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On the spectral properties of the exterior Calderón operator

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In this paper we characterize the spectrum of the exterior Calderón operator which maps the tangential scattered electric field to the tangential scattered magnetic field on the boundary of a scattering obstacle, Ω , which is assumed to be an open and bounded domain in \mathbb{R}^3 with a simply connected Lipschitz boundary Γ . We denote the simply connected exterior of the domain Ω by $\Omega_e = \mathbb{R}^3 \setminus \overline{\Omega}$. Consider the following exterior problem where the trace of the scattered electric field on the boundary is given by a fixed vector $\mathbf{m} \in H^{-1/2}(\operatorname{div}, \Gamma)$,

1)
$$(\boldsymbol{E}_{sc}, \boldsymbol{H}_{sc}) \in H_{loc}(\operatorname{curl}, \Omega_{e}) \times H_{loc}(\operatorname{curl}, \Omega_{e})$$

2) $\begin{cases} \nabla \times \boldsymbol{E}_{sc}(\boldsymbol{x}) = \operatorname{i} \boldsymbol{k} \boldsymbol{H}_{sc}(\boldsymbol{x}) \\ \nabla \times \boldsymbol{H}_{sc}(\boldsymbol{x}) = -\operatorname{i} \boldsymbol{k} \boldsymbol{E}_{sc}(\boldsymbol{x}) & \boldsymbol{x} \in \Omega_{e} \end{cases}$
3) $\begin{cases} \hat{\boldsymbol{x}} \times \boldsymbol{E}_{sc}(\boldsymbol{x}) - \boldsymbol{H}_{sc}(\boldsymbol{x}) = o(1/x) \\ \operatorname{or} & \operatorname{as} x \to \infty, \quad \text{uniformly w.r.t. } \hat{\boldsymbol{x}} \\ \hat{\boldsymbol{x}} \times \boldsymbol{H}_{sc}(\boldsymbol{x}) + \boldsymbol{E}_{sc}(\boldsymbol{x}) = o(1/x) \end{cases}$
4) $\boldsymbol{\gamma}(\boldsymbol{E}_{sc}) = \boldsymbol{m} \in H^{-1/2}(\operatorname{div}, \Gamma)$

where $x = |\mathbf{x}|$. The wave number $k = \omega/c$ is assumed to be a positive constant, where ω is the angular frequency of the fields, and *c* is the speed of light in the exterior medium. The trace operator $\boldsymbol{\gamma}$ on $C(\overline{\Omega_e}; \mathbb{C}^3)$ is given by $\boldsymbol{\gamma}(\boldsymbol{u}) = \hat{\boldsymbol{\nu}} \times \boldsymbol{u}|_{\partial\Omega}$. In the case that \boldsymbol{u} belongs to $H_{\text{loc}}(\text{curl}, \overline{\Omega_e})$, the fields have traces on Γ belonging to $H^{-1/2}(\text{div}, \Gamma)$, more precisely we have $(\boldsymbol{\gamma}(\boldsymbol{E}_{\text{sc}}), \boldsymbol{\gamma}(\boldsymbol{H}_{\text{sc}})) \in H^{-1/2}(\text{div}, \Gamma) \times H^{-1/2}(\text{div}, \Gamma)$, see [1] for the definition and the properties of the trace operators in $H_{\text{loc}}(\text{curl}, \overline{\Omega_e})$. Problem (1) has a unique solution [2, 3, 4] and the exterior Calderón operator \mathbf{C}^{e} is defined as

$$\mathbf{C}^{\mathbf{e}}: \boldsymbol{m} \mapsto \boldsymbol{\gamma}(\boldsymbol{H}_{\mathrm{sc}}), \qquad H^{-1/2}(\mathrm{div}, \Gamma) \to H^{-1/2}(\mathrm{div}, \Gamma),$$

where $\mathbf{m} = \mathbf{\gamma}(\mathbf{E}_{sc})$ and the fields \mathbf{E}_{sc} and \mathbf{H}_{sc} satisfy (1). The point spectrum of the exterior Calderón operator is $P\sigma(\mathbf{C}^{e}) = \{-i, i\}, i.e.$, eigenvalues $\lambda = \pm i$ and eigenvectors $\mathbf{m} \in H^{-1/2}(\operatorname{div}, \Gamma)$ satisfy

 $\mathbf{C}^{\mathbf{e}}\boldsymbol{m} = \lambda \boldsymbol{m}$

where $\boldsymbol{m}_{\tau n}^{\pm} = \boldsymbol{Y}_{\tau n} \mp i \mathbf{C}^{\mathbf{e}} \boldsymbol{Y}_{\tau n}$ and $\boldsymbol{Y}_{\tau n}$ are the generalized spherical harmonics defined in [5].

References

- [1] J.-C. Nédélec, Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems. Applied Mathematical Sciences — Vol. 144, Berlin: Springer-Verlag, 2001.
- [2] M. Artola, "Homogenization and electromagnetic wave propagation in composite media with high conductivity inclusions," in *Proceedings of the Second Workshop on Composite Media & Homogenization Theory* (G. Dal Maso and G. Dell'Antonio, eds.), (Singapore), pp. 1–16, World Scientific Publisher, 1995.
- [3] M. Cessenat, *Mathematical Methods in Electromagnetism*. Series on Advances in Mathematics for Applied Sciences — Vol. 41, Singapore: World Scientific Publisher, 1996.
- [4] A. Kirsch and F. Hettlich, *The Mathematical Theory of Time-Harmonic Maxwell's Equations*. Applied Mathematical Sciences Vol. 190, New York: Springer-Verlag, 2015.
- [5] G. Kristensson, I. G. Stratis, N. Wellander, and A. N. Yannacopoulos, "The exterior calderón operator for nonspherical objects," Tech. Rep. LUTEDX/(TEAT-7259)/1–43/(2017), Lund University, Department of Electrical and Information Technology, P.O. Box 118, S-221 00 Lund, Sweden, 2017. http://www.eit.lth.se.