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Lentmaier, Michael; Andriyanova, Iryna; Hassan, Najeeb; Fettweis, Gerhard

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Michael Lentmaier
Dept. of Electrical and
Information Technology
Lund University, Lund, Sweden
Email: michael.lentmaier@eit.lth.se

Iryna Andriyanova
ETIS group, ENSEA/UCP/CNRC-UMR8501
95014 Cergy-Pontoise, France
Email: iryna.andriyanova@ensea.fr

Najeeb ul Hassan
and Gerhard P. Fettweis
Vodafone Chair Mobile Comm. Systems
TU Dresden, Dresden, Germany
{najeeb_ul.hassan, fettweis}@tu-dresden.de

Abstract—**Spatially coupled codes are a class of capacity achieving channel codes which, like polar codes, have been studied within the NEWCOM# Network of Excellence. We present the concept of spatial coupling, discuss various features that makes it attractive and finally point out its potential for scenarios beyond channel coding and point-to-point communications.**

I. THE CONCEPT OF SPATIALLY COUPLED CODES

Consider transmission of a sequence of L codewords $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_L$ of length n. In conventional block coding, each of these codewords is encoded independently by means of some given code. Assuming, for example, a rate $R = n/k$ LDPC code defined by an $n - k \times n$ parity-check matrix $\mathbf{H}$, the codewords have to satisfy the condition $\mathbf{v}_t \cdot \mathbf{H}^T = \mathbf{0}$ for all $t = 1, \ldots, L$.

The fundamental idea of spatial coupling is that, instead of being encoded independently, the codewords $\mathbf{v}_t$ are interconnected (coupled) with their neighbors at times $t - 1, t - 2, \ldots, t - m$ during the encoding procedure. This is done in such a way that the sequence satisfies the condition

$$\mathbf{v}_t \cdot \mathbf{H}_0^T(t) + \mathbf{v}_{t-1} \cdot \mathbf{H}_1^T(t) + \cdots + \mathbf{v}_{t-m} \cdot \mathbf{H}_m^T(t) = \mathbf{0} \quad ,$$

where the matrices $\mathbf{H}_i(t)$, $i = 0, \ldots, m$ result from a decomposition of the original block code matrix, i.e., $\mathbf{H}_0(t) + \mathbf{H}_1(t) + \cdots + \mathbf{H}_m(t) = \mathbf{H} \ \forall t$.

It follows from the construction that spatially coupled LDPC (SC-LDPC) codes have a convolutional code structure, where the parameter m defines the corresponding memory. The decomposition of the parity-check matrix (i.e., the spreading of edges in the Tanner graph over different time instants) can be done in different ways, resulting in different ensembles of spatially coupled codes [1]–[3]. Fig. 1 illustrates the coupling of a $(3, 6)$-regular LDPC code ensemble based on a protograph.

It should be emphasized that spatially coupled codes are not just yet another particular code construction. Spatial coupling is a general concept that can be applied to different existing (and future) code constructions and it is not limited at all to binary LDPC codes. For example, this concept has been applied to non-binary LDPC codes [4], product codes [5], [6] and turbo codes (both serial and parallel concatenation) [7].
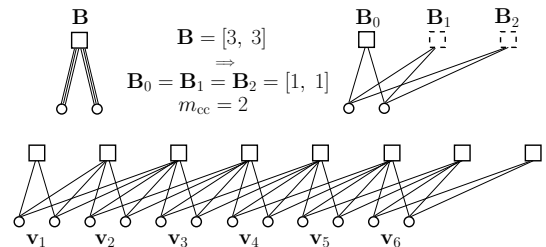
Fig. 1. Illustration of coupling: the protograph of a (3,6)-regular block code represented by a base matrix $\mathbf{B}$ is repeated $L = 6$ times and the edges are spread over time according to the component base matrices $\mathbf{B}_0$, $\mathbf{B}_1$, and $\mathbf{B}_2$.

