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# Pricing and timing of consolidated deliveries in presence of an express alternative – financial and environmental analysis

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## **Abstract**

Shipment consolidation has been advocated by researchers and politicians as a means to reduce cost and improve environmental performance of logistics activities. This paper investigates consolidated transport solutions with a common shipment frequency. When a service provider designs such a solution for its customers, she faces a trade-off: to have the most time-sensitive customers join the consolidated solution, the frequency must be high, which makes it difficult to gather enough demand to reach the scale economies of the solution; but by not having the most time-sensitive customers join, there will be less demand per time unit, which also makes it difficult to reach the scale economies. In this paper we investigate the service provider's pricing and timing problem and the environmental implications of the optimal policy. The service provider is responsible for  $N$  customers' transports, and offers all customers two long-term contracts at two different prices: direct express delivery with immediate dispatch at full cost, or consolidated delivery at a given frequency at a reduced cost. It is shown that the optimal policy is largely driven by customer heterogeneity: limited heterogeneity in customers' costs leads to very different optimal policies compared to large heterogeneity. We argue that the reason so many consolidation projects fail may be due to a strategic mismatch between heterogeneity and consolidation policy. We also show that even if the consolidated solution is implemented, it may lead to a larger environmental impact than direct deliveries due to inventory build-up or a higher-than-optimal frequency of the consolidated transport.

# 1 Introduction

Freight transportation is, on the operational level, characterized by strong economies of scale: larger shipment volumes lead to significantly lower transport costs per unit. However, for an individual shipper it is costly to realize these scale economies. Normally, two options exist: either, the shipper can purchase a less-than-truckload service (LTL), with longer transit time due to terminal handling; or he can purchase a full truck load service (FTL), but at a lower frequency. In both cases, the reduced flexibility leads to higher inventory costs. Since those costs are generally higher than the savings in transportation costs, scale economies often go unrealized, leading to operational and environmental inefficiencies. For instance, the EEA (2006) reports that the average vehicle load factor in Europe is below 50%.

Globally, freight transports account for approximately 8% of all energy-related GHG-emissions (IPCC, 2014). By increasing the vehicle load factor, the external costs associated with these emissions can be significantly reduced. Consequently, researchers and politicians have advocated different consolidation schemes, where shipments from several shippers are consolidated without the time consuming terminal handling of LTL-transports (see e.g. Arvidsson, 2013). In this paper we investigate one type of efforts: those where a common shipment frequency is used for all shippers who use the consolidated transport alternative. There are several examples of consolidation efforts where this is the case. Co-loading with set “sailing dates” (Tahe-rian, 2014), time-based consolidation in distribution systems (Marklund, 2011), and intermodal truck-train transports (Eng-Larsson and Kohn, 2012) all dispatch consolidated shipments at a certain frequency. Because such efforts would lead to larger volumes per shipment, they are often claimed to not just improve environmental performance but also reduce costs (e.g. Ülkü, 2012). For instance, according to the Executive Vice President of DHL Solutions & Innovations, “increasing the load factor of trucks is an attractive way to achieve more sustainability, as it not only improves the carbon footprint, but is also very appealing from an economic perspective” (Ehrhart, 2010).

In practice, most consolidation projects are initiated by a third party logistics service provider. One example is Volvo Logistics, who set up an intermodal truck-train solution between Sweden and Germany in 2008, called Viking Rail (see e.g. Eng-Larsson, 2012). Partly, this was to accommodate a request from a large customer to become more “climate-smart”. One objective of the new solution was therefore to reduce GHG-emission; the other was to avoid road tolls and traffic taxes in Central Europe. To initiate the project, a full train was contracted from DB Schenker. The train was to depart with 36 trailers in both directions. Thus, while the project was in part a response to a request from a customer, other customers were needed to reach enough scale to make the solution profitable. But when choosing which customers to include in the solution, the service provider faces a trade-off: to have the most time-sensitive customers join the

consolidated solution, the frequency must be high, which makes it difficult to gather enough demand to reach the scale economies of the solution; by *not* having the most time-sensitive customers join, there will be less demand per time unit, which also makes it difficult to reach the scale economies. So what customers should be included in the consolidated solution? At what frequency should the solution dispatch? And what is the optimal price of the new service? In this paper we analyze this problem, and evaluate the environmental implications of the optimal policy.

Shipment consolidation policies have been well researched before. But despite the fact that most transports are outsourced to service providers (Mellin and Sorkina, 2013; Jaafar and Rafiq, 2005; Hong et al., 2004), previous research largely ignores the fact that service providers interact strategically with their customers in these situations. In this research we make an attempt in capturing this by explicitly considering the contracting dynamics of a shipment consolidation setting. We investigate the problem by considering a non-asset service provider responsible for  $N$  customers' deliveries from one region to another. In our model, the service provider offers two long-term contracts at two different prices to each customer: direct express delivery with immediate dispatch at full cost, or consolidated delivery at a given dispatch frequency at a reduced cost. A customer then chooses the contract that leads to the lowest total expected cost. We consider two different pricing strategies: *individual prices* by which the service provider has the power to price discriminate customers based on their willingness-to-pay; and *standard prices*, by which the service provider offers the same price to all customers.

Although researchers and politicians seem confident in consolidation projects as a mean to reduce GHG-emissions, projects are rare in practice and often fail. For instance, there has been 150 consolidation projects in urban settings in Europe over the last 25 years, but only 5 of these have survived after subsidies were withdrawn SUGAR (2011). The Viking Rail project was discontinued after 5 years of operation<sup>1</sup>, and service providers in similar situations have reported difficulties in making their consolidation projects profitable, despite low operating costs on scale (see e.g. Lamngård, 2012). This paper makes an attempt in understanding why this is the case, by investigating what incentives the service provider has to offer a consolidated solution, and what incentives customers have to join the offered consolidated solution. It is shown that in many cases, the service provider's optimal policy is to not offer a consolidated solution. In cases where it is optimal to offer a consolidated solution, we show that the optimal policy depends on customer heterogeneity. This implies that to run a successful consolidated solution, the service provider must accurately match customer heterogeneity with the right policy. We argue that this may shed some light on why consolidation projects are rare in practice and often fail. First, it may be difficult to match heterogeneity with the right policy. Second, it may be difficult to adjust the policy to changes in heterogeneity once the policy is in place.

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<sup>1</sup><http://www.gp.se/ekonomi/1.2339085-volvo-lagger-ner-taget>

Lastly, we show that even if the consolidated solution is implemented, its environmental implications are not clear-cut. Consolidation may, in fact, lead to more environmental impact than direct deliveries, due to inventory build-up and a higher-than-optimal frequency of the consolidated transport.

The paper is structured in the following way. In the next section, related literature is reviewed, before our model is described in more detail in Section 3. In Section 4 we analyze the simultaneous pricing-timing decision of the service provider, and prove some underlying properties in three cases: 1) when customers are identical; 2) when customers are heterogeneous and can be charged individual prices; and 3) when customers are heterogeneous but are charged the same price. We show that in all cases, the underlying properties ensure that optimization can be done through a simple search procedure. The optimal policy's sensitivity to costs and customer heterogeneity are then investigated in Section 5. Next, in Section 6, we analyze the environmental implications of the optimal policy, and illustrate the difficulties of implementing freight consolidation with an example using realistic figures from a European context. Section 7 concludes the paper. All proofs are found in the appendix.

## 2 Literature

By investigating the pricing and timing of a consolidated transport service, this paper relates to literature on 1) transport pricing, 2) the timing of consolidated shipments, and 3) cost allocation in joint replenishment.

Since standard transport services are sold in a Bertrand-like fashion, with strong price competition, the price can often be seen as exogenous to the service provider. This is reflected in the transport planning literature. For instance, Kim and Van Wee (2011), Euchi et al. (2011), and Bolduc et al. (2007) all consider transport planning problems where the price of the transport services offered by service providers is assumed to be exogenous. We follow this literature and let the price for the direct delivery be exogenous to the service provider.

However, although the service provider has limited pricing power for standard services, there is more room to adjust the price for a customized or differentiated service. This is reflected in research on both cost-based pricing and relationship-specific pricing on the transport market. Cost-based pricing is discussed in Spasovic and Morlok (1993) and Yan et al. (1995). Spasovic and Morlok (1993) use a framework to determine marginal costs, which are then used to evaluate drayage rates in rail-truck intermodal services. Yan et al. (1995) use network flow models to estimate the alternative costs of intermodal transports to guide the pricing decisions of the service. Relationship-specific pricing, where the price is determined by a powerful customer, is discussed in Henig et al. (1997) and Alp et al. (2003), who analyze a buying firm's problem of how to simultaneously determine inventory policy and transport contract parameters in a periodic review inventory

model. Brusset and Temme (2005) and Berling and Eng-Larsson (2014) analyze the transport contract choice in a situation where the final prices of the contracts are outcomes of a game between the service provider and a single customer. The pricing mechanism in our model is most similar to the last perspective. We assume an exogenous price for the standard service (direct delivery), and based on this price, the service provider determines a rebate associated with the differentiated service (consolidated delivery) to cover any increase in cost at the customers. However, in contrast to previous research, we consider many heterogeneous customers. The service provider must thus set the price and design the service, while simultaneously deciding which customers to offer which transport service to.

