The coherent electromagnetic field by a particulate media - numerical implementation in a planar geometry

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1 Introduction

Electromagnetic scattering by randomly located particles is frequently encountered in science in terrestrial and atmospheric research, biomedical and life sciences, astrophysics, nanotechnology, just to mention a few topics. The theory is reviewed in [4–7]. The effective wave number approach is most commonly employed to solve the problem, and the effective wave number is obtained by solving a determinant relation. The new method presented in [1] solves the coherent transmitted and reflected fields from a finite or an infinite slab containing randomly located scatterers by the solution of a system of integral equations in the depth variable. In this paper, we present some numerical solutions with this method and also a comparison with the effective wave number method.

2 Theory

A plane wave impinges at normal incidence on the slab, $z \in [0, d]$, containing spherical dielectric particles of radius a. The domain of possible locations of local origins, $[z_1, z_2] = [a, d-a]$, is slightly smaller than the extent of the slab.

The transmitted and reflected coherent parts (ensemble average) of the total electric field on either side of the slab are

$$\boldsymbol{E}^{\pm}(\boldsymbol{r}) = \frac{3f}{2(ka)^3} \sum_n \mathrm{i}^{-l+\tau-1} \boldsymbol{A}_n(\pm\hat{\boldsymbol{z}}) k \int_{z_1}^{z_2} \mathrm{e}^{\pm \mathrm{i}kz'} f_n(z')\, \mathrm{d}z' \mathrm{e}^{\pm \mathrm{i}kz}, \quad \begin{cases} z > d \\ z < 0 \end{cases}$$

The summation over the multi-index $n = \{\tau, \sigma, m, l\}$ is over $\tau = 1, 2$, $\sigma = \mathrm{e}, \mathrm{o}$, $m = 0, 1, 2, \ldots, l$, and $l = 1, 2, 3 \ldots$, and the vector-valued functions $\boldsymbol{A}_n(\hat{\boldsymbol{k}}_\mathrm{i})$ are the vector spherical harmonics, see [3] for more details. The volume fraction of the spheres is denoted f. The coefficients $f_n(z)$ are the solution to a system of linear, one-dimensional integral equations in z, *viz.*,

$$f_n(z) = \mathrm{e}^{\mathrm{i}kz} \sum_{n'} T_{nn'} a_{n'} + k \int_{z_1}^{z_2} \sum_{n'} K_{nn'}(z - z') f_{n'}(z')\, \mathrm{d}z', \quad z \in [z_1, z_2]$$

The kernel entries, $K_{nn'}(z)$, can be computed analytically for the hole correction in terms of a series of spherical waves [2]. The particles are completely characterized by the transition matrix $T_{nn'}$, which for a spherical particle is diagonal in its (pairwise) indices. The expansion coefficients of the plane wave in terms of regular spherical vector waves are denoted a_n, see [3].

