Fracture as a Moving Boundary Problem

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Fracture as a Moving Boundary Problem

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ECF19, Kazan 2012

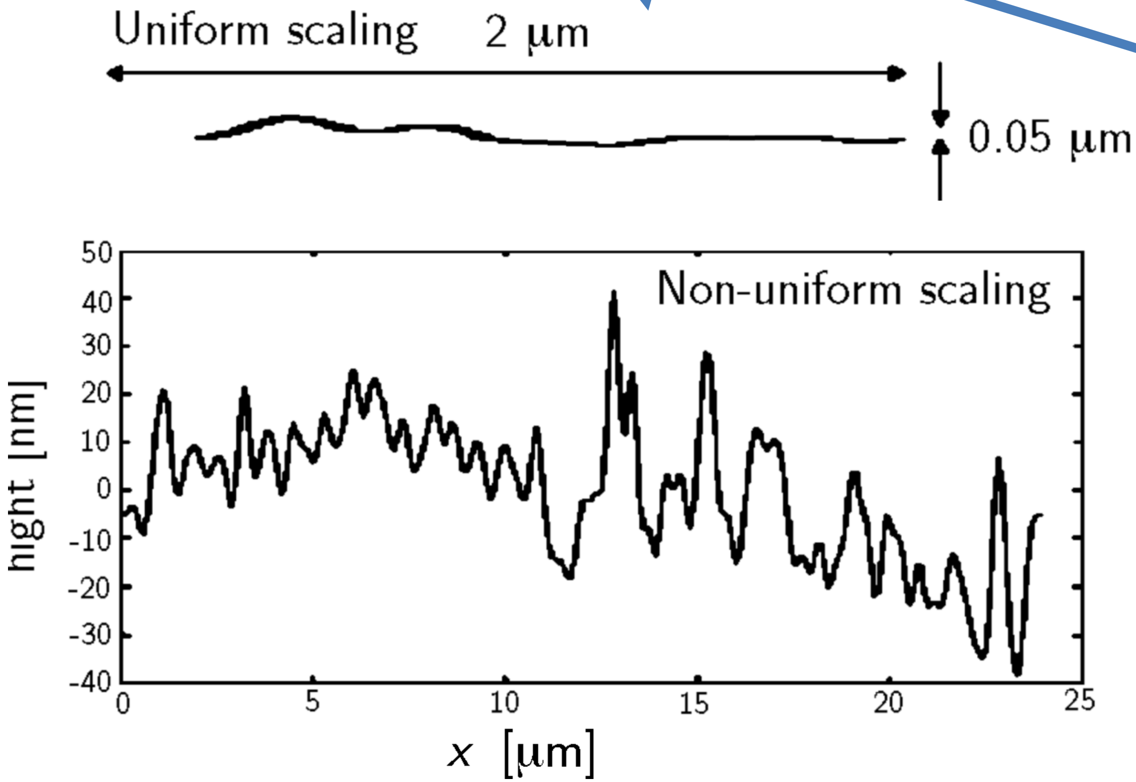
Corroding environment leads to:

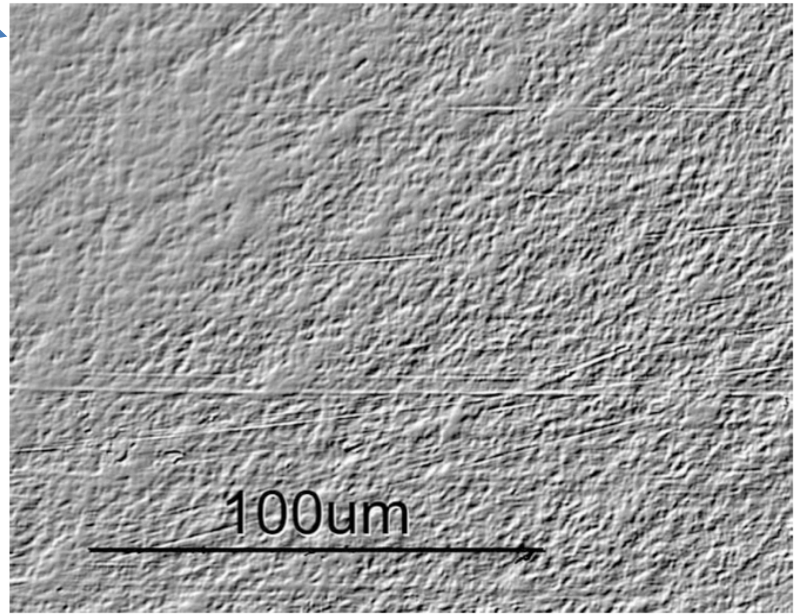
1. Continuous loss of mass
2. Pitting

... and with mechanical stress present

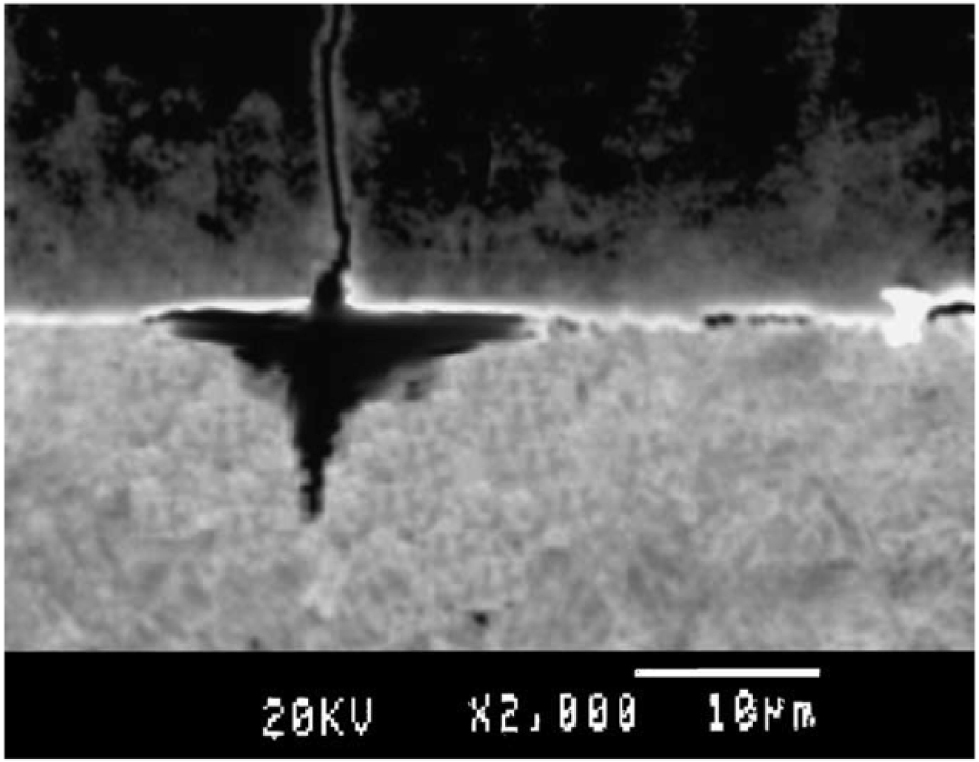
3. Surface roughening

(from Kung-Suk 2000)

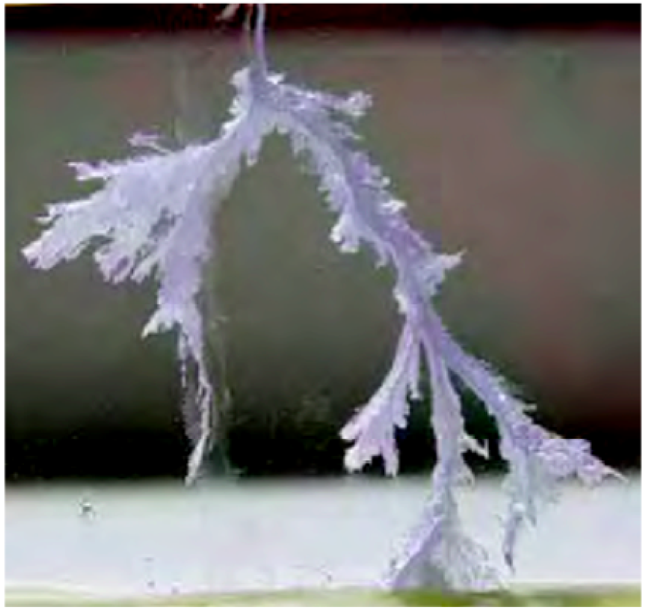




4. Evolving pits
5. Formation of cracks
6. Crack growth
7. Crack branching

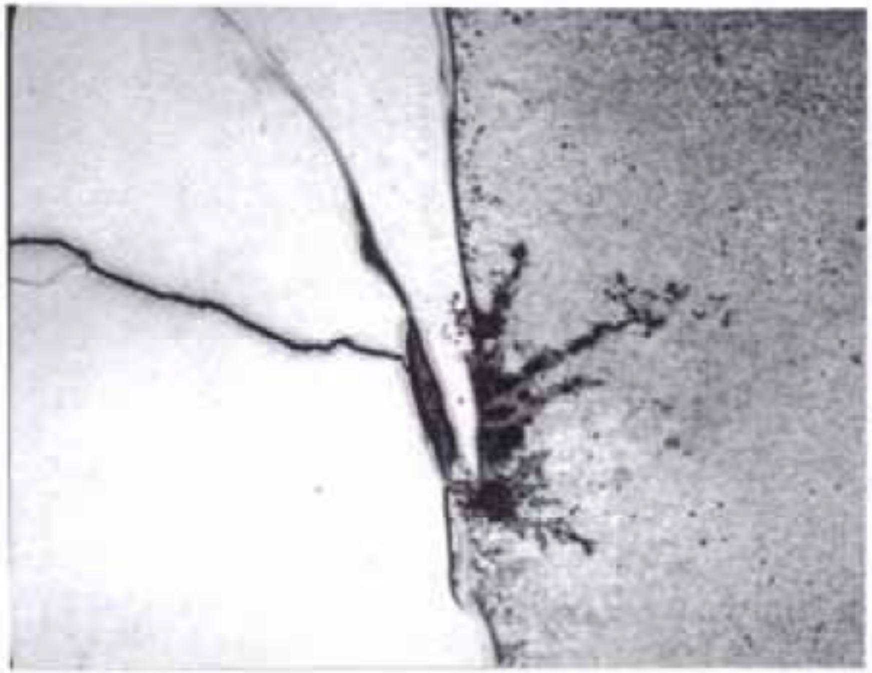


Cr/zone six charge related of land and groove substrate erosion through a micro-crack at the 12:00 bore origin. (Sopok *et al.* 2005)

