because the answers are independent, and the probability of a coincidence is low. If the probabilistic constraints are precise enough, the outcome of IBE can be uniquely determined, but involves (probabilistic) justification, and is definitely non-PD.

By contrast *abduction*, as Hintikka’s understands it, routinely occurs in PD contexts (or contexts that Inquirer still assumes to be PD). Abduction occurs when Inquirer anticipates a (possible) deductive derivation from some strengthened version of T , and attempts to steer the course of the investigation towards obtaining the answers that strengthen T in the desired fashion. Abduction thus depends on the ‘deductive insight’ that answers to some instrumental questions will strengthen T enough to reduce the admissible states to those in which some $q_i \in Q$ holds.

Unfortunately, abduction involving *yes-no* questions cannot always be fully rationalized. In particular, *yes-no* questions that do not ‘break down’ questions whose presuppositions are inferred from T and previous answers, involve what looks like ‘intuitive leaps’. The difficulty also affects probabilistic IBE: a relevant partition of cases, over which probabilities are distributed, depends on T , but on occasion must be imposed by ‘abductive’ *yes-no* questions (Genot and Jacot, 2012). With respect to our reconstruction of Gregory’s reasoning, asking about the dog would be as ‘abductive’ for the purpose IBE, as it is for reasoning deductively. Furthermore, introducing *yes-no* questions is, as we said, the only way to reveal inconsistencies, and can now be seen to be ‘abductive’ as well.

Let us illustrate that last point with the example of Holmes and Gregory. Either one accepts that Simpson is guilty, as Gregory does, or one does not, as Holmes does. Acceptance of an answer by a given inquirer i , given a set T_i of background assumptions for that inquirer, eliminates all the scenarios in $S(T_i)$ that are incompatible with that answer. If T_{Gregory} includes only scenarios where the watchdog is well-trained, then the answer to Holmes’ question about the dog rules out all the scenarios in which Simpson is guilty. Gregory could in principle bracket the assumption that the dog is well-trained, maintain acceptance of Simpson’s guilt, and reshuffle probabilities. The same holds *mutatis mutandis* about T_{Holmes} , although Holmes has not accepted Gregory’s guilt in the first place, and disregards (non-extreme) probabilities.

Whether one reasons deductively or probabilistically, *raising a yes-no* question about the dog depends on the insights into whatever effect the possible

answers to that questions would have—which scenario they would eliminate, or how they would affect the probabilities of certain events. We said earlier that Holmes’ question could be viewed as an implicit suggestion that Gregory should revise his current assumptions, and we can now be more precise: pointing out that the dog did not bark implicitly questions the grounds for the step at which Gregory ‘jumped’ from a high probability of Simpson’s guilt, to full acceptance. It shows that Simpson’s guilt may not be the best explanation of what has happened, after all. Holmes’ actually suggests to Gregory another, different reasoning line, by (subtly) manipulating Gregory’s range of attention.

Hintikka has explicitly reconstructed Holmes’ reasoning in *The Case of Silver Blaze* as an answer to a *why*-question (Hintikka, 2007, ch.7, §2). As we mentioned in § 3, answering a *why*-question compacts a line of inquiry. In that case, the statement that “a dog was kept in the stables, and yet [...] had not barked enough to arouse the two lads in the loft” answers the question: *Why is the thief the dog’s master?* Hintikka shows that the above statement can be extracted from the proof that the thief is the watchdog’s master—the only person that “the dog knew well” that could have visited the stables. In Hintikka’s reconstruction, this statement is an *interpolation formula*, i.e. a formula that follows from the premises, entails the conclusion, and comprises only vocabulary common to them. Hintikka’s reconstruction furthermore uses an extremely parsimonious first-order language, with two properties, one relation, and two names. Let us consider an informal equivalent of this reconstruction. The information (a) that no dog barked at the thief and (b) that there was a watchdog, provide *ad explanandum* conditions, alongside the general truth that watchdogs bark at strangers, but not at their masters. Once the stable-boys are ruled out—one had been drugged, and the other two were asleep in the loft—the only individual fitting the description ‘master to any watchdog kept in the stables’ is Silver Blaze’s trainer. Once Holmes has reached this conclusion, the principal question also changes. Learning about the dog incident makes Holmes ‘bracket’ his own expectations that the thief is an assassin (cf. n. 4).

