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On Interleaver Design for Serially Concatenated Convolutional Codes[1]

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Abstract—**Serially concatenated convolutional codes are considered. The free distance of this construction is shown to be lower-bounded by the product of the free distances of the outer and inner codes, if the precipices of the interleaver are sufficiently large. It is shown how to construct a convolutional scrambler with a given precipice.**

I. Introduction

An interleaver is a single input, single output, causal device which produces the output sequence $\boldsymbol{y} = \dots y_{-1}y_0y_1\dots = \dots x_{\pi(-1)}x_{\pi(0)}x_{\pi(1)}\dots$, that is, a permutation of the input sequence $\boldsymbol{x} = \dots x_{-1}x_0x_1\dots$. The invertible function π denotes the permutation on the input sequence indices, i.e., the output symbol y_j at depth j is the $\pi(j)th$ symbol $x_{\pi(j)}$ of the input sequence. The *interleaver delay* is given by $\delta = \max_j\{j - \pi(j)\}$.

The set of *separations* [1] (s,t) of an interleaver with permutation π is given by

$$\{(s,t) \mid |\pi(j) - \pi(j')| < s \Rightarrow |j - j'| \geq t,\ \forall j \neq j'\}$$

That is, two symbols positioned within an interval of length s in the input sequence are guaranteed to be separated by at least $t-1$ positions in the output sequence. Clearly, if the interleaver has the separation (s,t), then the corresponding deinterleaver has the separation (t,s). Furthermore, the *precipice* $(s,t)_p$ is a separation (s,t) such that neither $(s+1,t)$ nor $(s,t+1)$ do exist in the set of separations. In general, an interleaver can have several precipices.

We use the concept of *convolutional interleaving* to describe the interleaver by a *convolutional scrambler* [2].

Definition 1 *An infinite matrix $\boldsymbol{S} = (s_{ij}),\ i,j \in \mathbb{Z}$, that has one 1 in each row and one 1 in each column and that satisfies $s_{ij} = 0,\ i > j$ is called a convolutional scrambler.*

The interleaved sequence is then given by $\boldsymbol{y} = \boldsymbol{x}\boldsymbol{S}$.

II. Serially Concatenated Convolutional Codes

Consider a serial concatenation of two convolutional encoders with a convolutional scrambler in between.

Theorem 1 *Let d_{free} be the free distance of a serially concatenated convolutional code. If the interleaver has at least one precipice $(s,t)_p$ that satisfies the inequalities*

$$s \geq \min\{(j^{co}_{\text{free}} + 1)c_o,\ (j^{rco}_{\text{free}} + 1)c_o\}$$
$$t \geq j^{bi}_{2\text{free}} b_i$$

then

$$d_{\text{free}} \geq d^o_{\text{free}} d^i_{\text{free}}$$

where d^o_{free} and d^i_{free} denote the free distance of the outer and inner convolutional codes, respectively, and j^{co}_{free}, j^{rco}_{free}, and $j^{bi}_{2\text{free}}$ are derived from the active distances [3].

III. The (q,r) Convolutional Scrambler

A (q,r) convolutional scrambler is a convolutional scrambler $\boldsymbol{S}_{(q,r)} = (s_{ij})$ with

$$s_{ij} = 1, \quad j = i + R_r(iq),\ q+1 < r$$

where $\gcd(q+1,r) = 1$ and $R_{(q+1)}(r) = 1$. The period of this scrambler is $T = r$ and the delay is $\delta = r - 1$.

Theorem 2 *Given a (q,r) convolutional scrambler, then*

$$(s,t) = \left(\frac{r-1}{q+1},\ q+1\right)$$

is a precipice.

Example 1 *Consider the $(3,13)$ convolutional scrambler. It has period $T = 13$ and delay $\delta = 12$. There is one precipice at $(s,t)_p = (3,4)$. Thus, all symbols within a segment of size three in the input sequence are separated by at least three bits in the output sequence.*

The (q,r) convolutional scrambler provides the possibility to realize a convolutional scrambler for a given precipice $(s,t)_p$ by letting $q = t-1$ and $r = st+1$. This gives an interleaver with interleaver delay $\delta = st$, which is the minimal required interleaver delay for the considered precipice.

Example 2 *Consider a serially concatenated convolutional code generated with one inner rate $R_i = 1/2$ encoder with $d^i_{\text{free}} = 5$ and one outer rate $R_o = 2/3$ encoder with $d^i_{\text{free}} = 3$, both with overall constraint length $\nu = 2$. This construction gives a free distance of $d_{\text{free}} = 15$. The required precipice is $(s,t)_p = (13,12)$, hence, an interleaver with delay of $\delta = 156$ is required. This corresponds to 78 information bits.*

References

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