II. PROPERTIES OF SPATIALLY COUPLED CODES

A. Threshold Saturation

A belief propagation (BP) decoding analysis of SC-LDPC codes shows that the performance of the iterative decoder is improved significantly by spatial coupling. In fact, the results in [2] show that asymptotically, as L tends to infinity, the BP threshold is boosted to that of the optimal maximum a posteriori (MAP) decoder. Stimulated by these findings, Kudekar, Richardson and Urbanke developed an analytical proof of this *threshold saturation* phenomenon [3], [8]. Since the MAP thresholds of regular LDPC ensembles with increasing node degrees are known to converge to capacity, it follows that spatial coupling provides a new way of provably achieving capacity with low-complexity iterative BP decoding. Furthermore, the spatially coupled code ensembles inherit from the uncoupled counterparts the linearly increasing minimum distance property [9]. This combination of capacity achieving thresholds with low complexity decoding and linearly increasing distance is quite unique and has attracted a lot of interest in the research community.

B. Locality: Efficient Encoding and Decoding

One consequence of interconnecting a sequence of codewords is that the encoding and decoding cannot be performed independently anymore. It is tempting to treat a spatially coupled code as another block code of length $n_L = n \cdot L$. This interpretation is possible, but it does not take into account the locality property that is inherent in the code construction. From the convolutional code structure it follows that the minimum distance and the ML decoding performance depends on the block size n and the memory m, but not on the length L of
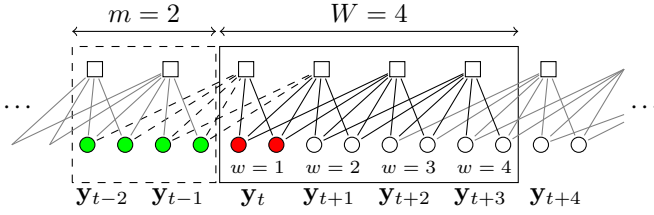
Fig. 2. Window decoder of size $W = 4$ at time t. The green variable nodes represent decoded blocks and the red variable nodes ($\mathbf{y}_t$) are the target block within the current window. The dashed lines represent the read access to the m previously decoded blocks.

the sequence, provided that L is significantly larger than m. The strength of a code increases with product $n \cdot m$, which determines the constraint length.

As a natural consequence, it is desirable to operate with encoder and decoder structures that are independent of L in terms of complexity, storage requirements and latency. This can be achieved by means of a sliding window decoder, which operates on a region of W codewords, i.e., $n \cdot W$ symbols. An example of a window decoder of size $W = 4$ is given in Fig. 2. It has been shown in [10] that for equal structural latency, SC-LDPC codes under window decoding outperform LDPC codes for short to long latency values and outperform convolutional codes from medium to long latency values. Another advantage of using a window decoder is the flexibility at the decoder. Since the window size W is a decoder parameter, it can be varied without changing the code, providing a flexible trade-off between performance and latency [10].

C. Universality and Robustness

Another remarkable feature of spatially coupled codes is their universality property, which means that a single code construction performs well for a large variety of channel conditions. For general binary-input memoryless symmetric channels the universality of SC-LDPC codes has been shown in [8]. The performance of SC-LDPC codes over the block fading channel was analyzed in [11]. It turns out that the diversity order of the code can be increased, without lowering the code rate, by simply increasing the coupling parameter (memory) of an SC-LDPC code. For a (3,6)-regular SC-LDPC code with rate $R = 1/2$ and memory $m = 4$ a remarkable diversity of $d = 10$ is achieved without the need for any specific code structure. The memory of the SC-LDPC codes makes them robust against a non-stationary mobile-radio environment.

III. SPATIAL COUPLING BEYOND CODING

Iterative algorithms are widely used for improving the performance of communication systems. Different locally operating components of the receiver exchange messages with each other in order to approximate the optimal global solution. The key is that the complexity of such a receiver is still in the order of the individual components, while an optimal receiver would be prohibitively complex. Whenever such an algorithm can be described by means of a graphical model, it is possible to apply the concept of spatial coupling on the corresponding graph. It turns out that the threshold saturation phenomenon and the universality and robustness of spatially coupled systems can be observed for a wide range of scenarios.

We conclude this short overview by naming a few examples that may inspire some readers to find applications of this concept within their own area of research. In [12] spatially coupled codes are considered for modulation and detection. In an EXIT chart analysis it is observed that the receiver becomes more robust against varying detector characteristics. Similar observations are made in [13] for a MIMO system with linear precoding. Regarding multi-user scenarios, the Gaussian multiple-access channel (MAC) is considered in [14]. It is shown that simple regular SC-LDPC codes achieve nearly-universal performance. In [15] a multiple access demodulation scheme is analyzed from an interference cancellation point of view and it is shown that spatial coupling can achieve nearly optimal performance.

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