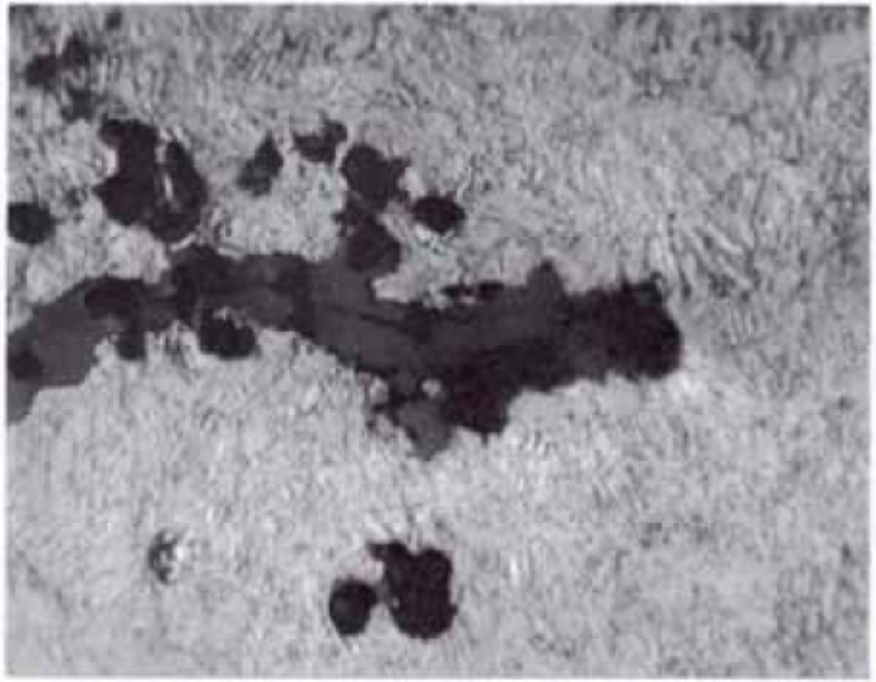


Growing crack in a polycarbonate exposed to acetone (Hejman 2011)

Corrosion Crack crossing a bi-material interface



Corrosion crack penetrating a bimaterial interface between austenitic and pressure vessel steel of type SA533C11.



The tip of one of the crack branches. Crack length 7 mm, notch width 10 μm. *Reproduced with permission from Vattenfall AB.*

Evolving Surface Morphology

Asaro-Tiller (1972), Grinfeld (1986, 1993), Srolovitz (1989), Freund (1995), Kim (2000)

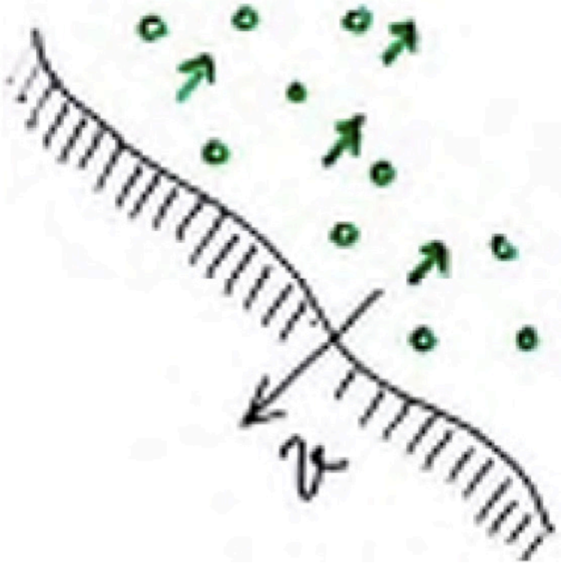
Gibb's free energy

$$\Phi = U_c + U_e$$

where

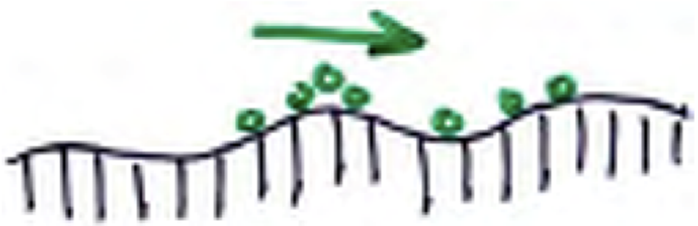
U_c is the free chemical energy and
U_e is the free elastic energy

Evaporation-condensation



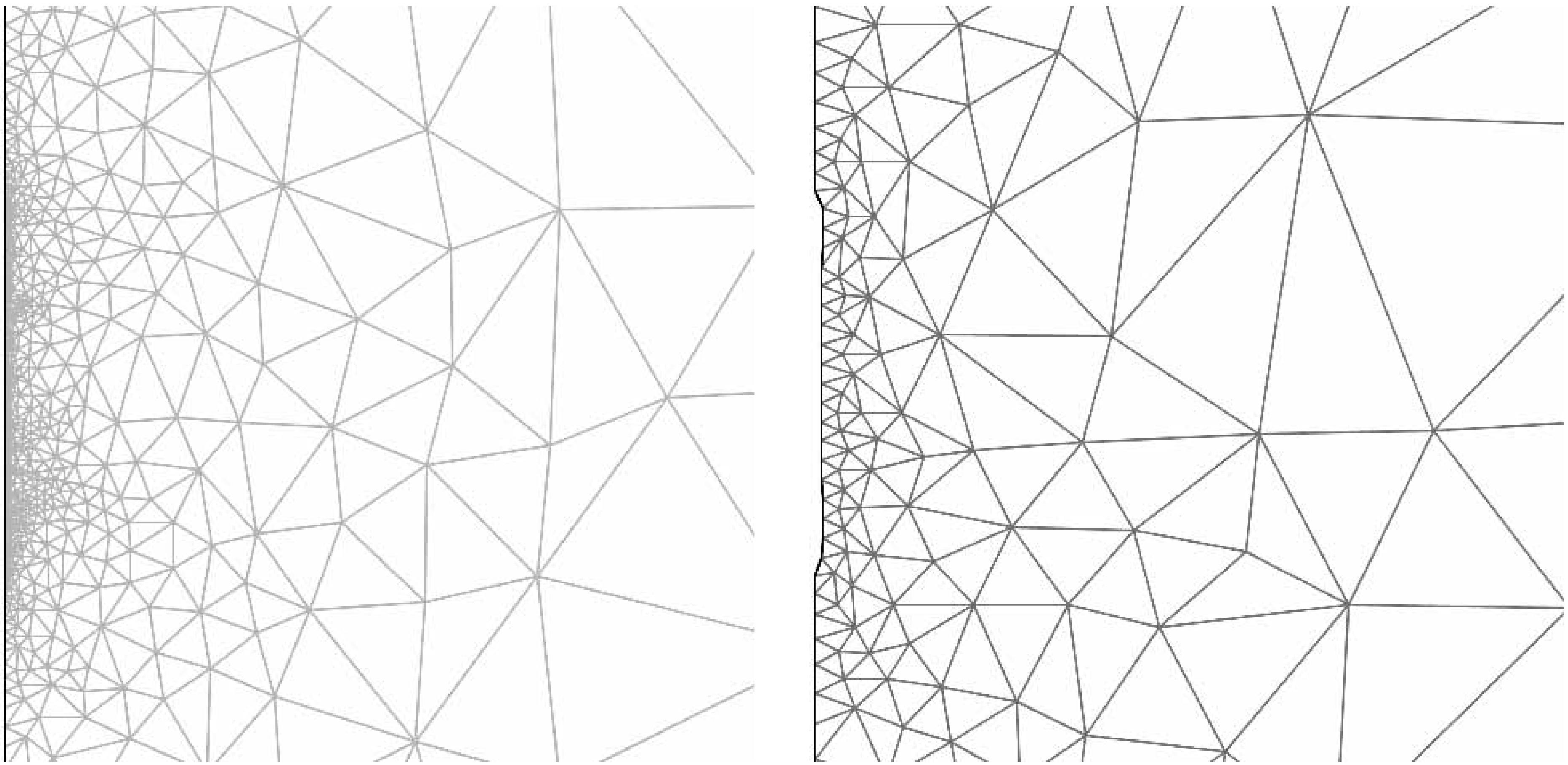
$$\frac{\partial h}{\partial t} = -L_1 \Phi$$