Apart from the pricing decision, the service provider must also determine the timing of the consolidated delivery. In the inventory control literature, shipment consolidation has been investigated with focus on finding optimal policies for inventory and vehicle dispatch in systems of one or several echelons. Çetinkaya and Lee (2000), Çetinkaya and Bookbinder (2003) and Ülkü (2012) consider coordination of inventory and transportation in single-stage systems, including time-based and quantity-based consolidation policies. In multi-stage inventory systems, quantity-based consolidation has been analyzed by Cheung and Lee (2002) and Kiesmüller and de Kok (2005). More closely related to our research is that on time-based shipment consolidation. This is considered in Marklund (2011), who analyzes a divergent supply chain with time-based consolidation. He provides an exact recursive procedure to determine the costs of the system, and two heuristics for calculating the optimal dispatch frequency when a first-come-first-serve allocation policy is used. Howard and Marklund (2011) extend this to also consider state-dependent myopic allocation policies, where allocation is determined some time after the order has arrived at the central warehouse. While this paper also considers a system where a firm is responsible for deliveries to downstream stocking points, we extend this literature by assuming that the upstream firm (the service provider) and the downstream stocking points (the customers) are different decision-makers. This implies that an optimal policy for the entire system cannot be implemented by one player directly, and that the final decisions are the outcome of a game. This complicates the problem significantly, which calls for a more stylized model. We therefore do not model inventory dynamics explicitly, but handle this implicitly through a general cost function, seen in the next section, that captures the increase in cost due to longer lead-times.

Lastly, a related stream of literature that does analyze shipment consolidation from a decentralized perspective is the literature on joint replenishment. Dror and Hartman (2007), Anily and Haviv (2007), and Zhang (2009) use concepts from cooperative game theory to analyze the allocation of the cost reduction from joint replenishment among retailers. They consider slightly different versions of the same infinite horizon problem, and show that the core of the game is nonempty. That is, there is a possible “fair” allocation of costs. It is implied with that approach that the final allocation rule is enforceable by some benevolent party.

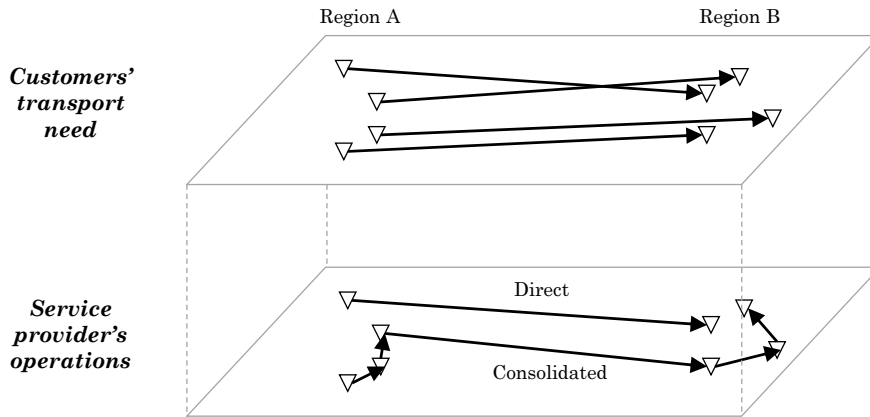


Figure 1: The service provider handles customers' transport need from one region to another . (Illustration, with  $N = 4$ .)

In our research, however, we assume that the upstream firm (the service provider) dictates which transport alternatives that are available on the market. Based on our observations, we believe that this captures several transport markets well.

### 3 Model

We consider two regions (see Figure 1) with a transportation need from one to the other by  $N$  different companies (from here on referred to as customers or “he”). The customers have outsourced their transports to a third-party-logistics service provider (from here on referred to as service provider, or “she”). The service provider offers two long-term contracts that the customers can choose from: direct transports with immediate dispatch, or consolidated transports that depart at a given dispatch frequency,  $1/\tau$  (to simplify notation, we will for the most part consider its inverse, the shipment interval  $\tau$ ). Each customer faces the problem of choosing the contract that minimizes his expected long-run cost. The service provider's problem is to design the contracts and transportation solution so that her expected long-run profit is maximized. We assume that all players have a cost/profit focus but that they choose the alternative that is perceived to be the more environmentally friendly (i.e. the consolidated solution) if the cost/profit is the same across the alternatives.

#### 3.1 The service provider

The service provider faces two possibilities for each customer: either the customer chooses the direct delivery contract, or the consolidated delivery contract.

If customer  $i$  chooses the first contract, the service provider will provide the customer with direct trans-

ports between the two regions. The service is sold at the exogenous price  $p$  per unit. We assume that one “service unit” corresponds to the transport of one “goods unit” from the first region to the second. Such a service unit, – e.g. the transport of a full truck load, a pallet, or a parcel – incurs a cost of  $c_D$  for the service provider. The lead time of this solution is the transportation time  $l$ . To simplify the notation, we let price, cost, and lead-time be the same for all customers. This should capture practice well, since the transport between the two regions likely make up the bulk of the cost and time. However, it is straightforward to extend the model to include customer-specific prices,  $p_i$ , costs,  $c_{D,i}$ , and lead-times,  $l_i$ .

If any customer  $i$  chooses the second contract, the service provider sets up a consolidated transport solution between the two regions. The vehicle(s) of the consolidated solution runs at every  $\tau$  time units. The customer’s lead-time with this solution is the transportation time  $l$  plus the time he has to wait for the common transport to be dispatched,  $[0, \tau)$ . Thus, this alternative reduces the customer’s flexibility, typically increases his lead-time, and leads to an additional cost, e.g. a cost for the additional safety stock needed to cope with the longer and more uncertain lead-times. Due to this additional cost, the contract is offered at a lower price. We refer to the *difference* in the price between the two contracts as the rebate,  $r_i$ . The service provider’s revenue for customer  $i$  will therefore only be  $p - r_i$  per transport. However, she can reduce her cost by consolidating the transports from several customers on the same consolidated solution. The cost for the consolidated transport, e.g. the cost to reserve and run the train-leg of an intermodal truck-train solution, is  $c_C$  in each period of length  $\tau$ , which includes the cost for back-hauls if applicable. We thus assume that there is no variable cost linked to the consolidated transport as it will not lead to any additional insights (but again it is straight forward to extend the model to incorporate this).

The service provider must decide on the length of the period,  $\tau$ , as well as what rebate,  $r_i$ , to give to each of the customers simultaneously. The expected profit is linked to both of these decisions. A longer time between the consolidated transports reduces the service provider’s cost for these transports, but leads to a need for larger rebates to keep customers in this solution, or more direct transports at a cost  $c_D$  each for the customers that leave the solution. Note that the choice of using direct or consolidated transports as well as the frequency of the consolidated solution influences the environmental performance of the system. This is further developed in later sections.

### 3.2 The customers

The customers face a choice between two contracts. The first contract provides direct transports at price  $p$ . The second contract provides consolidated transports at a lower price,  $p - r_i$ , but leads to a (stochastic) additional cost per time unit due to the lower flexibility. To make the exposition more clear and to facilitate



the calculations, we will make the following assumption regarding the expected value of the additional cost that a customer has to be compensated for.