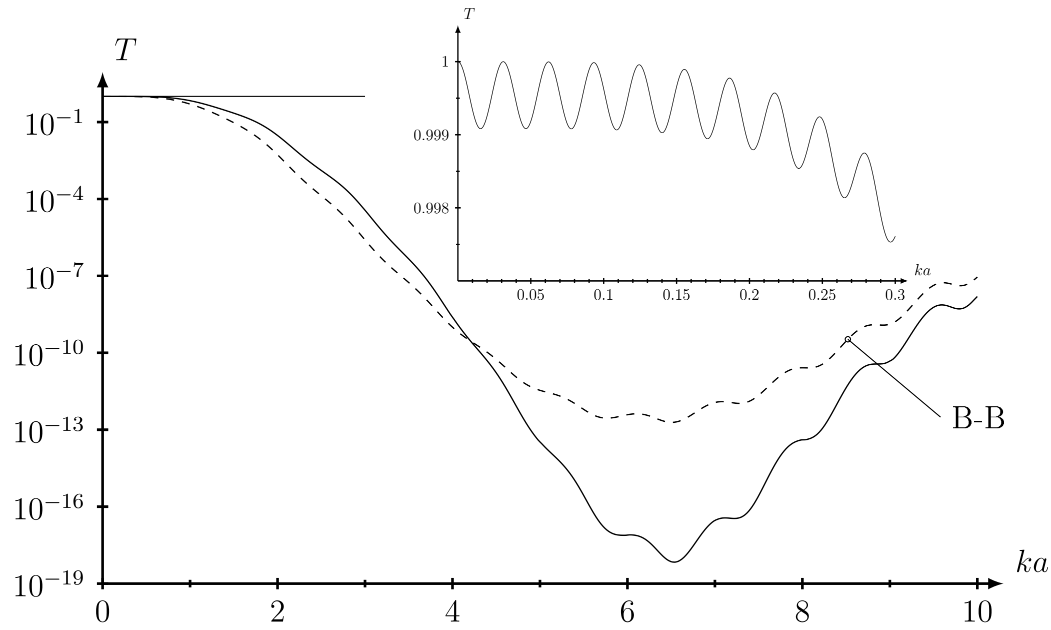

Figure 1: The transmissivity T (coherent part) as a function of the electrical size ka for a slab thickness of $d = 100a$ and constant volume fraction $f = 0.1$. The dashed line is the result obtained by the Bouguer-Beer law (B-B) computed with a slab thickness of $98a$. The insert shows the fine ripple that occurs at low frequencies.

3 Numerical results

In Figure 1, we compare the transmissivity as a function of ka with the transmissivity computed with Bouguer-Beer law (B-B) for a slab with thickness $98a$. The slab contains non-magnetic dielectric spheres of radius a and $\epsilon_{\mathrm{r}} = 1.33^2$.

There is a fine ripple in the transmissivity at low frequencies that is non-visible on the scale of the figure and hidden in the line thickness. This is illustrated in the insert in Figure 1. The effect diminishes at higher frequencies. The reason for this ripple is interference effects between the front and trailing end discontinuities in particle densities at $z = a$ and $z = d - a$. The period of the oscillation $\Delta(ka)$ is

$$\Delta(ka) = 2\pi \frac{k}{\operatorname{Re} k_{\mathrm{eff}}} \frac{a}{2D}$$

where k_{eff} is the effective wave number computed from the transmission data. The effective wave number is also compared with the existing technique of computation by the zeros of a determinant relation [5]. In Figure 2, the reflection coefficient in the complex plane is compared with the reflection coefficient for a homogeneous slab. Both the amplitude and the phase of the coherent reflection coefficient $r(ka)$ and the reflection coefficient of the homogenized slab agree perfectly at low frequencies. The homogenized slab has its left-hand sided located at $z = a$, and an additional phase of $2ka$ is added to compensate for this offset. This is an additional numerical

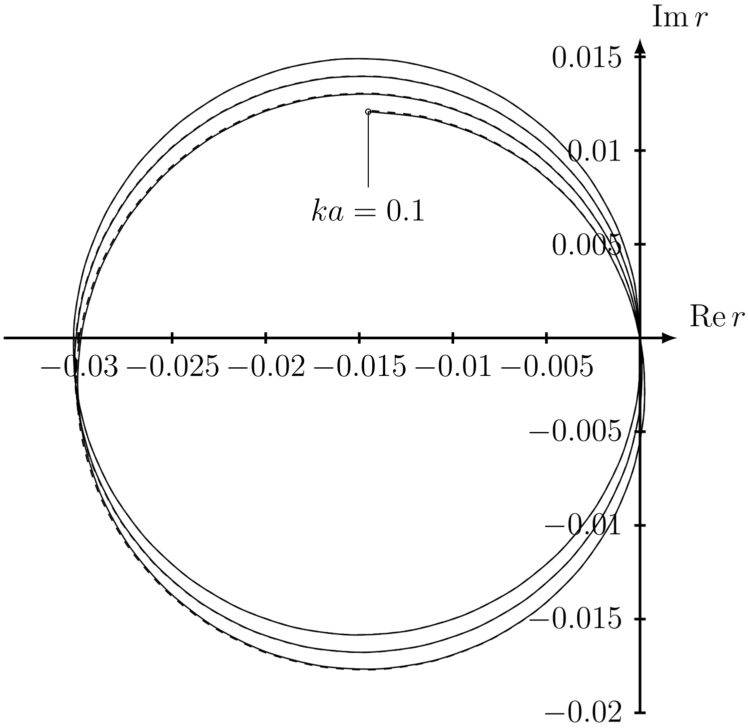

Figure 2: The black curve shows the complex-valued reflection coefficient, $r(ka)$, in the complex plane as a function of the electrical size $ka \in [0, 0.1]$ for a slab of thickness $d = 100a$ and volume fraction $f = 0.1$. The dashed curve shows the reflection coefficient for a homogenized slab with thickness $98a$ and left-hand side location at $z = a$.

verification that the correct location of the homogenized slab is $[a, d - a]$ if the original slab is located at $[0, d]$.

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