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## Fracture as a Moving Boundary Problem

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# Fracture as a Moving Boundary Problem

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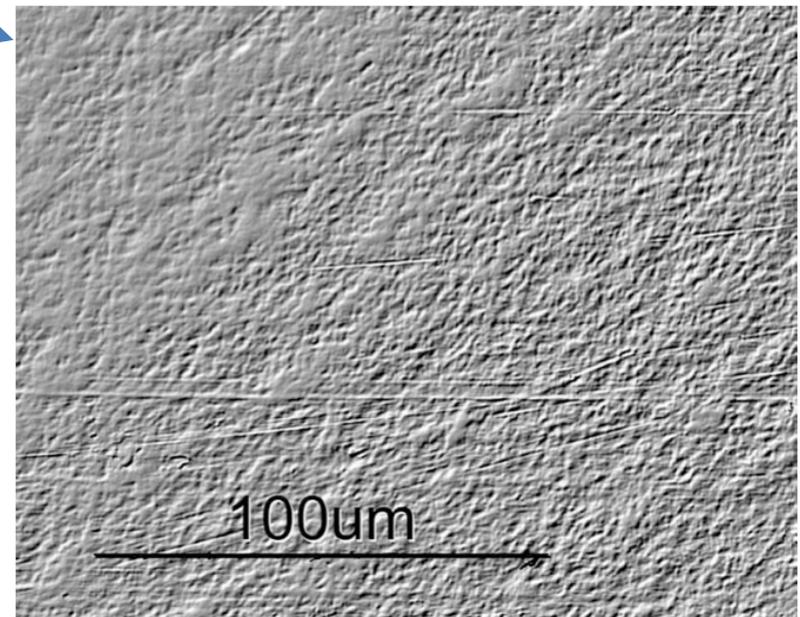
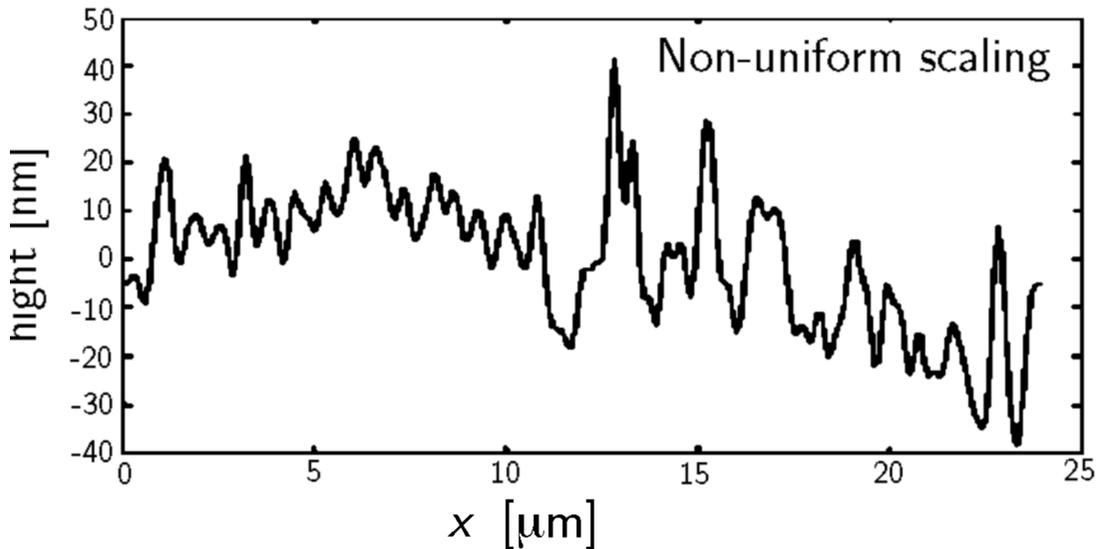
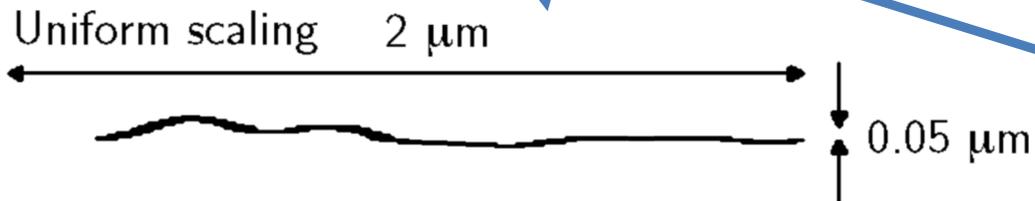
ECF19, Kazan 2012