**Assumption 1 (A1).** *The expected additional cost per time unit for customer  $i$  if he chooses the consolidated alternative,  $AC_i(\tau)$ , is*

- i. concave increasing in  $\tau$ , with*
- ii.  $\lim_{\tau \rightarrow 0} AC_i(\tau) = 0$ , and  $\lim_{\tau \rightarrow \infty} AC_i(\tau) > c_D \mu_i$ , and*
- iii. twice differentiable with  $\frac{d^2 AC_i(\tau)}{d\tau^2} \tau > -2 \frac{d AC_i(\tau)}{d\tau}$ .*

These are all reasonable assumptions<sup>2</sup>. Since  $\tau$  is directly linked to the lead-time, the first point in the assumption entails a concave increasing cost in the lead-time for the customer. The first limit,  $\lim_{\tau \rightarrow 0} AC_i(\tau) = 0$ , ensures that the additional cost  $AC_i(\tau)$  is continuous in  $\tau$  as  $AC_i(0)$  by definition is 0. This assumption along with the last condition ensures unimodality of the service provider's profit function for  $\tau > 0$  and homogeneous customers (see Lemma 1 in Section 4) which simplifies the search procedure significantly in other cases as well. The structural results derived later hold even if  $AC_i(\tau)$  is not twice differentiable as long as it is continuous. If  $AC_i(\tau)$  is discontinuous, that is, it shifts at certain  $\tau$ -values, the results hold in between these points of discontinuity. The second limit  $\lim_{\tau \rightarrow \infty} AC_i(\tau) > c_D \mu_i$  ensures that the trivial solution  $\tau = \infty$ , that is, to never ship, is not optimal. In fact, it is likely that  $\lim_{\tau \rightarrow \infty} AC_i(\tau) = \infty$  as it entails an infinite safety stock or shortages.

The solution to the customer's problem is then straightforward. The customer should choose the first contract if the rebate,  $r_i$ , is such that he does *not* get adequately compensated for the expected additional cost  $AC_i$  associated with the consolidated solution.

### 3.3 Sequence of events

The contract game is a one-period game, with full information, where the service provider is the price leader, and the contract is binding for an infinite future. The costs used during the contracting game are thus based on expectations in the long-run, assuming stationary inventory policies at the customers. First, the service provider observes the demand and all costs, and presents two contract alternatives to each customer

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<sup>2</sup>Assumption 1 is likely to be fulfilled in many systems. For instance, it is fulfilled in its entirety if a customer uses the transport to replenish warehouse using a traditional continuous review  $(R, Q)$ -policy and the reorder point  $R$  is determined using an  $\alpha$ -service level (i.e. probability of no stock-outs during an order cycle). The additional cost in this case is the cost for the additional safety stock needed. If the demand is normally distributed with mean  $\mu_i$  and standard deviation  $\sigma_i$  then the safety stock will be a safety factor,  $\Phi^{-1}(\alpha)$  times  $\sqrt{\sigma_i^2 \tau / 2 + \mu_i \tau^2 / 12}$  if  $l$  is negligible (see e.g. Axsäter 2006), which should be multiplied with the holding cost  $h_i$ . This function is clearly concave increasing and twice differentiable in  $\tau$  with the limits 0 and  $\infty$  respectively and it fulfills the last criterion. Note however that this general representation also captures switching costs and/or inertia to reduce transport quality, as indicated by Eng-Larsson and Kohn (2012) to exist in these situations.

simultaneously. The first contract is for direct transports and specifies just the price,  $p$ , per service unit. The second contract is for the consolidated solution and specifies a rebate,  $r_i$ , and a shipment interval,  $\tau$ . Each customer then, individually, chooses the alternative that minimizes his expected long-run costs. While the customer has a choice, clearly, the service provider is the leader and can set the price so that it is rational for a given customer to choose a given contract. The service provider then builds the consolidated solution if the profit of doing so is greater than serving all customers with direct transports.

## 4 Pricing and Timing of Consolidated Deliveries

In this section we derive the optimal rebate and shipment interval for the service provider's consolidated delivery-contract in three different scenarios. First we investigate the case of homogeneous customers, that is, when all customers face the same expected demand and has the same cost structure. We then turn to the case of heterogeneous customers, first with individual prices where each customer is offered a unique rebate, and then with standard prices where the same rebate is offered to all customers using the consolidated delivery option.

Independently of pricing scheme, a customer's participation constraint for using the consolidated delivery option is

$$AC_i(\tau) - \mu_i r_i \leq 0. \quad (1)$$

That is, the additional cost of using the consolidated delivery must be completely covered by the offered rebate. We assume that a customer prefers consolidated delivery as soon as the additional costs are covered. For the service provider, the participation constraint for offering a consolidated delivery is given by

$$\sum_{i \in IC} (r_i(\tau) - c_D) + \frac{c_C}{\tau} < 0, \quad (2)$$

where  $IC$  is the set of customers using the consolidated delivery option. The constraint simply ensures that the total costs from consolidating customers in  $IC$  are not larger than the costs of using direct delivery for the same customers.

Before moving on, we note from (1) that there exists a unique "accept rebate", that is, a lowest unit rebate that ensures customer  $i$ 's participation in the consolidated delivery at any shipment interval  $\tau$ . This is given by

$$\underline{r}_i(\tau) = AC_i(\tau) / \mu_i. \quad (3)$$

This rebate-function is of importance since, by offering this rebate or higher to customer  $i$ , the service

provider ensures  $i$ 's participation. If it is profitable to include  $i$  in the participation set, then the service provider should offer a rebate that is sufficiently large and extract the entire profit of his participation. Consequently, it is desirable for the service provider to offer this rebate whenever possible. However, as we will see, this is not always possible.

#### 4.1 Homogeneous Customers

When the service provider faces homogeneous customers, her problem can be reduced to a one-dimensional optimization problem over  $\tau$ . This is because the optimal rebate is the same for all customers (if the consolidated delivery option maximizes the expected profit). Offering a larger rebate is feasible, but will lead to an unnecessary decrease in the expected profit for the service provider. Offering a lower rebate will mean that the customer does not recover all costs and therefore prefers direct delivery. The service provider's long run expected profit per time unit is thus given by

$$\pi(\tau) = \begin{cases} N \cdot (p - c_D) \mu_i & \tau = 0, \\ N \cdot (p - \underline{r}_i(\tau)) \mu_i - \frac{c_C}{\tau}, & \tau > 0, \end{cases} \quad (4)$$

with the following property expressed as a lemma. All proofs are found in the appendix.

**Lemma 1.** *When customers are homogeneous and Assumption 1 holds, then the service provider's long-run expected profit per time unit,  $\pi(\tau)$ , is uni-modal increasing-decreasing in  $\tau > 0$ .*

Owing to the lemma, it is straightforward to find the service provider's optimal solution. First find the  $\tau$  that fulfills  $c_C - N \cdot AC'_i(\tau) \cdot \tau^2 = 0$  and the corresponding profit from (4), and then compare this to the profit when  $\tau = 0$ . Choose the  $\tau$  that provides the largest profit and the rebates according to (1).

Although having homogeneous customers may be unlikely, this simple situation still provides some insights about the mechanisms that govern a decentralized consolidation scheme and its environmental impact. This is illustrated in Figure 2. The  $AC_i(\tau)$ -curve shows the minimum loss in revenue necessary to achieve the customers' participation, as a function of  $\tau$ . The  $(\mu_i c_D - c_C/N\tau)$ -curve shows the cost savings associated with consolidated rather than direct transports. The profit is the difference between the cost-savings curve and the loss-in-profit curve. The left limit in Figure 2 is the most flexible solution (the shortest shipment interval) for which the service provider can make a profit, whereas the right limit is the most efficient solution (the longest shipment interval) for which the service provider can make a profit. Clearly, all feasible solutions are within this interval.

Simple comparative statics shows that the cost-savings are increasing in  $c_D$ ,  $N$ , and  $\mu$  and decreasing in  $c_C$ . The profit, and the distance between the lowest and highest  $\tau$  for which the consolidated alternative

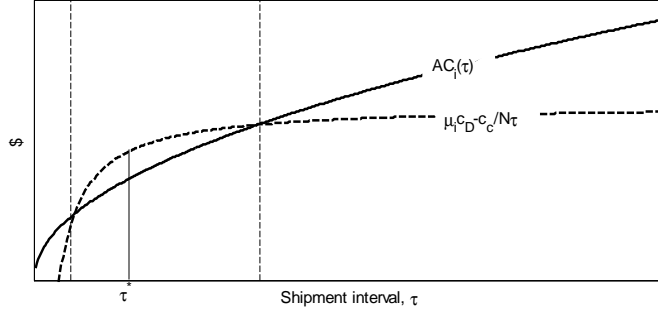


Figure 2: Homogeneous customers. Illustration of how the service provider's profit is influenced by shipment consolidation over shipment interval  $\tau$ ; solid line depicts the minimum rebate, the dashed line depicts the cost reduction.

is the most profitable, are affected in the same way. If  $c_C$  was to increase or any of the other parameters were to decrease, e.g. because of a loss in the customer base or due to lower fuel prices, the limits would quickly approach each other. Eventually, this would lead to a situation where it is impossible for the service provider to run a consolidated solution with a profit.