Surface diffusion

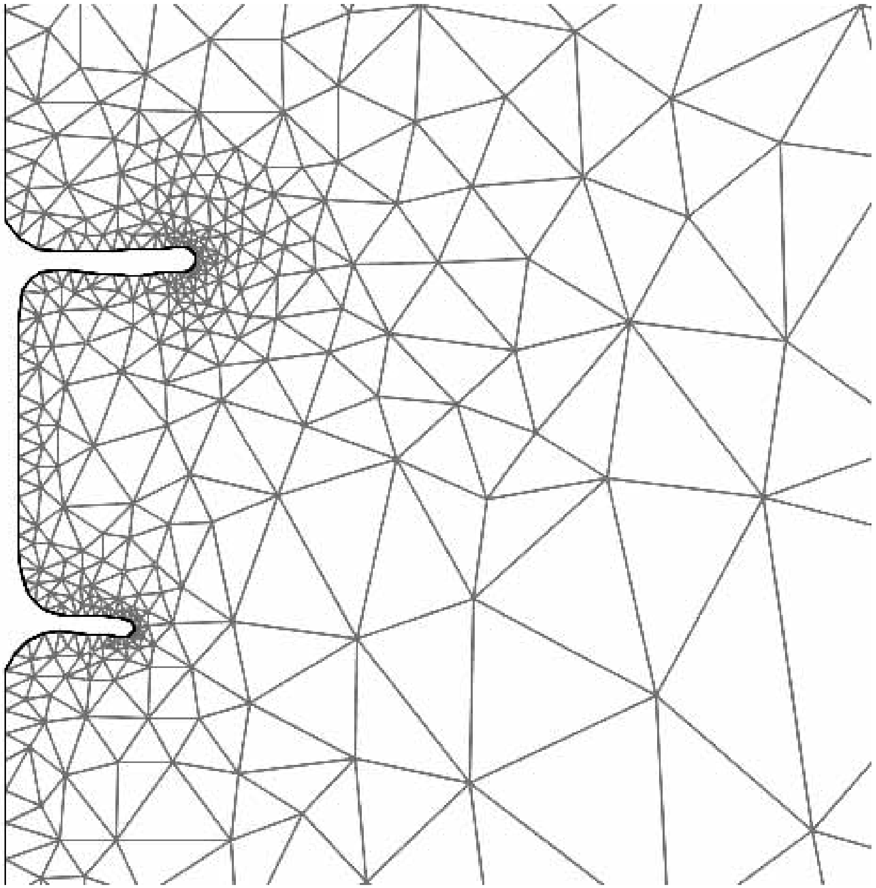
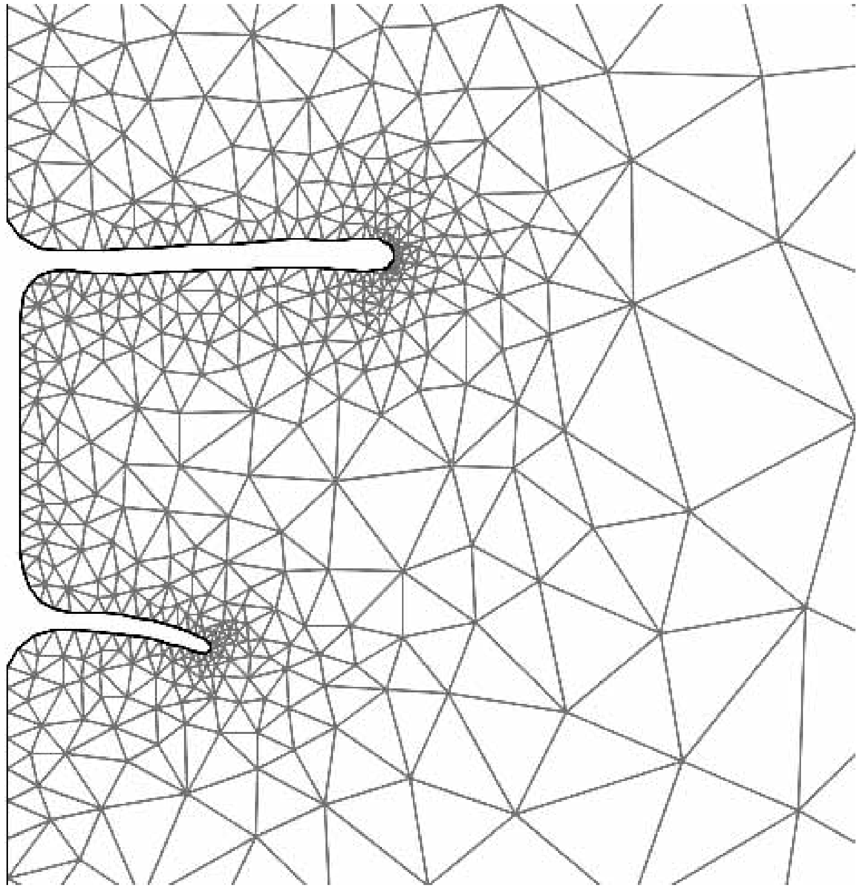