# Corroding environment leads to:

1. Continuous loss of mass
2. Pitting
- ... and with mechanical stress present
3. Surface roughening



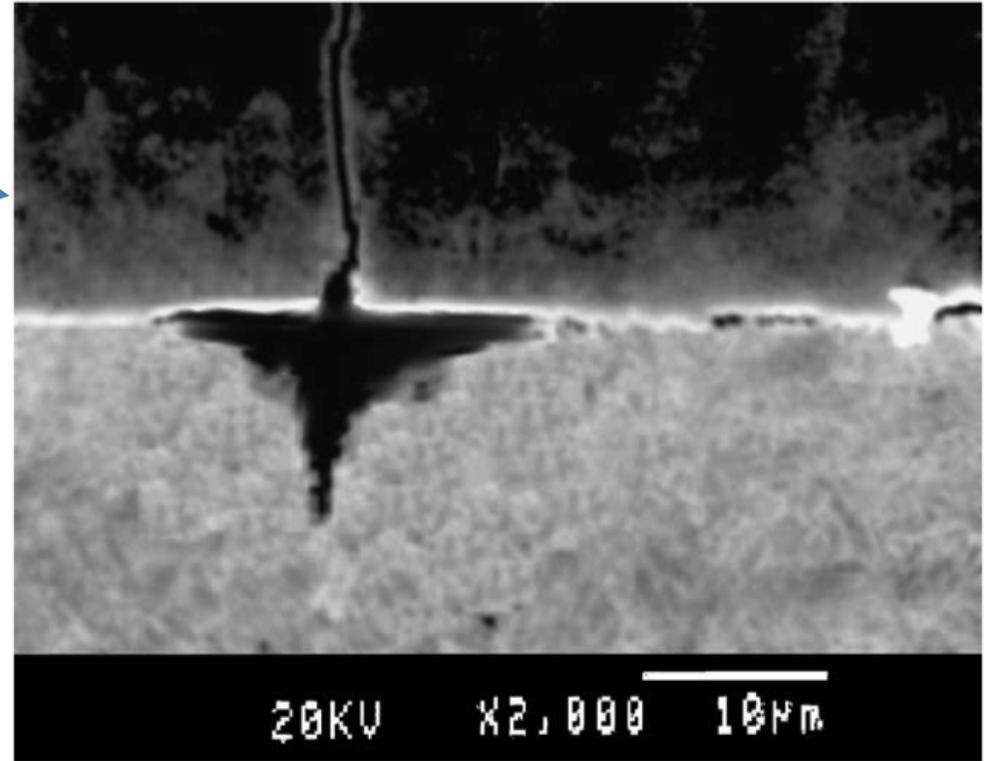
(from Kung-Suk 2000)



4. Evolving pits
5. Formation of cracks
6. Crack growth
7. Crack branching

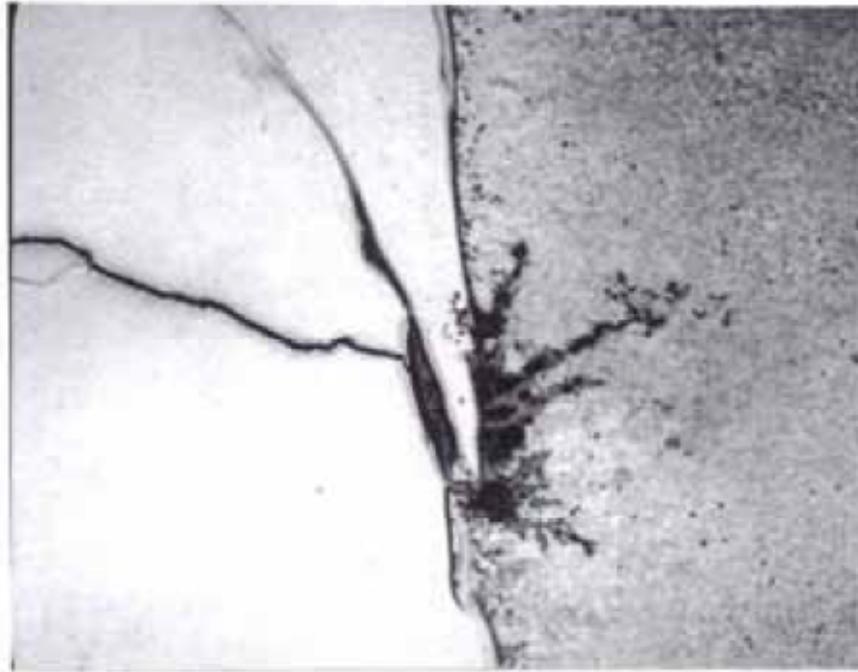


Growing crack in a polycarbonate exposed to acetone (Hejman 2011)



Cr/zon six charge related of land and groove substrate erosion through a micro-crack at the 12:00 bore origin. (Sopok *et al.* 2005)

# Corrosion Crack crossing a bi-material interface



Corrosion crack penetrating a bimaterial interface between austenitic and pressure vessel steel of type SA533C11.



The tip of one of the crack branches. Crack length 7 mm, notch width 10  $\mu\text{m}$ . *Reproduced with permission from Vattenfall AB.*

# Evolving Surface Morphology

Asaro-Tiller (1972), Grinfeld (1986, 1993), Srolovitz (1989), Freund (1995), Kim (2000)