## 4.2 Heterogeneous Customers, Individual Prices

We now consider the case of  $N$  heterogeneous customers with a common shipment interval for all customers using the consolidated solution, with a rebate that can be varied between the customers. That is, the service provider has the possibility to price discriminate customers. The service provider's long run expected profit per time unit is in this case given by

$$\pi(\tau, IC) = \begin{cases} \sum_{i=1}^N (p - c_D) \mu_i, & \tau = 0, \\ \sum_{i \in IC} (p - \underline{r}_i(\tau)) \mu_i + \sum_{i \notin IC} (p - c_D) \mu_i - \frac{c_C}{\tau}, & \tau > 0, \end{cases} \quad (5)$$

where  $IC$  is the participation set. As before, it is apparent that the service provider should never offer a larger rebate than necessary, so  $r_i = \underline{r}_i(\tau)$  for all customers in the participation set. From (5) we can see that if  $\underline{r}_i(\tau) > c_D$  it is better to not include customer  $i$  in the participation set, which is achieved by setting  $r_i < \underline{r}_i(\tau)$ , e.g. by letting  $r_i = c_D$ . Combined, this gives that  $r_i^*(\tau) = \min\{\underline{r}_i(\tau), c_D\}$  is an optimal rebate scheme (of course any rebate scheme with  $r_i(\tau) = \underline{r}_i(\tau)$  when  $\underline{r}_i(\tau) \leq c_D$  and  $r_i(\tau) < \underline{r}_i(\tau)$  otherwise will be optimal).

The increasing property of  $AC_i(\tau)$ , with  $\lim_{\tau \rightarrow \infty} AC_i(\tau) > c_D \mu_i$ , implies that there exists a break-point  $\tau_i$  above which it is never optimal to include customer  $i$  in the participation set (at  $\tau_i$  the service provider is indifferent between including the customer or not but if the rebate is given as described above he will be

included). This implies the following.

**Lemma 2.** *With individual prices, there are  $N$  plausible optimal participation sets. The size of the optimal participation set,  $|IC^*|$ , is decreasing in  $\tau > 0$ .*

The lemma proves the intuition that when the service quality (i.e. the frequency of consolidated shipments) decreases, it becomes increasingly costly to keep customers, which means that it is optimal to have less customers using the consolidated delivery solution. In other words, it can only be profitable for a service provider to run a consolidated delivery if she can guarantee a certain service quality.

The lemma also implies that the sum of the customer-specific costs,  $\sum_{i \in IC} \min\{\underline{r}_i(\tau), c_D\} \mu_i$ , is concave increasing in  $\tau$ . While this provides some structure to the problem, the concavity is kinked at those points where a customer leaves the participation set, so unlike the case of homogeneous customers, the service provider's maximum expected profit is not necessarily unimodal increasing-decreasing in  $\tau > 0$ . However, we have the following important property for each participation set.

**Lemma 3.** *If Assumption 1 holds for all  $i \in IC$ , then, for a given participation set  $IC$ , the service provider's long-run expected profit per time unit,  $\pi(\tau, IC)$ , is unimodal increasing-decreasing in  $\tau > 0$  with local optima  $\tau_1^*, \tau_2^*, \dots, \tau_N^*$ . Moreover, the global optimum,  $\tau^*$ , is always in a local optimum, i.e.  $\tau^* \in \{0, \tau_1^*, \tau_2^*, \dots, \tau_N^*\}$ .*

Using these lemmas, a simple search can be performed over plausible optimal participation sets to find the service provider's global optimum  $\tau^*$ . As a result, any customer with  $\tau_i \geq \tau^*$  will use the consolidated delivery. Since customers with higher coefficient of variation, service levels, and holding costs normally have lower  $\tau_i$ , the interpretation is intuitive; customers with lower valued products and more stable demand are more likely to be on the consolidated delivery<sup>3</sup>.

### 4.3 Heterogeneous Customers, Standard Prices

Finally, consider the case of  $N$  heterogeneous customers with a common shipment interval and standard rebate for all customers using the consolidated delivery. The service provider's long run expected profit per time unit is in this case given by

$$\pi(\tau, IC) = \begin{cases} \sum_{i=1}^N \mu_i \cdot (p - c_D) \mu_i, & \tau = 0, \\ \sum_{i \in IC} (p - \widehat{r}_{IC}(\tau)) \mu_i + \sum_{i \notin IC} (p - c_D) \mu_i - \frac{cc}{\tau} = & \tau > 0, \end{cases} \quad (6)$$

where  $IC$  is the participation set, and  $\widehat{r}_{IC}(\tau)$  is the lowest rebate that ensures that all  $i \in IC$  prefer the consolidated solution. This makes the situation more complicated than when individual prices can be used.

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<sup>3</sup>see previous footnote

For instance, since  $r_i(\tau)$  is a individual function of  $\tau$ , it is not sure that it is the same customer  $i$  that dictates  $\widehat{r}_{IC}(\tau)$  for all  $\tau$ . Moreover, it might be optimal to set a rebate that is less than the cost for direct transports,  $c_D$ , even though a larger rebate would lead to a larger participation set. This as the savings in transportation cost that can be achieved by extending the participation set from  $IC$  to  $IC \cup j$  can be outweighed by the loss in revenue from all customers in the current  $IC$ . Consequently, the service providers profit when including  $j$  in the participation set will only increase if

$$(c_D - r_j(\tau)) \mu_j - \sum_{i \in IC} (r_j(\tau) - \widehat{r}_{IC}(\tau)) \mu_i \geq 0. \quad (7)$$

To proceed, we first report the following important result regarding the number of potential participation sets.

**Lemma 4.** *With standard prices, there are  $\widehat{N} = N + \Upsilon$  plausible optimal participation sets, where  $\Upsilon$  is the number of  $\underline{r}_i(\tau)$ -curve and  $\underline{r}_j(\tau)$ -curve pairs, for customers  $i \neq j$ , that cross at least once.*

Next, we can show that, as in the case of individual prices, there are in fact certain unimodal properties that facilitate the search procedure.

**Lemma 5.** *If Assumption 1 holds for all  $i \in IC$ , then, for a given participation set  $IC$ , the service provider's long-run expected profit per time unit,  $\pi(\tau, IC)$  is unimodal increasing-decreasing in  $\tau > 0$ , with local optima  $\tau_1^*, \tau_2^*, \dots, \tau_{\widehat{N}}^*$ . Moreover, the global optimum,  $\tau^*$ , is always in a local optimum, i.e.  $\tau^* \in \{0, \tau_1^*, \tau_2^*, \dots, \tau_{\widehat{N}}^*\}$ .*

Based on these observations, a search can be conducted to solve the service provider's problem, similar to the search performed with individual prices (but over a larger set of plausible participation sets).

Based on this, it can be easily verified that except in the case of just one type of customer, individual prices lead to a higher long-run expected profit for the service provider than standard prices, and never to a smaller optimal participation set (but possibly to a shorter shipment interval). The fact that individual prices lead to higher service provider profits is intuitively straightforward. With price discrimination the service provider can charge each customer the maximum price he is willing to pay for a consolidated delivery. This means that the full benefit of using the consolidated delivery is rewarded to the service provider. Despite the fact that all customers face a higher (or the same) price when the service provider price discriminates, more customers will use the consolidated delivery under price discrimination, as any participation set with more than one customer will be more expensive for the service provider to reimburse with standard prices than with individual prices. This also implies that standard prices work as a type of cost-sharing mechanism, where some of the benefit of the consolidated delivery is rewarded to the customers. The most time-insensitive

customers receives the largest benefit. However, since any given set becomes more expensive (or costs the same), it is never optimal for the service provider to cater to a larger set with standard prices.

## 5 Sensitivity Analysis

In this section we focus on how the customers' expected additional costs from waiting,  $AC_i(\tau)$ , impacts the service provider's problem. In practice, these costs differ among customers depending on downstream characteristics such as the value of the transported goods, the demand uncertainty, service levels at the customer, and the perishability of the goods. For some service providers, customers are rather homogenous. A service provider catering to, for instance, the wood and forestry industry is one such example. For other service providers, customers may show a large spread in their expected additional costs. In this section we investigate how this heterogeneity influences the optimal solution.

For this analysis, we let the customers' alternative costs be given on the form  $AC_i(\tau) = K_i\tau^{\beta_i}$ . The scale parameter,  $K_i > 0$ , depends on the customer's cost of holding inventory, his optimal service level, as well as his demand volatility. The shape parameter,  $\beta_i \in (0, 1)$ , captures the time-multiplier, that is, the "factor" that determines how quickly costs increase as the shipment interval increases. This is typically linked to the auto-correlation of the demand and is likely to be close to 0.5 (see footnote in Section 3 and Axsäter, 2006). We let the  $K_i$ 's and the  $\beta_i$ 's be uniformly distributed among customers according to  $K_i = \bar{K} + \Delta(i - (N + 1)/2)$ , with  $\bar{K} = 3000$ , and  $\beta_i = \bar{\beta} + \Delta(i - (N + 1)/2)$ , with  $\bar{\beta} = 0.5$ . The spread,  $\Delta$ , is used to capture the level of customer heterogeneity. Although we are aware that the results for customer populations with non-uniform distributions may differ significantly, proceeding in this way is motivated by the fact that it allows us to make a rough assessment of the impact of customer heterogeneity on the results: larger  $\Delta$  simply means a greater spread in the parameter and thus larger heterogeneity.