$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2 \Phi}{\partial x^2}$$

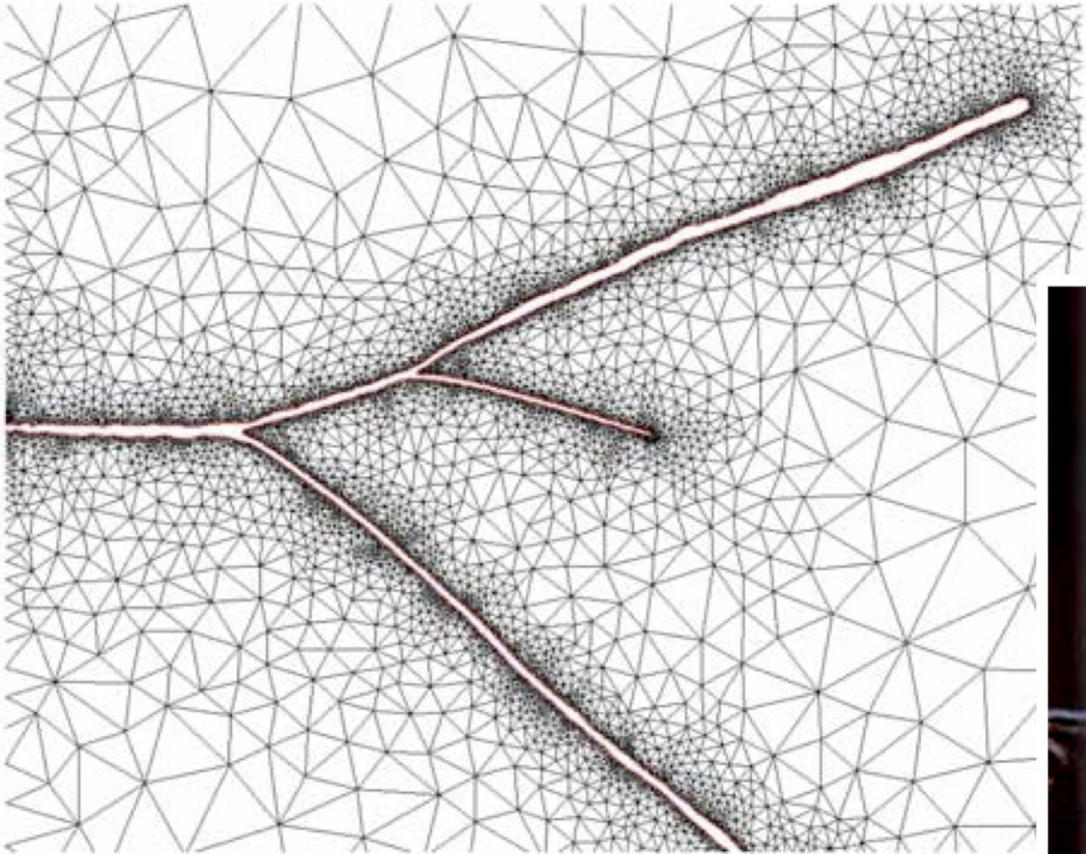
Corroding Surface

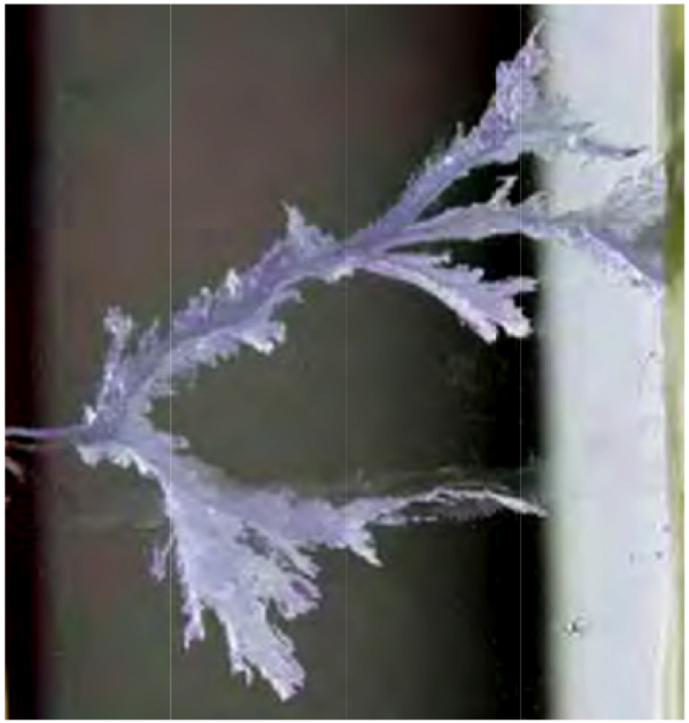


Corroding Surface

Branching





Landau potential:

$$\mathcal{F} = \mathcal{F}_c + \mathcal{F}_e + \mathcal{F}_{gr}$$; Ginzburg, Landau (50)