Gibb's free energy

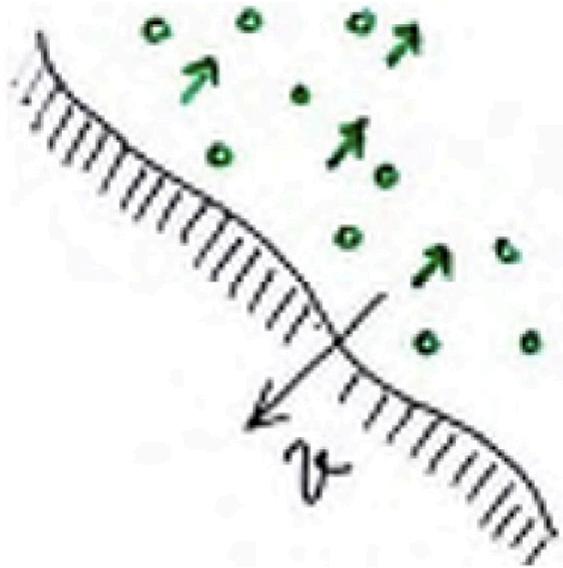
$$\Phi = U_c + U_e$$

where

$U_c$  is the free chemical energy and

$U_e$  is the free elastic energy

# Evaporation-condensation



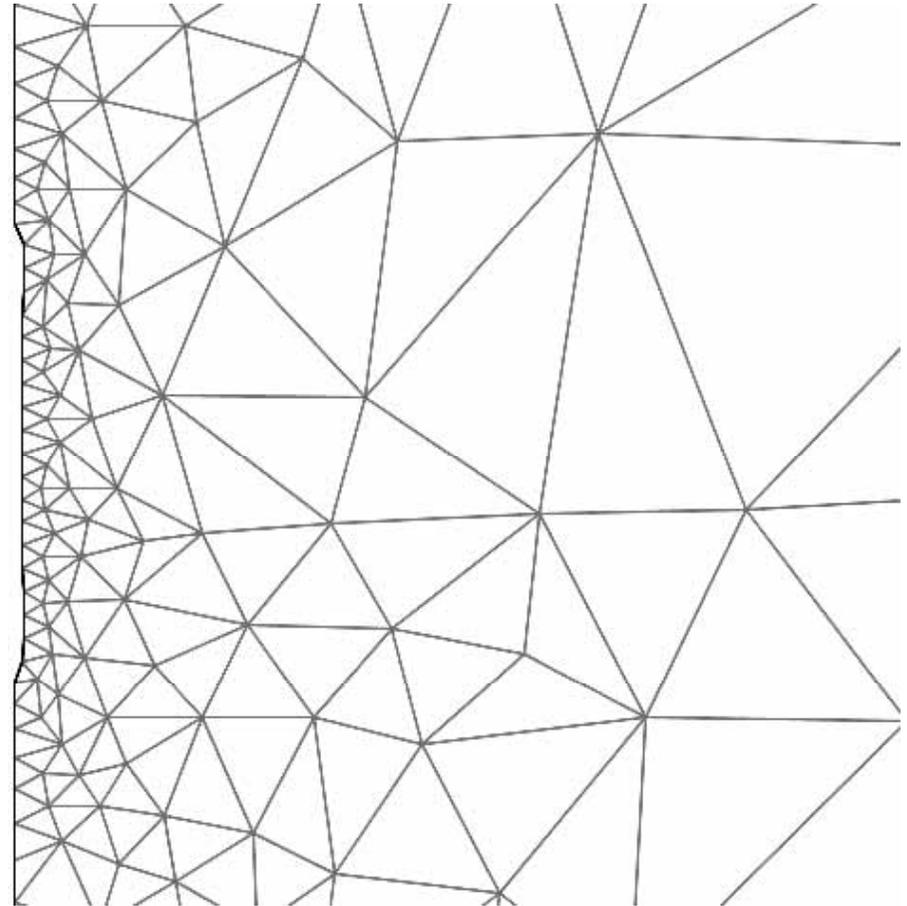
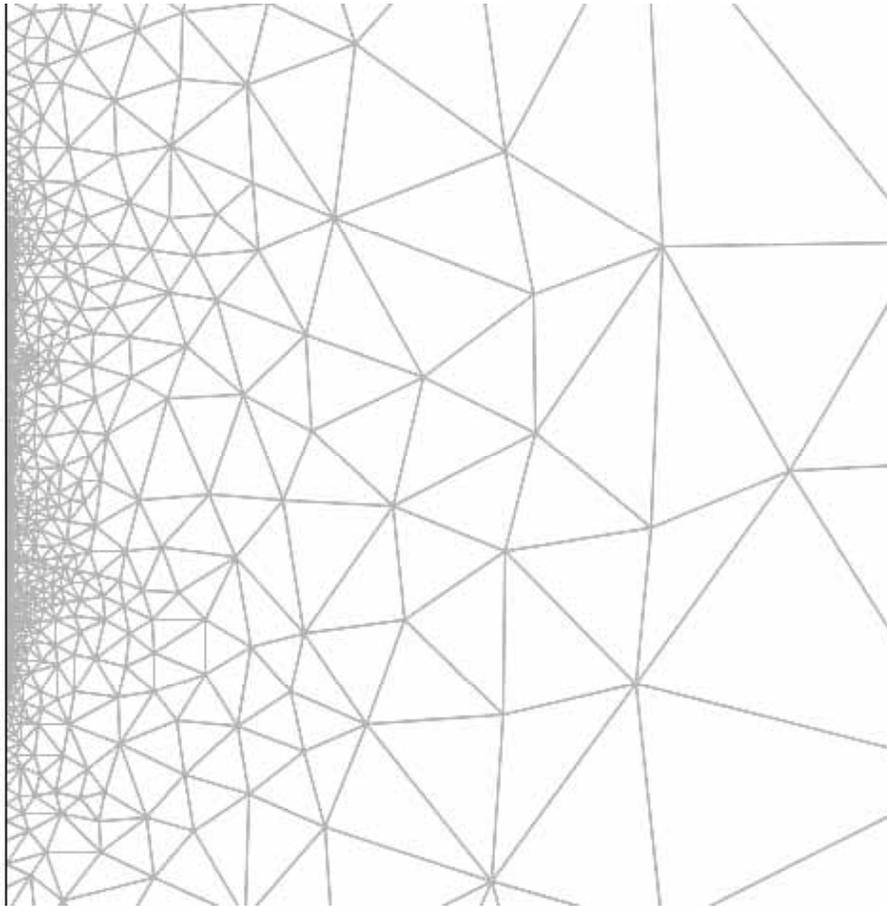
$$\frac{\partial h}{\partial t} = -L_1 \Phi$$

