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# Active Distances and Cascaded Convolutional Codes<sup>1</sup>

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**Abstract** — A family of active distances for convolutional codes is introduced. Lower bounds are derived for the ensemble of periodically time-varying convolutional codes.

## I. INTRODUCTION

The "extended distances" were introduced by Thommesen and Justesen [1] for unit memory (UM) convolutional codes. We present (non-trivial) extensions to encoder memories  $m \geq 1$  and call them *active distances* since they stay "active" in the sense that we consider only those codewords which do not pass two consecutive zero states [2].

## II. ACTIVE DISTANCES

Consider the ensemble of binary, rate  $R = b/c$ , periodically time-varying convolutional codes encoded by a polynomial generator matrix of memory  $m$  and period  $T$ ,

$$G = \begin{pmatrix} G_0(t) & \cdots & G_m(t+m) \\ & G_0(t+1) & \cdots & G_m(t+m+1) \\ & & \ddots & \\ & & & G_0(t+T-1) & \cdots & G_m(t+m+T-1) \end{pmatrix} \quad (1)$$

in which each digit in each of the matrices  $G_i(t+T)$  for  $0 \leq i \leq m$  and  $0 \leq t \leq T-1$ , is chosen independently and equally likely to be 0 and 1.

Let  $\mathcal{U}_{[t_1-m, t_2+m]}^r$  be the set of information sequences  $u_{t_1-m} \dots u_{t_2+m}$  such that the first  $m$  and the last  $m$  subblocks are zero and they do not contain  $m+1$  consecutive zero subblocks.

Let  $\mathcal{U}_{[t_1-m, t_2]}^c$  be the set of information sequences  $u_{t_1-m} \dots u_{t_2}$  such that the first  $m$  subblocks are zero and they do not contain  $m+1$  consecutive zero subblocks.

Let  $\mathcal{U}_{[t_1-m, t_2]}^s$  be the set of information sequences  $u_{t_1-m} \dots u_{t_2}$  such that at least one subblock is nonzero and they do not contain  $m+1$  consecutive zero subblocks.

Next we introduce the truncated time-varying generator matrix

$$G_{[t, t+j]} = \begin{pmatrix} G_m(t) & & \\ \vdots & \ddots & \\ G_0(t) & & G_m(t+j) \\ & \ddots & \vdots \\ & & G_0(t+j) \end{pmatrix}. \quad (2)$$

**Definition 1** Let  $C$  be a time-varying convolutional code encoded by a time-varying, polynomial generator matrix. Then the  $j$ th order active row distance is

$$a_j^r \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}_{[t-m, t+j+m]}^r} w_H(u_{[t-m, t+j+m]} G_{[t, t+j+m]}), \quad (3)$$

the  $j$ th order active column distance is

$$a_j^c \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}_{[t-m, t+j]}^c} w_H(u_{[t-m, t+j]} G_{[t, t+j]}), \quad (4)$$

and the  $j$ th order active segment distance is

$$a_j^s \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}_{[t-m, t+j]}^s} w_H(u_{[t-m, t+j]} G_{[t, t+j]}). \quad (5)$$

For a convolutional code encoded by a time-varying, non-catastrophic, polynomial generator matrix we define its free distance as  $d_{\text{free}} \stackrel{\text{def}}{=} \min_j a_j^r$ .

## III. CASCADED CODES

Consider a scheme with two convolutional codes in cascade.

**Theorem 1** There exist cascaded convolutional codes in the ensemble of periodically time-varying cascaded convolutional codes whose active distance satisfies

$$\delta_l^r \stackrel{\text{def}}{=} \frac{a_j^r}{mc} \geq (l+1)h^{-1} \left(1 - \frac{l}{l+1}R\right) - O\left(\frac{\log_2 m}{m}\right) \quad (6)$$

for  $l \geq l_0^r = O(\frac{1}{m})$ ,

$$\delta_l^c \stackrel{\text{def}}{=} \frac{a_j^c}{mc} \geq lh^{-1}(1-R) - O\left(\frac{\log_2 m}{m}\right) \quad (7)$$

for  $l \geq l_0^c = O(\frac{\log_2 m}{m})$ , and

$$\delta_l^s \stackrel{\text{def}}{=} \frac{a_j^s}{mc} \geq lh^{-1} \left(1 - \frac{l+1}{l}R\right) - O\left(\frac{\log_2 m}{m}\right) \quad (8)$$

for  $l \geq l_0^s = \frac{R}{1-R} + O(\frac{\log_2 m}{m})$ .

By minimizing the lower bound on the active row distance we obtain nothing but the main term in Costello's lower bound on the free distance, viz.,  $\frac{R}{-\log_2(2^{1-R}-1)}$ .

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