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Published in:
[Host publication title missing]

DOI:
[10.1109/ISIT.2001.936075](https://doi.org/10.1109/ISIT.2001.936075)

2001

[Link to publication](#)

Citation for published version (APA):

Jordan, R., Höst, S., & Johannesson, R. (2001). On interleaver design for serially concatenated convolutional codes. In [Host publication title missing] (pp. 212) <https://doi.org/10.1109/ISIT.2001.936075>

Total number of authors:
3

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On Interleaver Design for Serially Concatenated Convolutional Codes¹

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Abstract—Serially concatenated convolutional codes are considered. The free distance of this construction is shown to be lower-bounded by the product of the free distances of the outer and inner codes, if the precipices of the interleaver are sufficiently large. It is shown how to construct a convolutional scrambler with a given precipice.

I. INTRODUCTION

An interleaver is a single input, single output, causal device which produces the output sequence $\mathbf{y} = \dots y_{-1}y_0y_1\dots = \dots x_{\pi(-1)}x_{\pi(0)}x_{\pi(1)}\dots$, that is, a permutation of the input sequence $\mathbf{x} = \dots x_{-1}x_0x_1\dots$. The invertible function π denotes the permutation on the input sequence indices, i.e., the output symbol y_j at depth j is the $\pi(j)$ th symbol $x_{\pi(j)}$ of the input sequence. The interleaver delay is given by $\delta = \max_j \{j - \pi(j)\}$.

The set of separations [1] (s, t) of an interleaver with permutation π is given by

$$\{(s, t) \mid |\pi(j) - \pi(j')| < s \Rightarrow |j - j'| \geq t, \forall j \neq j'\}$$

That is, two symbols positioned within an interval of length s in the input sequence are guaranteed to be separated by at least $t - 1$ positions in the output sequence. Clearly, if the interleaver has the separation (s, t) , then the corresponding deinterleaver has the separation (t, s) . Furthermore, the precipice $(s, t)_p$ is a separation (s, t) such that neither $(s+1, t)$ nor $(s, t+1)$ do exist in the set of separations. In general, an interleaver can have several precipices.

We use the concept of *convolutional interleaving* to describe the interleaver by a *convolutional scrambler* [2].

Definition 1 An infinite matrix $\mathbf{S} = (s_{ij})$, $i, j \in \mathbb{Z}$, that has one 1 in each row and one 1 in each column and that satisfies $s_{ij} = 0$, $i > j$ is called a *convolutional scrambler*.

The interleaved sequence is then given by $\mathbf{y} = \mathbf{x}\mathbf{S}$.

II. SERIALY CONCATENATED CONVOLUTIONAL CODES

Consider a serial concatenation of two convolutional encoders with a convolutional scrambler in between.

Theorem 1 Let d_{free} be the free distance of a serially concatenated convolutional code. If the interleaver has at least one precipice $(s, t)_p$ that satisfies the inequalities

$$s \geq \min\{(j_{\text{free}}^{\text{co}} + 1)c_o, (j_{\text{free}}^{\text{rco}} + 1)c_o\}$$

$$t \geq j_{2\text{free}}^{\text{bi}}b_i$$

¹This work was supported in part by the Swedish Research Council for Engineering Sciences under Grant 97-723 and in part by the German Research Council Deutsche Forschungsgemeinschaft under Grant Bo 867/8.

then

$$d_{\text{free}} \geq d_{\text{free}}^o d_{\text{free}}^i$$

where d_{free}^o and d_{free}^i denote the free distance of the outer and inner convolutional codes, respectively, and $j_{\text{free}}^{\text{co}}$, $j_{\text{free}}^{\text{rco}}$, and $j_{2\text{free}}^{\text{bi}}$ are derived from the active distances [3].

III. THE (q, r) CONVOLUTIONAL SCRAMBLER

A (q, r) convolutional scrambler is a convolutional scrambler $\mathbf{S}_{(q,r)} = (s_{ij})$ with

$$s_{ij} = 1, \quad j = i + R_r(iq), \quad q + 1 < r$$

where $\gcd(q+1, r) = 1$ and $R_{(q+1)}(r) = 1$. The period of this scrambler is $T = r$ and the delay is $\delta = r - 1$.

Theorem 2 Given a (q, r) convolutional scrambler, then

$$(s, t) = \left(\frac{r-1}{q+1}, q+1 \right)$$

is a precipice.

Example 1 Consider the $(3, 13)$ convolutional scrambler. It has period $T = 13$ and delay $\delta = 12$. There is one precipice at $(s, t)_p = (3, 4)$. Thus, all symbols within a segment of size three in the input sequence are separated by at least three bits in the output sequence.

The (q, r) convolutional scrambler provides the possibility to realize a convolutional scrambler for a given precipice $(s, t)_p$ by letting $q = t - 1$ and $r = st + 1$. This gives an interleaver with interleaver delay $\delta = st$, which is the minimal required interleaver delay for the considered precipice.

Example 2 Consider a serially concatenated convolutional code generated with one inner rate $R_i = 1/2$ encoder with $d_{\text{free}}^i = 5$ and one outer rate $R_o = 2/3$ encoder with $d_{\text{free}}^o = 3$, both with overall constraint length $\nu = 2$. This construction gives a free distance of $d_{\text{free}} = 15$. The required precipice is $(s, t)_p = (13, 12)$, hence, an interleaver with delay of $\delta = 156$ is required. This corresponds to 78 information bits.

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