Figures 3a-f show the optimal shipment interval, the optimal participation set, and the service provider's profit as a function of the spread in  $K_i$  and  $\beta_i$ . Since this paper is inspired by the situation faced by a service provider setting up an intermodal link between two regions in Europe, the figures are based on cost parameters similar to those in Berling and Eng-Larsson (2014), who present a similar situation. Consequently, we let  $p = c_D = 1,750$ , and  $c_C = 45,000$ , which means the break-even in terms of transport costs is at a shipment size of 25 intermodal trailers. Note however that this is the break-even for the service provider only if customers accept to join the intermodal solution without any rebate. Judging from Lammgård (2012) and Eng-Larsson (2012) this seem to be a plausible construction.

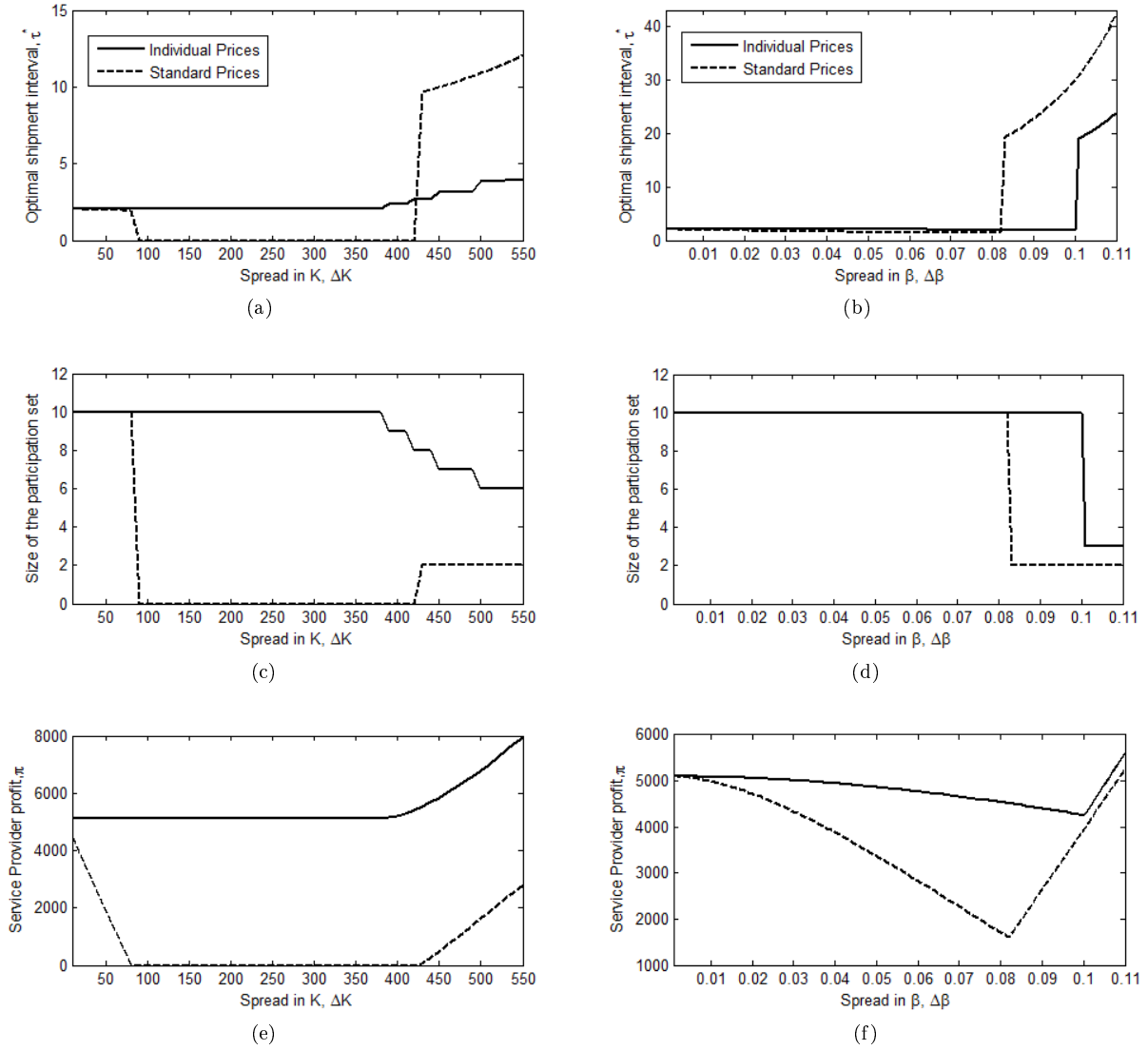


Figure 3: Optimal shipment interval, optimal participation set, and service provider profit as a function of the spread in the  $K$ -parameter and the  $\beta$ -parameter.  $N = 10$ ,  $\mu_i = 4$  for all  $i$ .



## 5.1 Shipment Interval and Participation Set

From Figures 3a-f it can be seen that two very different types of consolidation policies can be profitable for the service provider. For intuition, we refer to these extremes as consolidation over “space”, where several customers are consolidated on a solution with a short shipment interval, and consolidation over “time”, where only one or a few customers’ demand is consolidated on a solution with a long shipment interval. The first type of policy strives to produce a high-quality transport service. This is profitable only when several customers exhibits similar sensitivity to time, i.e. similar  $AC_i(\tau)/\mu_i$ . The second type of policy produces a transport service of sufficient quality for one or a few time-insensitive customer. This is profitable when customers are not homogenous, but there are one or a few very time-insensitive customers.

When individual prices can be used, the pure space and time policies are extremes on a continuum: as customers become more heterogeneous, the solution should be adjusted to fit the less time-sensitive customers. In Figure 3a, it is seen that as long as the optimal participation set  $IC$  is the full set of  $N$  customers, an increase in the spread of  $K_i$  will not increase the shipment interval. However, once it is optimal to exclude some customers from the participation set, there is a slow increase in  $\tau^*$ , with distinct shifts upwards when the optimal participation set is reduced. That is, the transport quality of the consolidated solution is adjusted to the less time-sensitive customers, while the more time sensitive customers are excluded. This follows directly from (4), where an optimization of the shipment interval renders

$$\tau^* = (c_C/\beta \sum_{i \in IC} K_i)^{1/(\beta+1)}, \quad (8)$$

where  $\sum_{i \in IC} K_i$  is constant if  $IC$  is the complete set, and decreasing otherwise, with distinct shifts downwards when  $IC$  is reduced.

If customers differ in the shape parameter  $\beta$  and individual prices are used, the optimal shipment interval will decrease when  $\beta$  increases for a given  $IC$  and vice versa. When the set is reduced, there will be a shift, as seen in Figure 3b. If the participation set is smaller than  $N/2$ , the participating customers become increasingly time-*insensitive* when the spread increases. Similar to the  $K$ -parameter, in these situations, an increase in  $\Delta$  will lead to an increase in the shipment interval.

When standard prices are used, the optimal policy is usually of an either-or-character: either all customers *or* a selected few should be on the solution (or none whatsoever). To see why, remember that it is the largest  $K$ -value among customers in the participation set that determines the optimal shipment interval, i.e.

$$\tau^* = (c_C/\beta \sum_{i \in IC} \widehat{K}_{IC})^{1/(\beta+1)}. \quad (9)$$

This value is initially increasing in the spread of  $K$ , as all customers are included in the participation set. This leads to a gradual reduction in  $\tau^*$  for this pricing strategy, as seen in Figure 3a. When the optimal participation set is reduced, there is a shift up as  $\widehat{K}_{IC}$  and the number of customers over which the summation is carried out is reduced, unless the consolidated solution is abandoned, which leads to  $\tau^* = 0$ . If the participation set is smaller than  $N/2$ , then  $\widehat{K}_{IC}$  is decreasing and  $\tau^*$  is increasing with the spread with distinct shifts up when the set is reduced. For the  $\beta$ -parameter, the same arguments apply to standard prices as for individual prices, although it is only the  $\beta$ -value of the most time sensitive customer that determines the optimal shipment interval. Which customer that is the most time-sensitive will differ with  $\tau$ . If  $\tau > 1$ , the most time sensitive customer is the customer in  $IC$  with the largest  $\beta$ -value, and if  $\tau < 1$  it is the customer with the smallest  $\beta$ -value. The optimal shipment interval can thus be either increasing or decreasing in the spread of  $\beta$ . This explains the gradual decrease in  $\tau^*$  seen in Figure 3b. Distinctive shifts up occur when the participation set is reduced as  $\widehat{\beta}_{IC}$  as well as  $\sum_{i \in IC} K_i$  is reduced.