# Surface diffusion

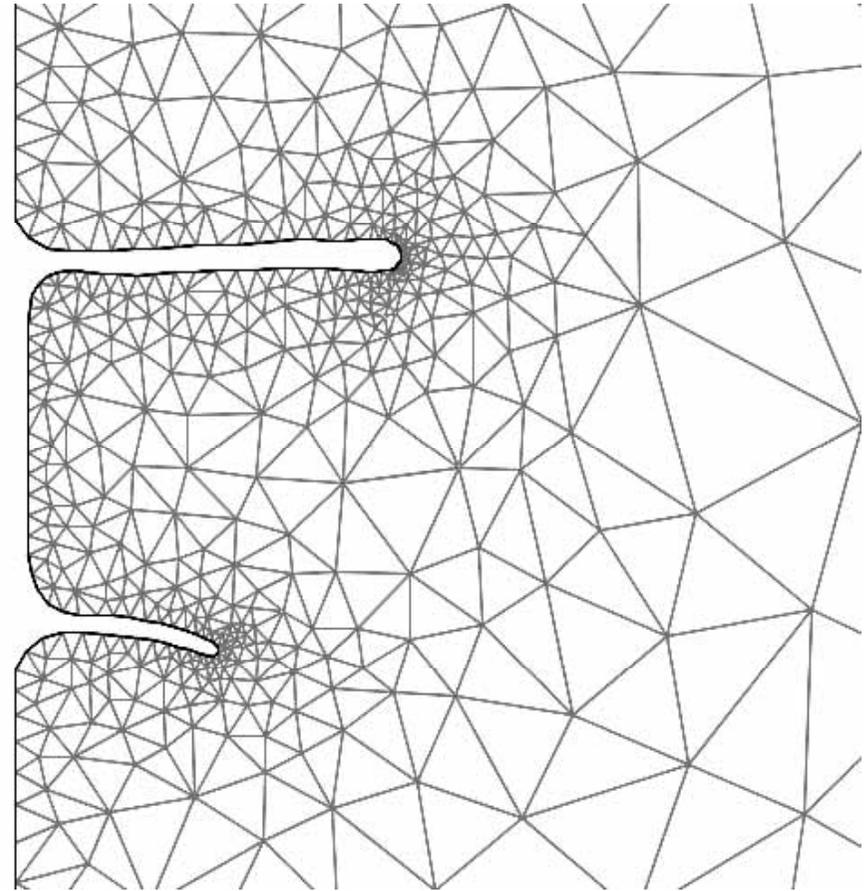
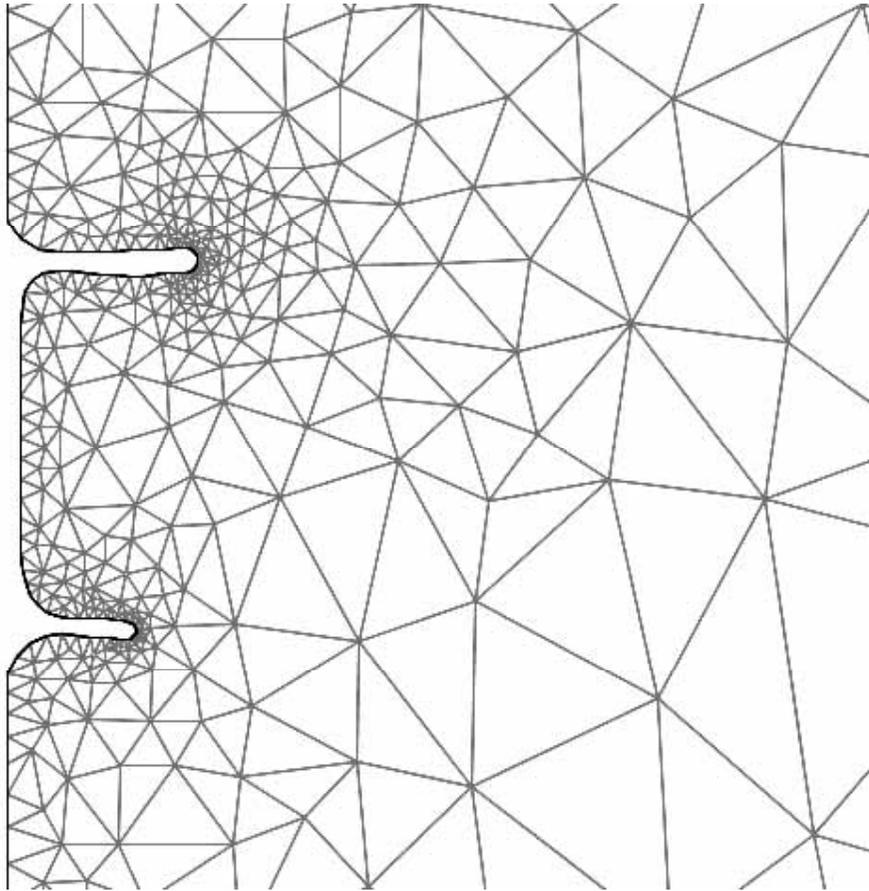