The crux of Holmes’ interrogative reasoning is how he picks premises (a) and (b). Since (a) is vacuously true (and uninformative) if no dog is indeed kept in the stables, one needs (b) to draw a useful conclusion. Holmes explains that the incident with the dog barking at Simpson in the evening implied (b), and that then learning (a) implied, together with the general truth that watchdogs bark at strangers, that the thief was not a stranger. And while the Strategy Theorem allows to reconstruct Holmes’ line of reasoning, it does so vacuously, because it depends on a *yes-no* question that enters Holmes’ range of attention (but not in Gregory’s) without being inferred from his background information. Actually Holmes’ picking premise (a) and anticipating its effect also depends on anticipating the answer to that question. Although (a) is part of the common ground that Holmes, Gregory and Watson share, its usefulness (as constraint on the information partition) is only revealed after (b) is learned. The same goes for the ‘general truth’ that watchdogs abstain from barking at their masters alone.

5 Abduction and collaborative learning

5.1 The CSILE study and its conclusions

The study reported by Hakkarainen and Sintonen (hereafter HS) in (HS, 2002) followed elementary school pupils, who had to complete science projects presented as broad questions. Examples of such questions given by HS are: “how to explain gravity?”, “how did the universe begin, and how will it evolve?” and “how do cells and the circulatory system in the human body work?”. In order to foster collaboration, but also to gather process data, the pupils were tutored in the use of the CSILE software environment, that lets users register notes in a database, with either informative or interrogative content. Once registered, a note is accessible to all other users, even if it is addressed by one user to another specifically. Informative content is revisable, and constitutes a knowledge base for the group. Interrogative content is registered in notes labeled either as “Problem” or “I Need To Understand” (HS, 2002, p. 32).

How-questions are similar to *why*-questions, and presuppose that what has to be explained is indeed the case. But children had first to make sense of the presuppositions of the *how*-questions submitted to them. Specifically, they had to recover definitions for terms such as “gravity” and “cells”, identify what the definite description “the circulatory system” refers to, theories articulating those definitions, as well as a theory entailing that the universe has an evolution. Since the CSILE knowledge base was initially empty, the pupils’ first entries were of interrogative content, aimed at breaking down the presuppositions of the broad *how*-questions into manageable ‘small’ questions, without a definite idea of the meaning of those presuppositions. And according to HS, one distinctive advantage of the IMI over other epistemological models, is its ability to capture the dynamics engendered by such circumstances:

[In] actual problem-solving situations, an agent has to start generating questions and theories before all necessary information is available. In the interrogative process, initially very general, unspecified and “fuzzy” questions are transformed to a series of more specific questions. As a consequence, the process of inquiry often has to start with a ‘theory to work with’ that is transformed into a more sophisticated one as the process goes on. [...] The dynamic nature of inquiry is, further, based on the fact that new questions emerge in the process of inquiry that could not be anticipated when the principal question was first raised. (Hakkarainen and Sintonen, 2002, p.39)

The methodology of HS’s study tracks the co-evolution of theories and questioning strategies, from the data collected within the CSILE environment. First, children’s questions, entered in the “Problem” and “I Need To Understand” categories of the CSILE database, were classified as *principal* and *instrumental*. Principal questions included the initial questions of the science project, and questions that triggered subordinated lines of inquiry. Second,

the evolution of the knowledge produced, i.e. the proposed answers to the initial questions—entered through notes category in the CSILE database—was correlated to the formulation of questions (both principal and instrumental). Third, a “*deepening of explanation scale*” was defined, which assigned scores to students based on whether “in-depth advancement in a student’s search for explanatory scientific information” (HS, 2002, p.33) was observed. Finally, the scores were validated by appeal to experts, namely “three internationally regarded philosophers of science from well-known Canadian and Finnish universities” (HS, 2002, p.34). The aim of this evaluation was to determine whether children had moved from “initial intuitive theories [to] a new conceptual understanding” (HS, 2002, p. 38) mirroring the scientific theories describing the phenomena they had to explain. Individual reasoning strategies were not explicitly studied, but the CSILE environment allowed to track how children monitored each others’ questions. HS express the general conclusion of their study as follows:

