What is wrong with pure mode I and II? A lot it seems - Discussion of fracture paper #16

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It is common practice when solving boundary value problems to split the solution into a symmetric and an antisymmetric part to temporarily reduce the number of variables and the mathematical administration. As soon as the symmetric problem is solved, the antisymmetric problem, or vice versa, is almost solving itself. Any problem can be split into a symmetric and an antisymmetric part which is a relief for anyone who analyses mixed cases.
It gives a clearer view but it is an academic exercise while nature usually doesn't have any comprehension of symmetry and antisymmetry. Fracture is no exception. The fracture processes will be activated when sufficient conditions are fulfilled. Even the smallest deviation from the pure mode I or II caused by geometry or load will not affect the conditions at the crack tip in any decisive way. Everything is almost pure mode I or II and it may be convenient ignore the small deviation and still treat the problem as a pure case. This seems simple enough but the paper reviewed tells that it has been a tripwire for many. The selected paper is the recently published:
"An improved definition for mode I and mode II crack problems" by M.R. Ayatollahi, M. Zakeri in Engineering Fracture Mechanics 175 (2017) 235–246.
The authors examine a power series expansion for an Airy stress function about the crack tip. The series give stress as a sum of powers r-1/2, 1, r1/2, r, etc. of the distance to the crack tip. Each term has an known angular dependence. The application is to a plane crack with any in-plane load. The series starts with a square root singular term while it is assumed that the crack tip is sharp and the material is linear elastic. The assumption requires that the geometrical features of the crack tip and the nonlinear region is not visible from where the expansion with some accuracy describes the stress field. The problem that the authors emphasise is that the splitting in symmetric and antisymmetric modes that leads to two similar expansions of the radial power functions with symmetric and antisymmetric angular functions. The representations so far has been called pure if the solution is strictly symmetric or strictly antisymmetric, i.e. the notation has been pure mode I and pure mode II. The problem is that not only seldom, has a vanishing mode I stress

intensity factor misled investigators to drop all symmetric terms of the series expansion. Also mode II has been unfairly treated in the same way. The most striking problem is of course when the constant stress acting along the crack plane, the T-stress, by mistake is neglected. The authors are doing a nice work sorting this out. They describe a range of cases where one stress intensity factor vanishes but for sure the crack tip stress state is neither strictly symmetric nor strictly antisymmetric. They also provide quite many examples to demonstrate the necessity to consider the T-stress even if the mode I singular stress term is absent. I commend the authors for doing a conscientious work.

If I should bring up something where different positions may be assumed it would be the selection of the series. The powers of r^{-1}, $r^{-3/2}$, r^{-2} etc are never mentioned and I agree that it is not always necessary. It should be commonly known that a sharp crack, a linear elastic material and traction free crack surfaces says it. There cannot be any stronger singularities than $r^{-1/2}$. However, isn't one consequence that close enough to the crack tip any constant stress should be insignificant as compared to the singular stress terms. If so, it should not have any significant effect on the stresses closest to the crack tip and neither affect the fracture processes nor the selection of crack path. On the other hand, if the constant term has a real influence on the course of events, that would as far as I understand mean that the nonlinear region has to have a substantial extent so that its state is given by both singular terms and the T-stress. The contradiction is then that the stronger singular terms r^{-1}, $r^{-3/2}$, etc. cannot be neglected. These terms are there. Already the r^{-1} term seems obvious if the crack has grown because of the residual stress caused by plastic strain along the crack surface that in the wake region behind the crack tip.

Also, the region of convergence, which is at most the length of the crack, is another pothole. Outside the convergence region a different series or an analytical continuation, may be used. For the series expansions the symmetric and antisymmetric solutions have to be treated as well, with the difference that there are constant stresses in both symmetric and antisymmetric modes that have to be included.

It would be interesting to hear if there are any thoughts regarding this.

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P.S. On the courtesy of Elsevier there is a 3 month promotional access to the latest article in the blog, meaning the articles are freely available to everyone. Now everyone who wishes to comment or discuss the paper here can do so. (Dr. Kumar, I hope you are reading this).