$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2 \Phi}{\partial x^2}$$

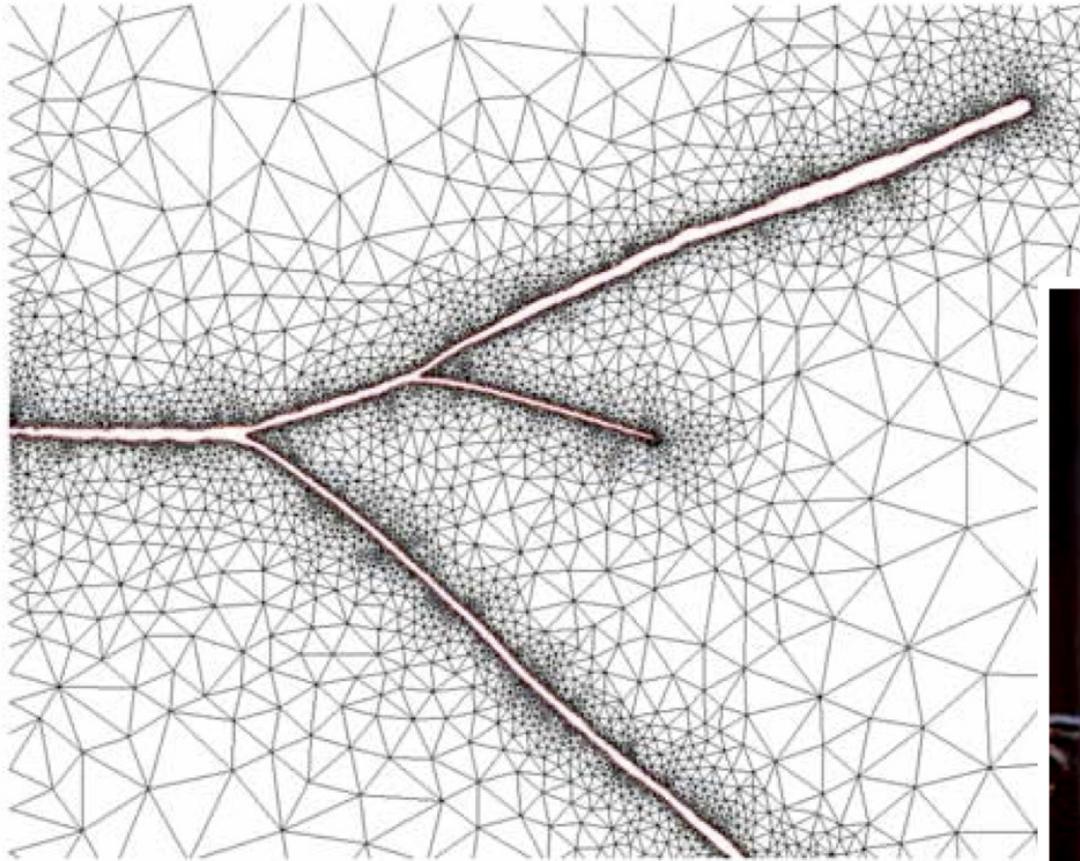
# Corroding Surface



# Corroding Surface



# Branching



Landau potential:

$$\mathcal{F} = \mathcal{F}_c + \mathcal{F}_e + \mathcal{F}_{gr} ; \text{ Ginzburg, Landau (50)}$$

with

$$\mathcal{F}_e = \int \frac{G(\psi)}{2} (\nabla w)^2 dV$$

$$\mathcal{F}_c = \int U(\psi) dV$$

$$\mathcal{F}_{gr} = \int \frac{g_b}{2} (\nabla \psi)^2 dV$$

Antiplane deformation  $\Rightarrow$  Two free variables

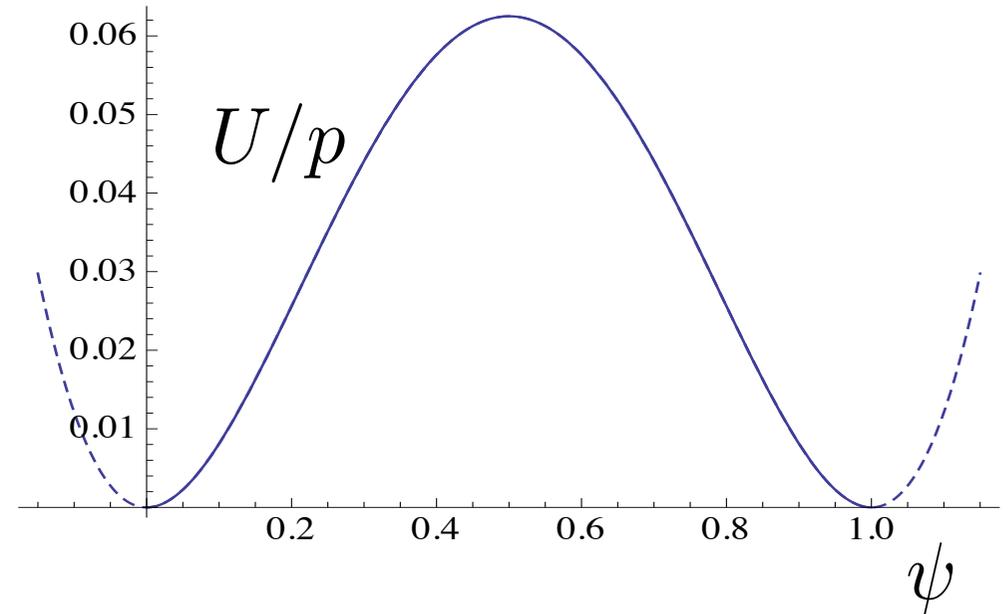
Displacements  $w$  and phase (density)  $\psi$

$$\frac{\partial \psi}{\partial t} = -L_{\psi} \frac{\delta \mathcal{F}}{\delta \psi} \quad , \quad \frac{\partial w}{\partial t} = -L_w \frac{\delta \mathcal{F}}{\delta w}$$

Cahn, Hilliard (58)