The study indicated that CSILE students participated in extended processes of question-driven inquiry and systematically generated their own intuitive theories. The epistemic value of CSILE students’ knowledge-seeking inquiry seems partially to be based on a process in which social communication pushed a student to pursue question-driven inquiry further than he or she might originally have been able to go. [CSILE] appeared to foster engagement in higher-level practices of inquiry [and] epistemological awareness concerning the process of inquiry. (HS, 2002, p.38–39)

Based on these conclusions, HS take the IMI to be empirically validated, as descriptively adequate. Furthermore, they express the view that the IMI should be considered a methodological basis by educators who often insist on the importance of encouraging questioning, and mined for suggestions on how to develop pedagogical models and didactic tools:

It appears to us that what is new about the interrogative approach is to emphasize question-transformation as the very foundation of scientific inquiry. [...] [W]e do not have well-developed culture of question asking at school and it is very difficult to get students to follow the questions that emerge through their process of inquiry. In this regard, pedagogical models and computer tools elaborated by relying on the interrogative approach appear to be very valuable. (HS, 2002, p. 41)

HS do not give further suggestions as to what kind of features those tools should include, that could facilitate question-transformation, and the evolution of theory-formation. This lack of specific suggestion is actually not surprising, for the IMI cannot actually back any general recommendation.

5.2 Range of Attention and Serendipity

The conclusions that HS draw with respect to inquiry learning in general, actually generalize the experts' opinion about the outcome of the CSILE study. HS summarize the outcome of the expert evaluation as follows:

According to the experts overall evaluation, CSILE students research questions were at a high level of sophistication, and, if successfully answered, were likely to produce new conceptual understanding. Moreover, two out of the three experts noticed the student-generated research questions formed a pattern, which allowed the students to answer their main research questions by generating a series of more specific questions. Although the third expert agreed with the other experts that many of the CSILE students research questions were valuable, he criticized some of the questions as being based on wrong presuppositions. (HS, 2002, p. 39)

Our analysis of the case of Silver Blaze should make conspicuous that criticism of questions based on "wrong presuppositions" can be mitigated by adopting a means-end analysis of the process of discovery and problem-solving. Indeed, we have seen that Holmes' instrumental questions, in the case of Silver Blaze, were based on the wrong presupposition that some unique individual was both a thief and a murderer. But they nonetheless lead to a reformulation of the principal question and a revision of the initial assumptions. And once the problem redefined, answers to these questions crucially contributed to solve it. Similarly, the "pattern" that allowed children to answer the main research questions crucially included re-formulations of questions, that promoted better understanding. Interestingly, HS insist on the fact that such reformulations often occurred in the CSILE study under the influence of others:

Many comments by others were apparently intended to show that a student did not genuinely focus on his or her principal research question but wandered unproductively around peripheral areas of the topic. Through social interaction pointing out inadequate presuppositions, these students were guided to focus on more productive research questions, for example: "I think that you should describe and tell more in your theory about how the UNIVERSE will change in the future, and less about how the people will change in the future and how they will know more about the universe in the future because that is not really the question you are researching" (Hakkarainen and Sintonen, 2002, pp. 38-39)

HS view such circumstances as an aspect that the IMI is conceptually better equipped to capture than other models. They are right, insofar as the above example, and similar observed cases, introduce explicit suggestions to alter the course of inquiry by formulating different instrumental questions. The effect of such interventions is similar to the intended effect of Holmes' mention of "the curious incident. . ." of which we have indeed shown that the

IMI captures the strategic import. However, the descriptive adequacy of the IMI does not warrant the methodological conclusion that “pedagogical models and computer tools elaborated by relying on the interrogative approach appear to be very valuable”. More accurately, the conclusion is unwarranted if “valuable” insights pertain to the ability of a group of inquirers to auto-regulate the course of inquiry. Let us see why.