For both pricing strategies, the general implications are the same independently of which parameter that control heterogeneity: the service provider must accurately match consolidation policy with customer heterogeneity. As heterogeneity changes, the optimal policy may change incrementally or drastically depending on the pricing strategy. This must be taken into account to run a successful consolidated solution.

## 5.2 Service provider's profit

With a consolidated solution, the service provider's customer specific cost for customer  $i$  is  $\min\{r_i(\tau), c_D\}$ : if customer  $i$  is in the participation set  $IC$ , the cost is  $r_i(\tau)$ ; if he is not, the cost is  $c_D$ . The option to include customer  $i$  in the set or not creates an added value for the service provider. One would therefore expect that the value of the option to be non-decreasing in the variability of the customers' cost parameters. This is also observed under individual prices in Figure 3e, i.e. when the customers differ in  $K_i$ . However, it is not seen for any of the other cases. In fact Figures 3e and 3f reveal that the profit is initially decreasing with the spread of the customers' cost parameters in the other cases. This can be explained by the fact that, in all the other cases, a larger spread implies a larger cost *on average* due to larger rebates, a decreasing participation set and/or more frequent shipments. This is most apparent when a standard price is used, since the most time-sensitive customer dictates the rebate in these cases. An increase in the spread then increases the price-setting customer's additional cost for a given  $\tau$ , and hence the required rebate,  $\widehat{r}_{IC}$  – if the number of customers in the participation set is larger than  $N/2$ . For a participation set smaller than  $N/2$ , an increase in the spread will instead lead to a reduction in the rebate and an increase in profit.

For a smaller participation set, the intershipment interval must be longer to consolidate a sufficient

number of transports over time to recover the cost of setting up the system. The consolidated alternative will thus only be offered when the  $K_i$ -values and  $\beta_i$ -values of the customers in the smaller participation set is sufficiently low. This is seen in Figure 3c and Figure 3e: in the intermediate range, there is no consolidated solution set up, but for larger values of  $\Delta$  one is set up for the less time-sensitive customers. The shipment interval for the smaller set of customers is much longer than for the larger set that contains all or almost all customers. In the examples considered where the customers differ in  $\beta$ , no such intermediate range exists but for others it may. In these examples a distinct shift when the service provider stops including all customers and starts to include only the least time-sensitive ones can still be seen, though. The fact that the average rebate must be higher, or the shipment interval shorter, to keep all customers in the solution when the spread of the  $\beta$ -values is increasing (and  $\tau \neq 1$ ) even under individual prices are less apparent. It is however straightforward to show, and is due to the fact that  $\sum_{i \in IC} AC_i$  becomes more concave if the spread of the  $\beta$ -values increases. This explains the initial decrease in profit seen in Figure 3b even under individual prices.

## 6 Environmental Implications

The previous sections focused on the profit/cost and how different cost parameters, customer heterogeneity and the possibility to price discriminate influence these and the optimal solution. In this section we investigate the environmental implication of the policy resulting from a given cost structure.

From the previous analyses it is apparent that the consolidated alternative is more likely to be a viable solution if the cost of the direct delivery,  $c_D$ , is high and/or the cost of the consolidated delivery solution,  $c_C$ , is low. Further, a reduction in  $c_C$  will typically reduce the optimal shipment interval  $\tau^*$ . From an environmental perspective, changes in transport costs is of interest to investigate in greater detail, since a regulator can influence these costs directly through taxes, fees or subsidies. It shall be noted that we assume that the players' decisions are made based on cost/profit considerations, and that any environmental effect is an outcome of these decisions.

To analyze the environmental implications we consider three environmental cost components. These are the environmental cost for the transports as well as the environmental cost for the additional environmental impact caused by the reduced flexibility of the consolidated solution, e.g. the environmental cost for the additional safety stock. We will denote these costs,  $e_D$  (direct transports),  $e_C$  (consolidated transports) and  $AE_i(\tau)$  (reduced flexibility), respectively. These are the external cost faced by society per transport ( $e_D$  and  $e_C$ ) or per time unit ( $AE_i(\tau)$ ), but it can also be seen as an arbitrary measure of environmental impact reported by either player, e.g. greenhouse gas emissions.

The transport related environmental costs are increasing in the number of transports. So, one would

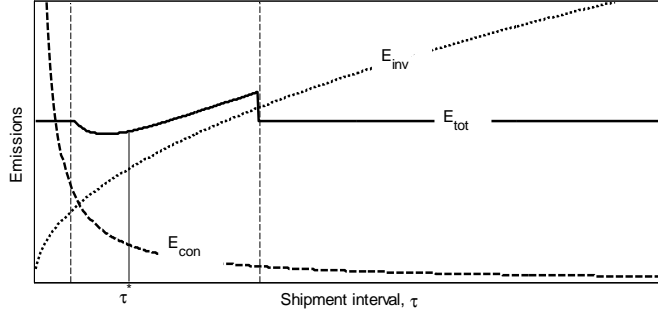


Figure 4: Homogeneous customers. Illustration of how the decentralized decision-making may lead to sub-optimal environmental performance even when the consolidated delivery option is used: dotted line depicts emissions per unit from reduced flexibility among customers ( $E_{inv}$ ), dashed line depicts emissions per unit from transportation ( $E_{con}$ ), and the solid line depicts the total emissions under the service provider’s optimal policy.

expect that the more customers that choose the consolidated solution and the more infrequently it runs the better. However, from an environmental perspective, it is not desirable to have too infrequent deliveries as the reduction in transport related environmental costs,  $e_C/\tau$ , does not compensate for the increase in environmental cost due to the reduced flexibility,  $AE_i(\tau)$ <sup>4</sup>. An increase in  $\tau$  will also lead to fewer customers being attracted to the consolidated solution. Therefore, it might be better to abolish it all together if it is not designed (price and shipment interval) to attract a sufficient number of customers, or if the frequency is so high that the transport-related environmental costs are not reduced significantly compared to direct transports. This is illustrated by Figure 4, which shows the total environmental costs for a customer using the consolidated solution in a system with homogeneous customers. Three types of environmental costs are depicted; 1) costs from emissions from the consolidated transports, and 2) costs from emissions associated with the reduced flexibility, and 3) the costs from the total emissions, which is the sum of the two types of emissions within the interval where consolidation is possible, and the emissions from direct transports outside of this interval. The corresponding cost/profit curves for this example can be seen in Figure 2. The figures illustrates how the decentralized decision-making may lead to environmentally sub-optimal solutions, even when the consolidated solution is used. If  $\tau^* < \tau_e^*$  the consolidated solution is designed with a *too short* shipment interval, leading to too many consolidated deliveries with too low utilization to reduce transport emissions sufficiently. In fact, the transport emissions in this case might even be higher than when direct transports are used. If instead  $\tau^* > \tau_e^*$  then the solution is designed with a *too long* shipment interval, leading to inventory build-up that offsets the reduction in transport emissions.

For the special case of homogeneous customers it is straightforward to see how the cost structure can be

<sup>4</sup>The environmental cost associated with reduced flexibility can be substantial which is indicated by the report by WEF (2009) that estimate that warehousing accounts for roughly 10% of all logistics-related emissions or McKinnon et al. (2012) who argue that warehousing accounts for 2-3% of the world’s total energy related emissions.

manipulated so the optimal solution from an environmental perspective coincides with the solution preferred from a cost/profit perspective. When the customers are not homogeneous, the problem becomes significantly more complicated.

The intuition might be that an improvement in environmental performance is simply achieved by increasing the cost of direct deliveries,  $c_D$ , and/or decreasing cost of the consolidated delivery solution,  $c_C$ . Carbon taxes would achieve the former, whereas subsidies such as the EU's Marco Polo project would achieve the latter. However, the answer is not that simple, as the impact of a change in the cost structure is not apparent, and an environmental cost structure adds an additional layer of complexity.

In the following, we first provide some insights regarding how  $c_D$  and  $c_C$  influence on the system, and why it is difficult to predict how a change in any of these will change the optimal solution. We will then numerically investigate the issue, based on the same base line parameters as in previous section.