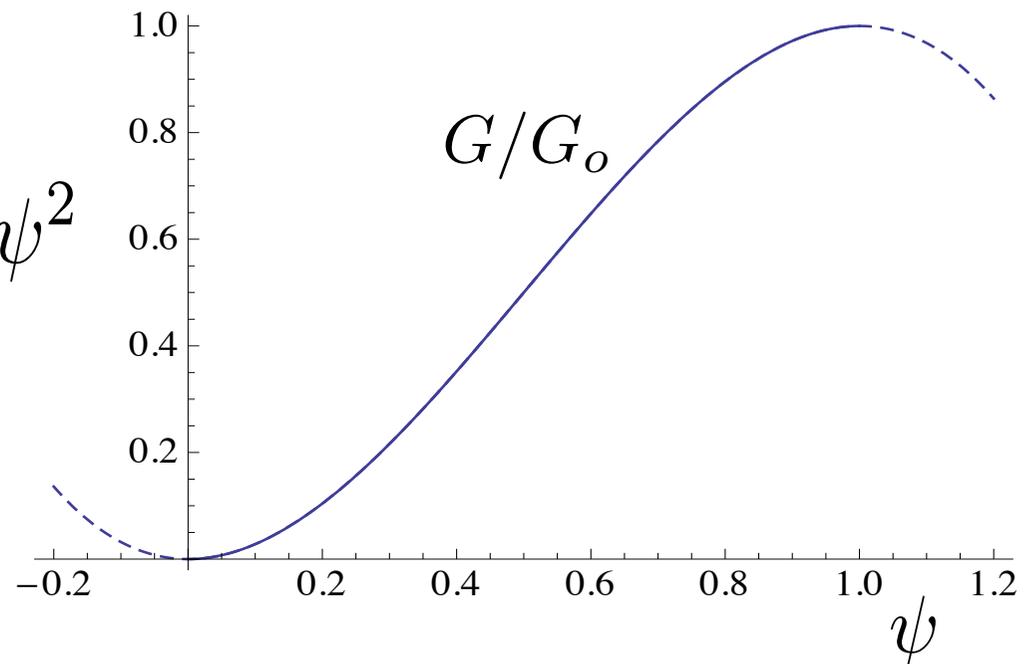
Double-well  
chemical potential

$$U(\psi) = p \psi^2 (1 - \psi)^2$$



Shear modulus

$$G(\psi) = G_o(\psi)(-2\psi + 3)\psi^2$$



Lagrangian expansion:

$$\frac{\delta \mathcal{F}}{\delta \psi} = \frac{\partial \mathcal{F}}{\partial \psi} - \nabla \cdot \frac{\partial \mathcal{F}}{\partial (\nabla \psi)}$$

$$\frac{\delta \mathcal{F}}{\delta w} = \frac{\partial \mathcal{F}}{\partial w} - \nabla \cdot \frac{\partial \mathcal{F}}{\partial (\nabla w)} \quad \Rightarrow$$

$$\frac{\partial \psi}{\partial t} = -L_\psi \left[ \frac{1}{2} G'(\psi) (\nabla w)^2 + p\psi(\psi^2 - 1) - g_b \nabla^2 \psi \right]$$

$$\nabla \cdot [G(\psi) \nabla w] = 0 \quad (\text{equilibrium})$$

(Ginzburg-Landau 1950)

Initial conditions

$$\psi = -\tanh\left(\frac{x_2 + \omega \sin(x_1\pi/\lambda)}{\epsilon}\right)$$

Boundary conditions

$$\text{A: } \frac{\partial\psi}{\partial x_1} = 0 \quad \text{and} \quad w = \frac{\lambda\tau_o}{G_o}$$

$$\text{B: } \psi \rightarrow 1 \quad \text{and} \quad \frac{\partial w}{\partial x_2} \rightarrow 0$$

$$\text{C: } \frac{\partial\psi}{\partial x_1} = 0 \quad \text{and} \quad w = -\frac{\lambda\tau_o}{G_o}$$

$$\text{D: } \psi \rightarrow -1 \quad \text{and} \quad \frac{\partial w}{\partial x_2} \rightarrow 0$$