The trigger of Holmes’ line of reasoning can be characterized descriptively as an instance of *serendipity*, defined as “observing an unanticipated, anomalous, and strategic datum which becomes the occasion for developing a new or extending an existing theory” (Barber and Merton, 2004, p.260). The concept of serendipity has been introduced in the sociology of science in order to overcome the descriptive limitations of epistemological models which leave out discovery. Interestingly, the IMI is able to characterize *loci* of serendipity, and even to further qualify inferentially their “strategic” nature, which is generally taken to be self-evident, and is left largely unexplained, in sociological discussions. In our example, Gregory is not aware of the ‘datum’ that the dog did not bark, while Holmes is. However, there does not seem to be any discernible reason explaining why Holmes becomes aware of that datum by himself, or explaining why Gregory needs Holmes. Holmes suggests at one point that the issue might be Gregory’s lack of imagination,⁸ which can hardly be fixed by some systematic method. Holmes’ own reasoning strategy can be vindicated on purely deductive grounds, and his ‘abducted’ deduction can be analyzed with the formal model of the IMI. But the model also supports the view that Holmes’ discovery is not the outcome of some systematic method. Introducing *yes-no* questions is indeed informed by one’s deductive skills, yet these skills cannot be mechanized in general.

The latter has some important consequences for the conclusions drawn by HS. The general case under which fall the problems they studied is the case of a problem $\langle T, Q \rangle$ where T is empty, Q receives a formulation in a language whose interpretation is not yet fixed for the Inquirer, and A_s is such that Q may actually be solved —i.e. one of the answers to Q is derivable from some subset A'_s of A_s . Because T is empty, the only presuppositions that can be derived from it are presuppositions of *yes-no* questions. $\langle T, Q \rangle$ will be solved when some subset A'_s of A_s is obtained, such that one of the answers to Q can be derived from that subset together with T and that answer is actually derived. In such cases, how T is strengthened depends exclusively on the range of attention of the Inquirer, which is in turn determined by the associations that the Inquirer will make on the basis of the linguistic formulations of Q —in Holmes’ words, the Inquirer’s imagination.

In a multi-agent setting, the interaction between inquirers induces a dynamics in the inquirers’ ranges of attention that is absent from the single-agent case. And indeed this is why the likes of Gregory and Lestrade are willing to

⁸ “Inspector Gregory, to whom the case has been committed, is an extremely competent officer. Were he but gifted with imagination he might rise to great heights in his profession. On his arrival he promptly found and arrested the man upon whom suspicion naturally rested.” (Conan Doyle, 1986, p. 527)

consult Sherlock Holmes, as his presence more often than not results in corrections in the course of inquiries in which they stall. However, transitioning from single-agent inquiry to multi-agent inquiry incurs no guarantee that pooling the ranges of attention of all the inquirers will result in an auto-regulated ‘collective’ range of attention sufficient to recover the set A'_s from which one then could derive an answer to Q in a solvable problem $\langle T, Q \rangle$ with empty T . To put it differently, increasing the number of inquirers does not mechanically increase the odds that serendipitous *yes-no* questions will enter the range of attention of inquiry learners reasoning from initially insufficiently specified theories. The IMI offers no formal vindication of the pre-theoretic intuition that collaborative inquiry improves upon solitary inquiry for the purpose of raising appropriate questions.

6 Conclusion

The formal results of the IMI can certainly help to understand *post hoc* the children’s inferences in the CSILE study, but the Strategy Theorem cannot rationalize (in general) the occurrence of *yes-no* questions that one needs to complete an initially empty background knowledge. The methodological import of the IMI is not noticeably better than the methodological import of less dynamic epistemological models. The IMI certainly supports the conclusion that “science educators [should] focus more on engaging students in sustained processes of question-driven inquiry than just examining contents of their current beliefs so as to facilitate their conceptual advancement” (Hakkarainen and Sintonen, 2002, p. 41). However, it offers little guidance regarding the design of collaborative-learning environments that would promote and nurture the development of successful interrogative learning strategies and successful interrogative problem-solving strategies.