with

$$\mathcal{F}_e = \int \frac{G(\psi)}{2} (\nabla w)^2 \mathrm{d}V$$

$$\mathcal{F}_c = \int U(\psi) \, \mathrm{d}V$$

$$\mathcal{F}_{gr} = \int \frac{g_b}{2} (\nabla \psi)^2 \mathrm{d}V$$

Antiplane deformation => Two free variables

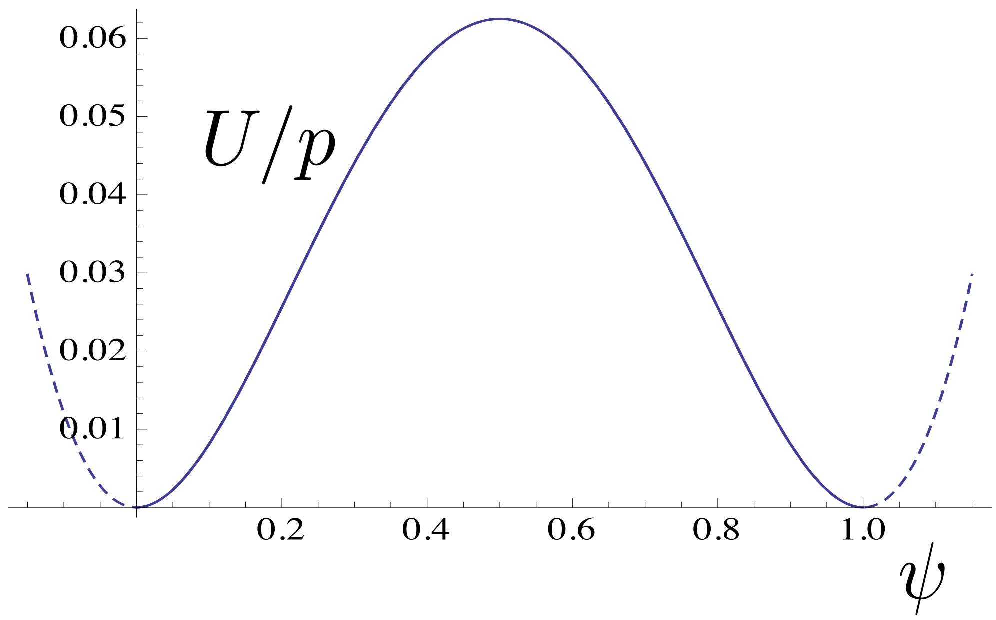
Displacements w and phase (density) ψ

$$\frac{\partial \psi}{\partial t} = -L_{\psi} \frac{\delta \mathcal{F}}{\delta \psi} \quad , \qquad \frac{\partial w}{\partial t} = -L_{w} \frac{\delta \mathcal{F}}{\delta w}$$

Cahn, Hilliard (58)

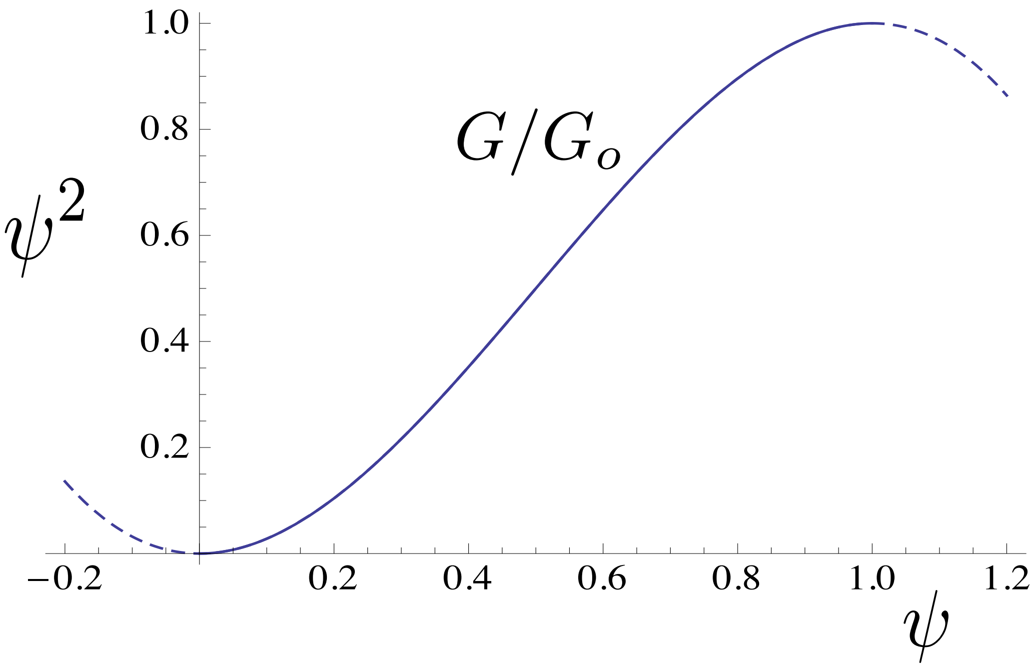
Double-well chemical potential

$$U(\psi) = p\,\psi^2(1-\psi)^2$$



Shear modulus

$$G(\psi) = G_o(\psi)(-2\psi + 3)\psi^2$$



Lagranian expansion:

$$\frac{\delta \mathcal{F}}{\delta \psi} = \frac{\partial \mathcal{F}}{\partial \psi} - \nabla \cdot \frac{\partial \mathcal{F}}{\partial (\nabla \psi)}$$

$$\frac{\delta \mathcal{F}}{\delta w} = \frac{\partial \mathcal{F}}{\partial w} - \nabla \cdot \frac{\partial \mathcal{F}}{\partial (\nabla w)} \qquad => $$

$$\frac{\partial \psi}{\partial t} = -L_\psi [\frac{1}{2} G'(\psi)(\nabla w)^2 + p\psi(\psi^2 - 1) - g_b \nabla^2 \psi]$$

$$\nabla \cdot [G(\psi) \nabla w] = 0 \quad \text{(equilibrium)}$$

(Ginzburg-Landau 1950)

Initial conditions

$$\psi = -\tanh(\frac{x_2 + \omega \sin(x_1 \pi / \lambda)}{\epsilon})$$

Boundary conditions

A: $\frac{\partial \psi}{\partial x_1} = 0$ and $w = \frac{\lambda \tau_o}{G_o}$

B: $\psi \to 1$ and $\frac{\partial w}{\partial x_2} \to 0$

C: $\frac{\partial \psi}{\partial x_1} = 0$ and $w = -\frac{\lambda \tau_o}{G_o}$