$$\frac{1}{2}f'' - \beta f' + (\kappa - f)(f^2 - 1) = 0$$

$$\hat{x}_2 = \alpha x_2 + ct$$

$$\alpha = \sqrt{\frac{p}{2g_b}} \quad \kappa = \frac{3G_o(\nabla w)^2}{4p}$$

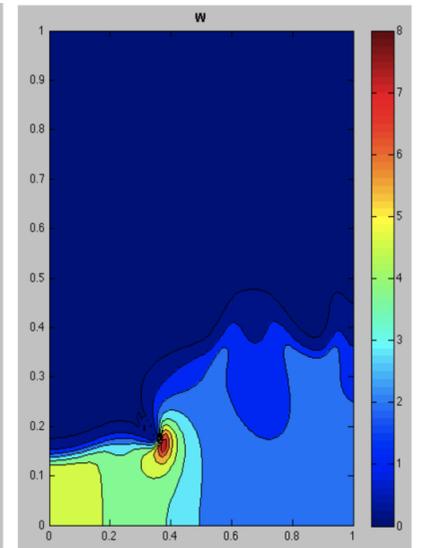
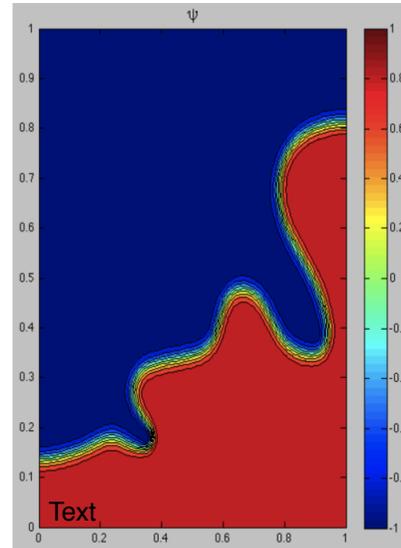
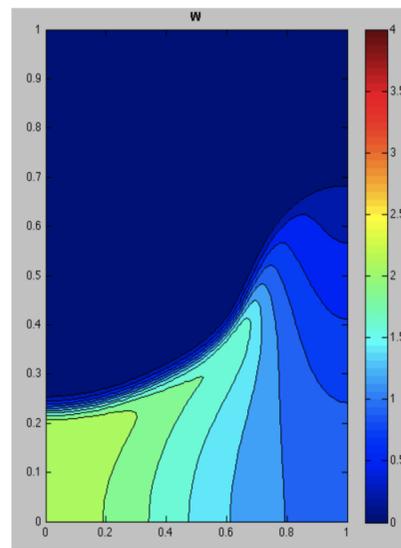
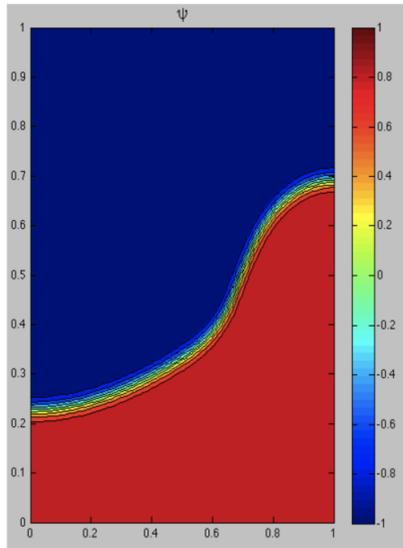
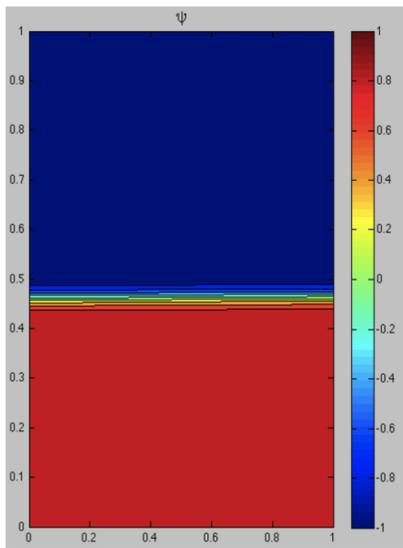
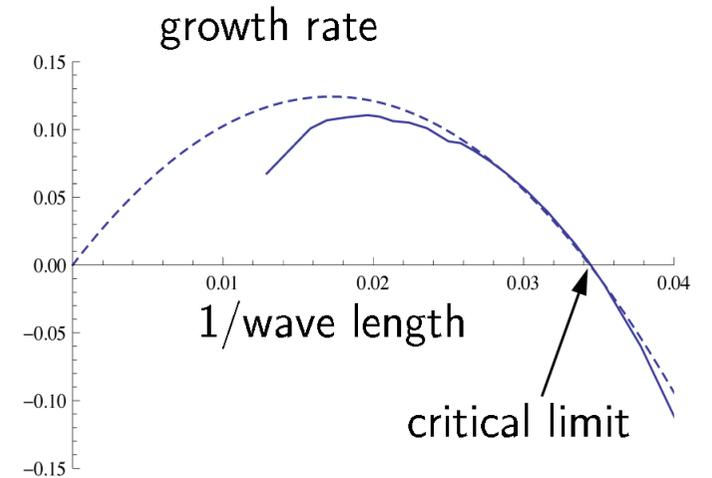
$$\beta = \frac{c}{pL_\psi} \sqrt{\frac{p}{2g_b}}$$

