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Some Distance Properties of Tailbiting Codes¹

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Abstract — The active tailbiting segment distance for convolutional codes is introduced. Together with the earlier defined active burst distance, it describes the error correcting capability of a tailbiting code encoded by a convolutional encoder. Lower bounds on the new active distance as well as an upper bound on the ratio between tailbiting length and memory of encoder such that its minimum distance d_{\min} equals the free distance d_{free} of the corresponding convolutional code are presented.

I. INTRODUCTION

Tailbiting codes can be obtained by terminating rate $R = b/c$ convolutional codes into block codes of length l c -tuples [1]. For simplicity, we consider only binary codes. The error correcting capability of a block code is estimated via d_{\min} . There is no description which error patterns with more than $\lfloor \frac{d_{\min}-1}{2} \rfloor$ errors can be corrected. In order to describe the error correcting capability of a tailbiting code beyond the minimum distance argument, we define the active tailbiting segment distance a_j^{tbs} . We also give an upper bound on the length of a tailbiting code so that d_{\min} equals d_{free} of the corresponding convolutional code. This is useful when analyzing concatenated coding schemes containing tailbiting encoders.

II. THE ACTIVE TAILBITING SEGMENT DISTANCE

Consider the convolutional code \mathcal{C} encoded by a rate $R = b/c$ encoder. The binary matrix σ_t denotes the encoder state at time t . Let $S_{[t_1, t_2]}^{\sigma_s, \sigma_e}$, $0 \leq t_1 < t_2$, be the set of state sequences $\sigma_{[t_1, t_2]} = \sigma_{t_1} \dots \sigma_{t_2}$ that start in state σ_s and terminate in state σ_e , and do not have two consecutive zero states with zero input in between. Let j_b be the smallest positive integer such that $S_{[0, j_b+1]}^{0,0} \neq \emptyset$. The j th order active burst distance [2] is $a_j^b \triangleq \min_{S_{[0, j+1]}^{0,0}} \{w_H(\mathbf{v}_{[0, j]})\}$, $j \geq j_b$. For any code \mathcal{C} , a_j^b is invariant over the set of its canonical encoders [2]. It is lower-bounded by a linearly increasing function $a_j^b \geq \alpha j + \beta^b$, $j \geq j_b$, where α is the asymptotic slope, and β^b is chosen as large as possible. Let j_r be the smallest number of steps that, starting in the allzero state, take us to any reachable state.

Definition 1 The j th order active tailbiting segment distance is $a_j^{tbs} \triangleq \min_{S_{[j_r, j_r+j+1]}^{\sigma, \sigma}} \{w_H(\mathbf{v}_{[j_r, j_r+j]})\}$, where σ denotes any possible encoder state.

Theorem 1 The active tailbiting segment distance is lower-bounded by $a_j^{tbs} \geq \alpha(j+1)$, for all $j \geq 0$.

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III. PROPERTIES OF TAILBITING CODES VIA THE ACTIVE DISTANCES

We define an *incorrect path* at the receiver to be any trellis path differing from the *transmitted path*. For any $k_1, k_2 < l$, let $e_{[k_1, k_2]}$ denote the Hamming weight of the error pattern $e_{[k_1, k_2]} = e_{k_1} e_{k_1+1} \dots e_{k_2}$, where e_i , $0 \leq i < l$, are c -tuples and all indices are evaluated modulo l . Then, we have

Theorem 2 Consider a tailbiting code \mathcal{C}^{tb} of length l c -tuples encoded by a convolutional encoder. Then, for $j_b < l$, a maximum likelihood (ML) decoder corrects all error patterns $e_{[0, l-1]}$ that satisfy $e_{[t, t+j \bmod l]} < \min\{a_j^b/2, a_{l-1}^{tbs}/2\}$ for $0 \leq t < l$, $j_b \leq j < l$. For $j_b \geq l$, all error patterns that satisfy $e_{[0, l-1]} < a_{l-1}^{tbs}/2$ are corrected.

Example 1 A tailbiting code of length $l = 18$ 2-tuples encoded by $G(D) = (1 + D + D^2 \quad 1 + D^2)$ with $d_{\min} = 5$ is used on a binary symmetric channel. From Theorem 2 follows that

$$e_{[0, 17]} = 10\ 00\ 00\ 01\ 00\ 00\ 00\ 00\ 01\ 00\ 00\ 00\ 10\ 00\ 00\ 00\ 00\ 00$$

is corrected by an ML decoder although the error pattern contains four channel errors which exceeds $\lfloor \frac{d_{\min}-1}{2} \rfloor$.

Consider a tailbiting code of length l c -tuples with minimum distance d_{\min} . The free tailbiting length l_{free} is the shortest length l for which d_{\min} will remain equal to d_{free} of the corresponding convolutional code for all tailbiting lengths greater than or equal to l_{free} . The free tailbiting length is upper-bounded by $l_{\text{free}} \leq \lfloor d_{\text{free}}/\alpha \rfloor$.

IV. ENSEMBLE PROPERTIES OF THE ACTIVE TAILBITING SEGMENT DISTANCE

The concept of the active distances can be generalized to time-varying convolutional encoders.

Theorem 3 There exists a rate $R = b/c$ convolutional code \mathcal{C} encoded by a time-varying encoder of memory m such that $a_j^{tbs} > \rho(j+1)c + O(\log m)$, for $j = O(m) \geq j_0$, $m \rightarrow \infty$

where $\rho = h^{-1}(1-R)$ is the Gilbert-Varshamov parameter, $h(\cdot)$ is the binary entropy function, and j_0 is the smallest integer satisfying $(1-R)(j+1)c \geq 4 \log m$.

Using the Heller asymptotic bound we obtain

Theorem 4 There exists a tailbiting code \mathcal{C}^{tb} encoded by a time-varying encoder of memory m , such that $\lim_{m \rightarrow \infty} l_{\text{free}}/m \leq \frac{1}{2\rho}$.

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