D: $\psi \to -1$ and $\frac{\partial w}{\partial x_2} \to 0$

$$\frac{1}{2} f'' - \beta f' + (\kappa - f)(f^2 - 1) = 0$$

$$\hat{x}_2 = \alpha x_2 + ct$$

$$\alpha = \sqrt{\frac{p}{2 g_b}} \quad \kappa = \frac{3 G_o (\nabla w)^2}{4p}$$

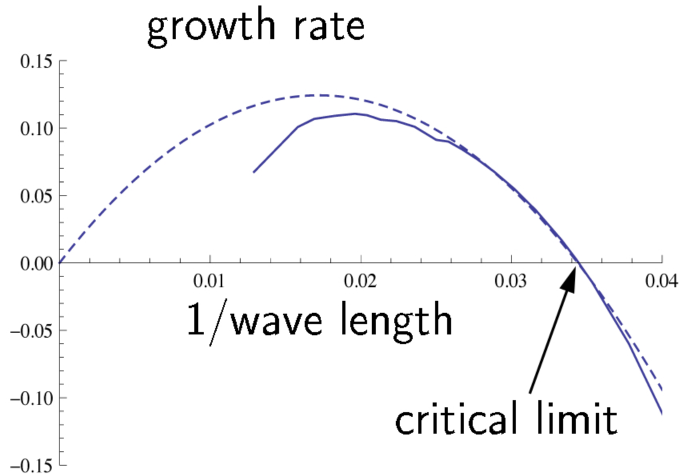
$$\beta = \frac{c}{p L_\psi} \sqrt{\frac{p}{2 g_b}}$$

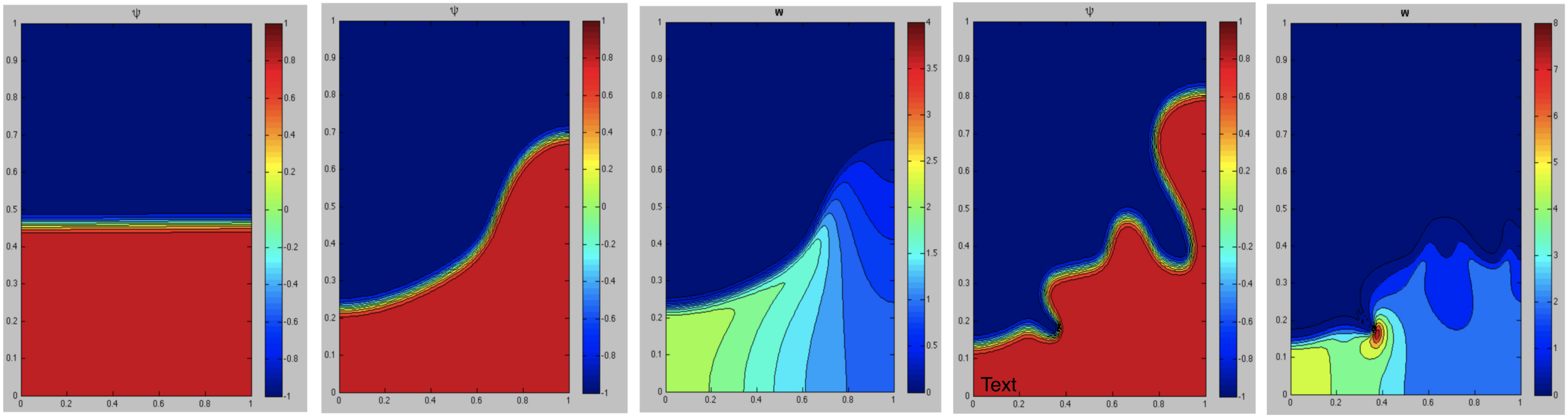
Put $\beta = \kappa$ =>

$$c_o = \frac{3}{4} L_\psi G_o (\nabla w)^2 \sqrt{\frac{2 g_b}{p}}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}x_2} - 2\beta\right)(f' + f^2 - 1) = 0$$

$$\psi = -\tanh\left(\sqrt{\frac{p}{2g_b}}x_2 + \frac{3}{4}L_\psi G_o(\nabla w)^2\sqrt{\frac{2g_b}{p}}t\right)$$