The IMI does however warrant the following conclusion: a guaranty that an inquiry learner will be able to solve interrogatively a problem can always be obtained by manipulating the learner’s range of attention. But manipulation of the learners’ range of attention is tantamount to the transmission of strategic knowledge. A paradigmatic example of this manipulation, is how Socrates teaches Meno’s slave all the geometry the illiterate boy needs to demonstrate that the diagonal of the square is incommensurable to its side (Hintikka, 2007, ch.4, §8). Socrates uses only *yes-no* questions when doing so, and nonetheless manages to convey the required knowledge in geometry. Each time the slave is probing the consequences of an erroneous guess, he could well be said to be progressing in the demonstration. But his progress can only converge to the correct solution if monitored by Socrates. Socrates’ own range of attention builds upon deductive skills, namely his ability to find the best derivation of the solution to the problem at hand. The transmission of Socrates’ strategic knowledge suffices for the slave to complete the demonstration, because the derivation is constructive, and Socrates transmits the skills required to perform the construction.

Thus, the IMI makes conspicuous the conundrum that the inquiry learning methodology has faced since Socrates. On the one hand, increased guidance—transmission of strategic knowledge—can help learners reach solutions, but incurs the risk that they will wait for tutors to formulate questions. On the other hand, lack of guidance favors autonomy, but incurs the risk of unproductive research, that proceeds from poorly grounded questions. *Pace* Hakkarainen and Sintonen, the conceptual apparatus of the IMI cannot warrant more substantial methodological conclusions. However, perhaps more surprisingly, the IMI can offer some insights as to why one cannot in general leverage epistemological models to obtain general pedagogical and didactic principles applicable to inquiry learning. These insights are made possible by a deeper conceptual understanding into the nature and strategic role of serendipity. Indeed, it follows from the Strategy Theorem and the Deduction Theorem that, short of pre-existing knowledge, interrogative problem-solving requires not only deductive skills, but also ingenuity and good luck.

References

- Barber, E. and Merton, R. K. (2004). *The Travels and Adventures of Serendipity*. Princeton University Press.
- Conan Doyle, A. (1986). *Sherlock Holmes: The Complete Novel and Stories*, volume 1. Bantam Books.
- Genot, E. J. (2009). The Game of Inquiry. The Interrogative Approach to Inquiry and Belief Revision Theory. *Synthese*, 171:271–289.
- Genot, E. J. and Jacot, J. (2012). How can yes-or-no questions be informative before they are answered? *Episteme*, 9(2).
- Hakkarainen, K. and Sintonen, M. (2002). The Interrogative Model of Inquiry and Computer-Supported Collaborative Learning. *Science & Education*, 11(1):25–43.
- Hintikka, J. (1986). Reasoning About Knowledge in Philosophy: the Paradigm of Epistemic Logic. In *TARK '86: Proceedings of the 1986 conference on Theoretical aspects of reasoning about knowledge*, pages 63–80. Morgan Kaufmann.
- Hintikka, J. (1987). The Interrogative Approach to Inquiry and Probabilistic Inference. *Erkenntnis*, 26:3:429–442.
- Hintikka, J. (1988). What is Abduction? The Fundamental Problem of Contemporary Epistemology. *Transactions of the Charles S. Peirce Society*, 34:503–533.
- Hintikka, J. (1992). The Concept of Induction in the Light of the Interrogative Approach to Inquiry. In Earman, J., editor, *Inference, Explanation, and Other Philosophical Frustrations*, pages 23–43. University of California Press.
- Hintikka, J. (2007). *Socratic Epistemology*. Cambridge University Press.
- Hintikka, J. and Halonen, I. (1997). Semantics and Pragmatics for Why-Questions. *The Journal of Philosophy*, 92:636–657.

- Hintikka, J., Halonen, I., and Mutanen, A. (2002). Interrogative logic as a general theory of reasoning. In Gabbay, Johnson, Ohlbach, and Woods, editors, *Handbook of the Logic of Argument and Inference*, volume 1, pages 295–337. Elsevier.
- Kelly, K. T. (2004). Uncomputability: the problem of induction internalized. *Theoretical Computer Science*, 317(1):227–249.
- Martin, E. and Osherson, D. (1998). *Elements of Scientific Inquiry*. The MIT Press.