Put  $\beta = \kappa \Rightarrow$

$$c_o = \frac{3}{4}L_\psi G_o(\nabla w)^2 \sqrt{\frac{2g_b}{p}}$$

$$\left(\frac{d}{dx_2} - 2\beta\right)(f' + f^2 - 1) = 0$$

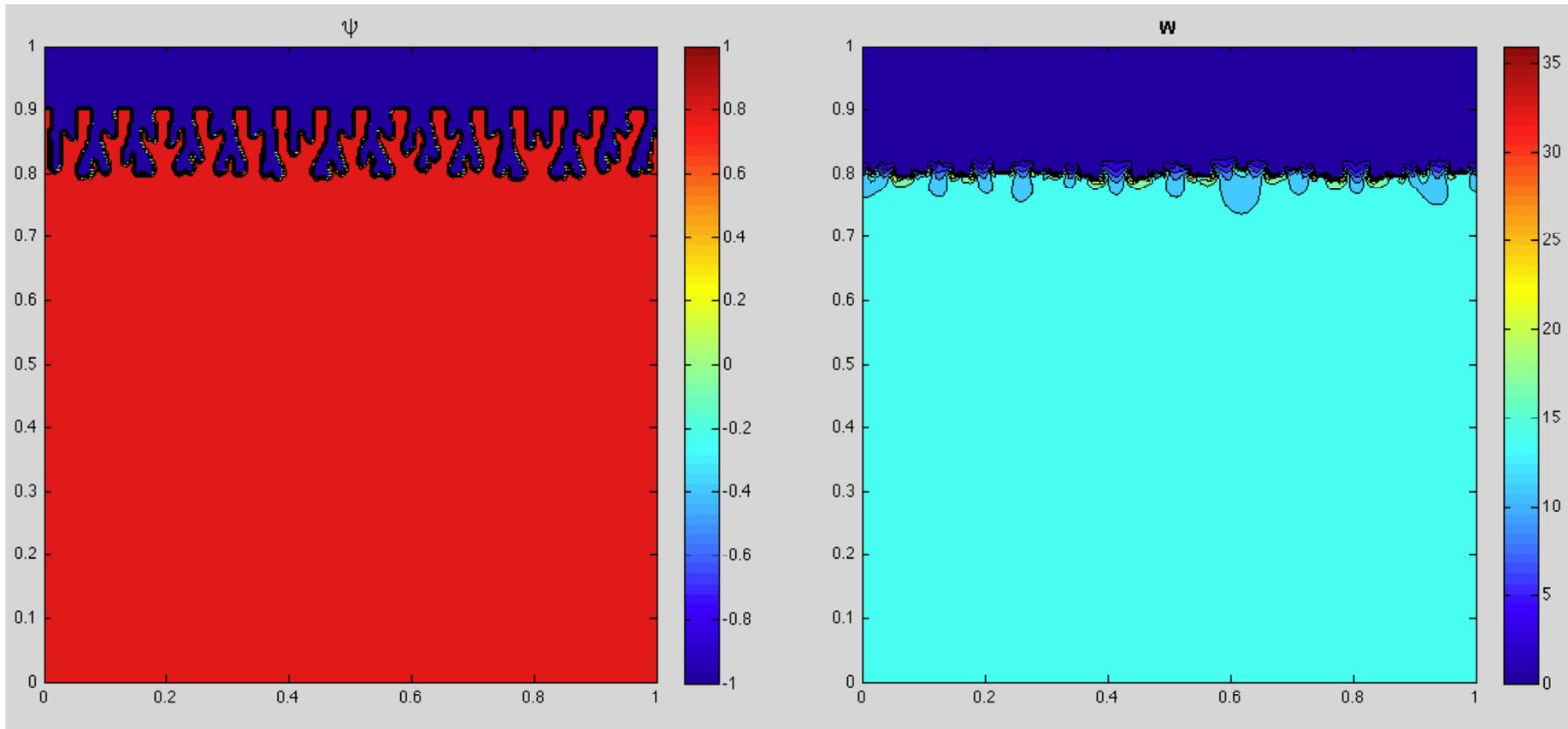
$$\psi = -\tanh\left(\sqrt{\frac{p}{2g_b}}x_2 + \frac{3}{4}L_\psi G_o(\nabla w)^2\sqrt{\frac{2g_b}{p}}t\right)$$



Red is remaining material

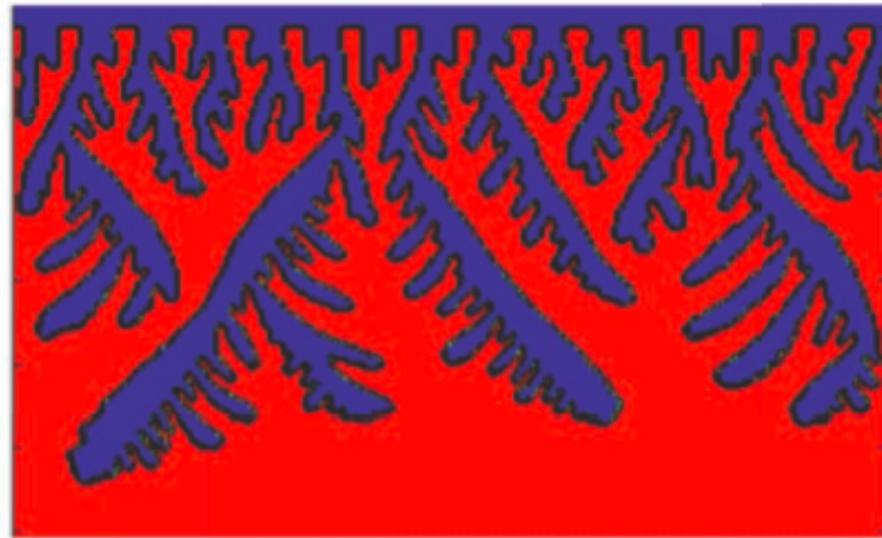
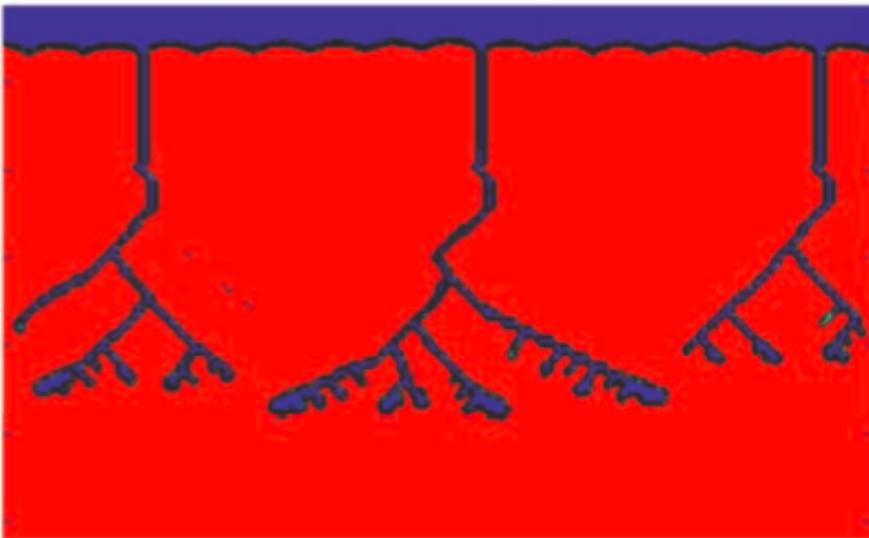
Effective Stress

Effective Stress



Red is remaining material

Effective stress



Without general corrosion

with general corrosion

# Summary

Phase field modelling simplifies stress corrosion analyses

Time dependent solution to Ginzburg-Landau eq. obtained

Crack initiation growth and branching in one simulation