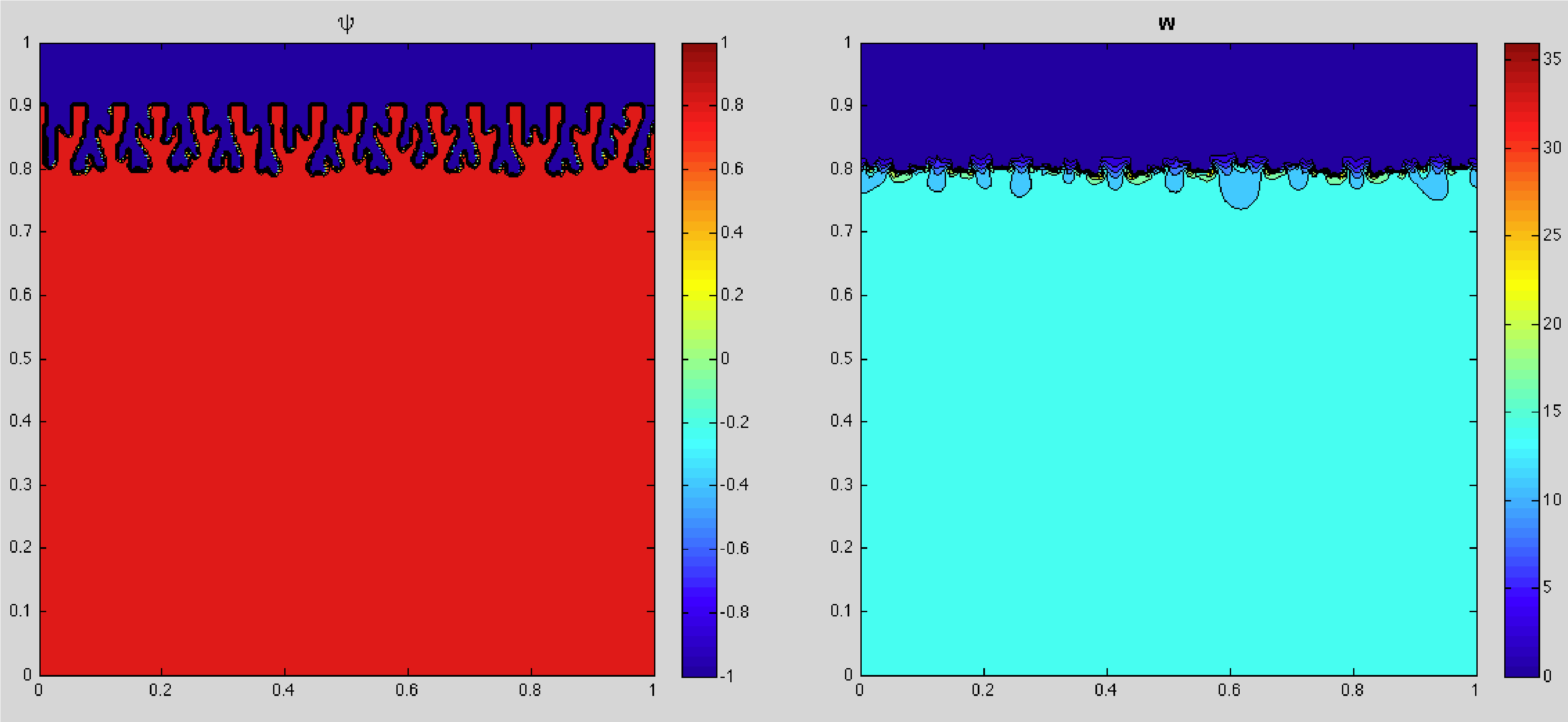




Red is remaining material

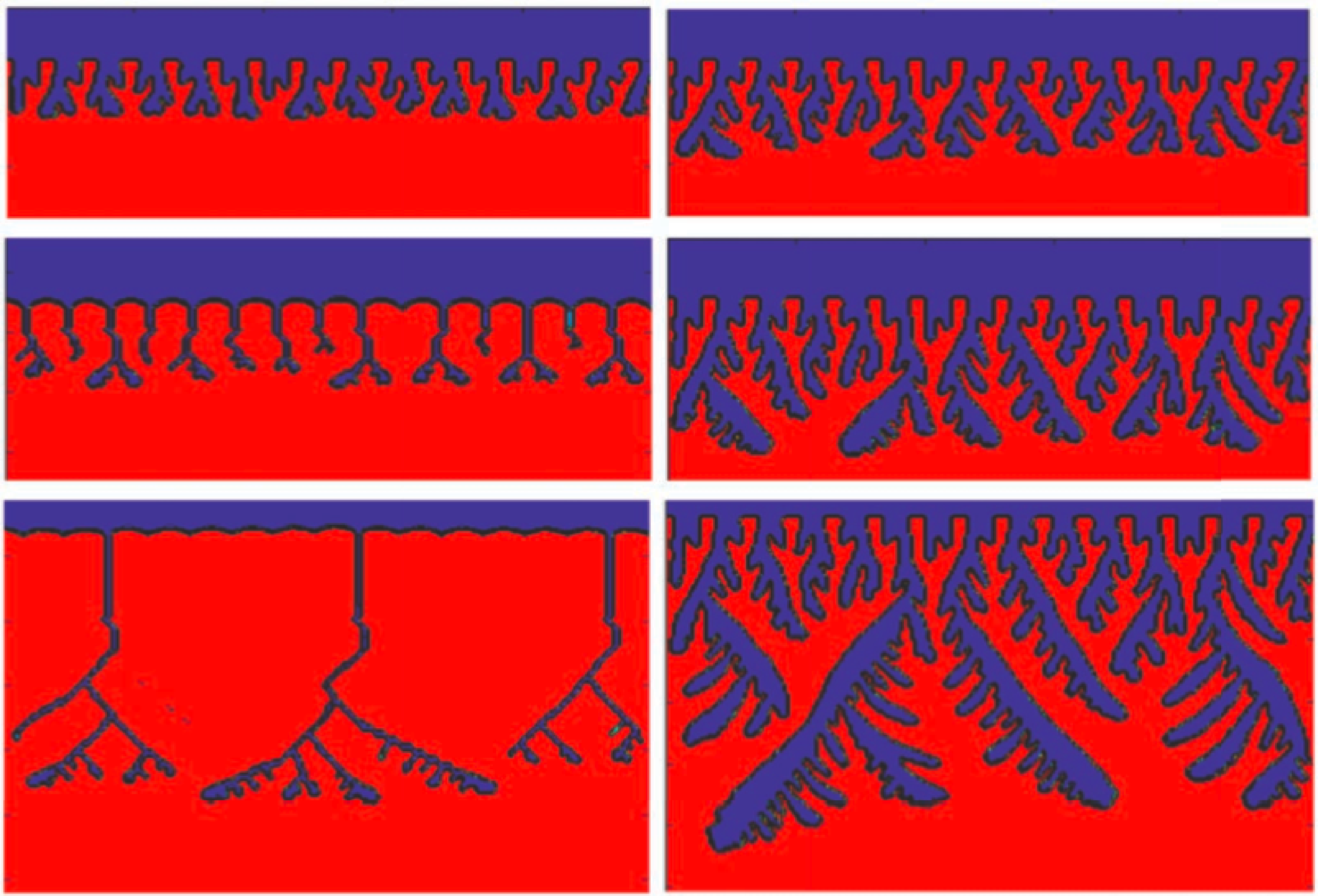
Effective Stress

Effective Stress



Red is remaining material

Effective stress



Without general corrosion with general corrosion

Summary

Phase field modelling simplifies stress corrosion analyses

Time dependent solution to Ginzburg-Landau eq. obtained

Crack initiation growth and branching in one simulation