**Proposition 1.** *Changes in transport costs have the following effects.*

- i. Under individual prices the size of the optimal participation set,  $|IC^*|$ , is non-decreasing in  $c_D$  and the optimal shipment interval,  $\tau^*$ , is non-increasing. For standard prices, the total demand of the optimal participation set,  $\sum_{i \in IC^*} \mu_i$ , is non-decreasing in  $c_D$ .*
- ii. Under individual prices, all local optima  $\tau_i^*$  increase in  $c_C$ ; with standard prices, all local optima  $\tau_{IC_{1,i}}^*, \tau_{IC_{2,i}}^*, \dots, \tau_{IC_{\hat{N},i}}^*$  are increasing in  $c_C$  but all intersections,  $\tau_{IC}^* \neq \tau_{IC,i,j}$ , are unaffected.*

From the proposition it is clear that taxation to increase the price of normal truck transport,  $c_D$ , leads to a situation where it is optimal to have more customers using consolidated transports. If it possible to use individual prices, these transports will be performed at a higher frequency. One can expect that this is often the case also when standard prices are used, but it depends on the how the “accept rebate” for the added customer,  $AC_j(\tau)/\mu_j$ , evolves compared to the one for the customer that set the rebate in the smaller set,  $AC_i(\tau)/\mu_i$ . While the situation is similar when subsidies are used to reduce the cost for consolidated transports,  $c_C$ , more mechanisms are at play, which makes the situation more difficult to analyze. What could be said is that as long as individual prices are used, subsidies have the same qualitative effect as taxes: more and better consolidated transports.

However, as discussed above, Proposition 1 does not necessarily imply that an increase in  $c_D$  or a decrease in  $c_C$  leads to better environmental performance. To find the environmental impact of these adjustment, another level of complexity must be added; there is no simple relationship between  $\tau^*$  and the environmental performance. In fact, even if one unit of consolidated transport is environmentally better than one unit of regular transport, that is, if  $e_C \leq e_D$ , it can not be ascertained that regulation through taxes or subsidies will improve the environmental performance (i.e. reduce the environmental cost).

In Figure 5, the environmental cost is depicted for individual prices and standard prices as functions of the transport costs, for a group of heterogeneous customers also considered in Section 5. The environmental costs for transports between the two regions are based on the NTM-calc tool<sup>5</sup>. We approximate and normalize these costs for the intermodal case to  $e_C = 7.5$  and  $e_D = 1$ . Note that these costs are expressed per transport and not per unit, which means that the expected environmental cost per unit from using the consolidated transport depends on the shipment interval,  $\tau$ . The environmental cost of the reduced flexibility is assumed to be proportional to the alternative cost, and given by  $AE_i(\tau) = 0.001 \cdot AC_i(\tau)$ .

Figure 5 illustrates what we know from Proposition 1, and depicts the environmental implication of this. Starting with Figure 5a, we see the effect of changes in the cost of the direct delivery, which, for example, would increase if a heavier carbon tax was implemented. In the figure we see that this leads to a situation where the service provider goes from no consolidated transports, to offering a consolidated solution at a price which makes it attractive for all customers. That is, it becomes optimal for the service provider to offer a consolidated delivery solution of the first type – to consolidate over “space”. But, even though this leads to more consolidated transports (as predicted by Proposition 1) with higher vehicle load factors, we actually see an increase in the environmental cost. This occurs since the frequency of the consolidated solution is not high enough for the reductions in transport related environmental costs to offset the increase in environmental costs from the reduced flexibility of all customers.

Figure 5b depicts changes in the consolidated transport cost. Note that the shipment interval of the consolidated delivery depends on the consolidated transport cost. Consequently, as seen in Figure 5b, when this cost is reduced, the transports run more frequently, which means that less inventory is needed and the environmental costs decrease. As the cost of the consolidated solution keeps decreasing, so do environmental costs, until a point where the consolidated transports depart at such a high frequency that there are more transports than environmentally optimal, leading to quick increases in total environmental costs. As seen in the figure, the same dynamics exist independently of pricing strategy, although break-points and actual values differ between the two.

The figures also illustrate the fact that when the cost of direct deliveries increases, the total environmental impact increase or decrease in discrete steps, whereas when the cost of the consolidated transport decreases, total environmental impact increase or decrease continuously, with discrete shifts at certain points. This means that the marginal impact of a subsidy is, often, larger. The figures also clearly indicate that introducing intermodal transports does not always lead to an environmental improvement. For this to happen, the shipment interval cannot be too short nor too large. Clearly, in the situation depicted in this small study, this is only achieved under very peculiar circumstances, which may or may not be feasible in practice due

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<sup>5</sup><http://www.ntmcalc.com>

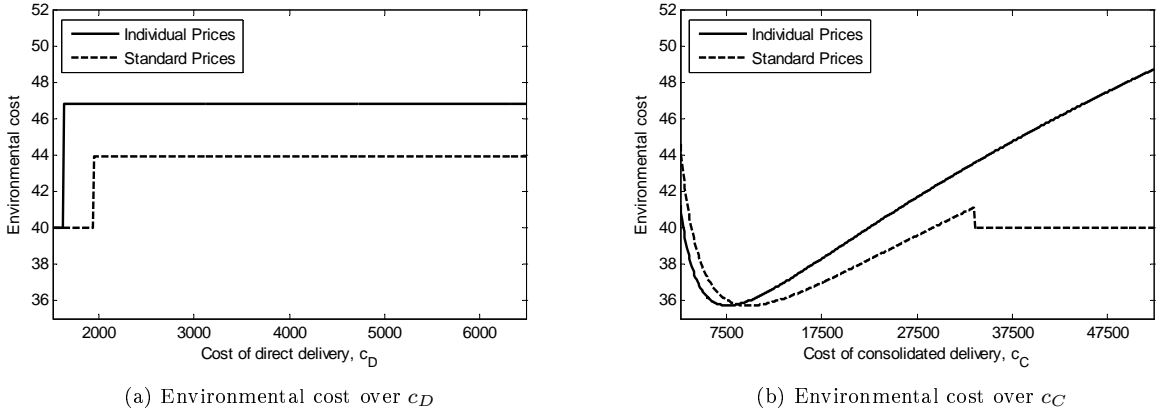


Figure 5: Total environmental cost (e.g. expected emissions) per time unit for the two pricing strategies as functions of the cost of direct delivery,  $c_D$ , and the cost of consolidated delivery,  $c_C$ . Environmental cost parameters;  $e_D = 1$ ,  $e_C = 7.5$ , and  $AE_i(\tau) = 0.001 \cdot AC_i(\tau)$ . Scenario with  $N = 10$  customers that differ in the scale parameter  $K_i$  with  $\Delta = 200$  and  $\mu_i = 4$  for all  $i$ .

to regulations regarding train length.

## 7 Conclusion

In this paper we have investigated a third party logistics provider’s (service provider) problem of pricing and timing a consolidated solution for  $N$  customers’ deliveries from one region to another. We showed how the service provider’s simultaneous pricing and timing problem can be reduced and solved to optimality using a simple search, for both standard and individual prices. While we have referred to the players as service provider and customers, the model also captures the more general case where the service provider is any type of firm selling goods to customers being responsible for the transports (e.g. goods are sold according to Incoterm 2010 DDP or DAP).

Depending on customer heterogeneity, very different consolidation policies are optimal. If, for instance, customer heterogeneity is low, it is optimal for the service provider to produce a high-quality transport service that includes all or most of the customers. If, on the other hand, customer heterogeneity is high, it is optimal to produce a transport service of just sufficiently high quality for one or a few time-insensitive customer. Both types of policies have been observed in practice. While many researchers argue for ways to make consolidated solutions of higher transport quality, as observed by Eng-Larsson and Kohn (2012) in their study of successful consolidation projects: “Instead of aiming for the best transport quality, successful cases have aimed for just the right amount of transport quality for their logistics system.” In other words, the service provider must accurately match consolidation policy with customer heterogeneity. This match

may be difficult to do in practice, since it requires knowledge of all customers' willingness to pay. Perhaps more importantly, as heterogeneity changes (e.g. due to demand shocks), the optimal policy may change while, in practice, a solution's design is often fixed for several years because of long-term investments. Also, the change in heterogeneity may lead to a situation where the optimal policy is to not have a consolidated solution. As seen in the paper, these situations are not uncommon. Taken together, this may provide some insights as to why shipment consolidation is rare in practice and why so many consolidation projects have failed. Volvo, for instance, claim a reduction in demand as the main reason for abolishing their solution. This affects the customer-base, which calls for a redesign. However, a more detailed investigation of the impact of demand uncertainty over time is an interesting extension for future research.

The paper shows that the environmental benefit of shipment consolidation is not always clear-cut. Owing to inventory build-up or higher-than-optimal dispatch frequency, implementing such a solution may lead to adverse effects. Clearly, the actual effect on environmental costs depend on the type of transport technology as well as the type of good being transported. We also saw that the effects of regulation there is ambiguous. A more thorough welfare analysis, where the market interactions are taken into account, would be an interesting extension but calls for a slightly different framework.

This research has used a very stylized model to reach initial insights to the effect of the market interactions in affecting the incentives to offer consolidated transports to shippers. Depending on the purpose, the model can be extended to capture other aspects that may be of importance, for instance the service provider's capacity decision: if she is to offer a consolidated transport solution, how much transport capacity should she invest in? This, too, is an interesting venue for further research.

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## 8 Appendix

### 8.1 Proof of Lemma 1

*Proof.* If we differentiate  $\pi(\tau)$  with respect to  $\tau > 0$  we get  $\frac{d\pi(\tau)}{d\tau} = -AC_i'(\tau) + \frac{c_C}{\tau^2} = \frac{c_C - \tau^2 AC_i'(\tau)}{\tau^2}$  and it is apparent that  $c_C > 0$  and that the numerator is decreasing in  $\tau$  if criteria iii) in Assumption 1 is fulfilled. Consequently,  $\pi(\tau)$  is uni-modal increasing-decreasing (or possibly just decreasing if  $\lim_{\tau \rightarrow 0} \tau^2 AC_i'(\tau) > c_C$ ).  $\square$

### 8.2 Proof of Lemma 3

*Proof.* The first part of the proof is a simple extension of the proof to Lemma 1 as  $\sum_{i \notin IC} c_D \mu_i$  is independent of  $\tau$ .

For the next part, let  $\tau_i$  denote the  $\tau$  where  $\underline{r}_i(\tau) = c_D$ , i.e. the point where  $i$  leaves the optimal participation set  $IC$ . Let us presume that  $\pi(\tau, IC)$  is increasing in  $\tau$  at  $\tau_i$ . By definition (equation 5) we have that  $\pi(\tau_i, IC \setminus \{i\}) = \pi(\tau_i, IC)$  and if  $\pi(\tau, IC)$  is increasing in  $\tau$  at  $\tau_i$ ,  $\pi(\tau, IC \setminus \{i\})$  must increase faster or it will be optimal to keep  $i$  in the participation set. Thus there exists a  $\tau > \tau_i$  where  $\pi(\tau, IC \setminus \{i\}) > \pi(\tau_i, IC)$  so  $\tau_i \neq \tau^*$ . If  $\pi(\tau, IC)$  is instead decreasing in  $\tau$  at  $\tau_i$  then it is obvious that there exists a  $\tau < \tau_i$  where  $\pi(\tau, IC) > \pi(\tau_i, IC)$  and again  $\tau_i \neq \tau^*$ . The corresponding argument can be made at the  $\tau (< \tau_i)$  where the optimal participation set becomes  $IC$ . Ergo, the optimal solution must be a local optimum.  $\square$

### 8.3 Proof of Lemma 4

*Proof.* As before, we have a participation set with  $1, 2, \dots, N$  customers (plus the empty set, which we disregard). However if the  $\underline{r}_i(\tau)$ - and  $\underline{r}_j(\tau)$ -curves cross each other, then the participation set where they are the last customers to be included or excluded will depend on  $\tau$ . Because of the crossing, there are two plausible participation sets with the same number of customers and not just one, the first set including  $i$  but not  $j$  (when  $\underline{r}_i(\tau) < \underline{r}_j(\tau)$ ) and the other with the inverted selection. This brings us to the given number of plausible optimal participation set.  $\square$

### 8.4 Proof of Lemma 5

*Proof.* To prove the first part let us use  $\pi_i(\tau, IC)$ , the expected profit under the assumption that customer  $i$  dictates  $\widehat{r}_{IC}(\tau)$  for all  $\tau > 0$ . That is the expected profit given that  $\widehat{r}_{IC}(\tau) = \underline{r}_i(\tau)$  and all customers  $j \in IC$  remains in  $IC$  even if  $\widehat{r}_{IC}(\tau) < \underline{r}_j(\tau)$ . A simple extension of Lemma 1 shows that this profit is unimodal

increasing-decreasing in  $\tau > 0$  with a maximum at  $\tau_{(IC,i)}^*$ . However,  $\pi_i(\tau, IC)$  is not necessarily a plausible profit for all  $\tau$  as customer  $j$  will leave the participation set if  $\underline{r}_i(\tau) < \underline{r}_j(\tau)$ . Let  $\tau_{(IC,i,j)}$  be the point in  $\tau$  where  $\widehat{r}_{IC}(\tau)$  shifts from  $\underline{r}_i(\tau)$  to  $\underline{r}_j(\tau)$ . At this point we have  $\underline{r}_j'(\tau) > \underline{r}_i'(\tau)$  since  $AC_i(\tau)$  and  $AC_j(\tau)$  are differentiable and thus continuous and by definition  $\underline{r}_i(\tau + d\tau) < \underline{r}_j(\tau + d\tau)$ . If  $\pi_i(\tau, IC)$  is decreasing in  $\tau$  at  $\tau_{(IC,i,j)}$  then  $\pi_j(\tau, IC)$  will be decreasing in  $\tau$  at  $\tau_{(IC,i,j)}$  as well and at a faster rate since  $\underline{r}_j'(\tau) > \underline{r}_i'(\tau)$ . Since  $\pi_j(\tau, IC)$  is unimodal increasing-decreasing,  $\pi(\tau, IC)$  will be decreasing for all  $\tau > \tau_{(IC,i,j)}$ . This since the same argument can be repeated at all  $\tau > \tau_{(IC,i,j)}$  where the customer determining  $\widehat{r}_{IC}(\tau)$  shifts. This proves that once  $\pi(\tau, IC)$  start to decrease in  $\tau$  it will continue to do so and hence that it is unimodal increasing-decreasing in  $\tau > 0$ . The optimal  $\tau$  is either a  $\tau_{(IC,i)}^*$  or a  $\tau_{(IC,i,j)}$ . The former is apparent, the latter would be the case if  $\underline{r}_j'(\tau_{(IC,i,j)})$  is significantly larger than  $\underline{r}_i'(\tau_{(IC,i,j)})$ .  $\square$

## 8.5 Proof of Proposition 1

*Proof.* Part 1. For any set,  $IC_k \neq \emptyset$ , the local optimum,  $\tau_k^*$ , is unaffected by changes in  $c_D$  since the cost for direct transports are not a function of  $\tau$ . The revenue and the cost for direct transports increases with  $c_D$  though. The resulting shift in the profit  $\Delta\pi(\tau_k^*, IC_k)$  for participation set  $IC_k$  is  $\sum_{i \in IC_k} \mu_i \Delta c_D$ . Consequently,  $\Delta\pi(\tau_k^*, IC_k) > \Delta\pi(\tau_j^*, IC_j)$  if  $\sum_{i \in IC_k} \mu_i > \sum_{i \in IC_j} \mu_i$ , so a large enough increase in  $c_D$  will cause a shift from the set  $IC_j$  with the lower demand to set  $IC_k$  with a higher demand.

Under individual prices one include all customers for which  $\underline{r}_i(\tau) \leq c_D$  and if  $\tau_k^* \leq \tau_j^*$  then all customers in  $IC_j$  will also be customers in  $IC_k$  and the demand grows by adding customer  $k$  to the participation set, i.e. the set is increasing. To prove that  $\tau_k^* \leq \tau_j^*$  let us consider  $\underline{c}_D = \underline{r}_k(\tau_j^*)$ , i.e. the lowest  $c_D$  where the service provider is indifferent of including customer  $k$  in the participation set. If  $\tau_k^* > \tau_j^*$  the service providers profit can be increased by increasing the intershipment interval. However, if this is done she has to increase the rebate to all customers in the participation set as  $\underline{r}_i(\tau)$  is increasing in  $\tau$  for all  $i$ . In particular she has to give customer  $k$  a rebate  $r_k(\tau_j^*) > c_D$ . Such a high rebate is of-course non-optimal and thus we have that  $\tau_k^* \leq \tau_j^*$ .

Under standard price we cannot say anything with certainty as the ‘‘accept rebate’’ curves of the different customers might cross and differ in shape so an increase in  $c_D$  can lead to a new customer setting the rebate and shipping frequency and the new shipping frequency might lead to some other customers dropping out of the solution.

Part 2. An increase in  $c_C$  leads to an increase in  $\pi'(\tau, IC_i)$  for all  $i$ . Consequently, any  $\tau_i^*$  that is an internal point shifts to the right.  $\square$